Assignment #1

Instructions:
- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}\left(\hat{\beta}_1\right)$ and $\operatorname{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

a) $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 21.03 - (-0.1585)(12.2) = 21.03 + 1.9337 = 22.9637$ #

$\hat{\beta}_2 = \dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} = \dfrac{-174.2}{1098.8} = -0.1585$ #

$\hat{Y} = 22.9637 - 0.1585 X_i$ #

The model show that when $X = 0$, Y would be 22.9637. While if X increase by 1 Y would decrease by 0.1585 and vice versa. #

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b) $r^2 = 1 - \dfrac{\Sigma \hat{u}_i^2}{\Sigma(Y_i - \bar{Y})^2} = 1 - \dfrac{873.14}{882.97} = 1 - 0.9889 = 0.0111$ #

R^2 measure how close the data are to the regression line. In other word it explained how much Y can be explained by the regression line. In this case the $R^2 = 0.0111$ which is very close to 0, show that a lots of data are spread out of the regression line. #

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c) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 (X) = 22.9637 - (0.1585)(5) = 22.9637 - 0.7925 = 22.1712$ #

Initially at $X = 0$, $Y = 22.9637$ but as X goes up by 5 units $\hat{Y}$ goes down by $(0.1585)(5) = (-0.7925)$ units. Which cancel out with 22.9637, thus bring $\hat{Y}$ back to 22.1712 #

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d) $Var(U_i) = \dfrac{\Sigma \hat{u}_i^2}{n-k} = \dfrac{873.14}{30-2} = \dfrac{873.14}{28} = 31.1836$ #

$Var(\hat{\beta}_1) = \dfrac{\Sigma x_i^2}{n \Sigma x_i^2} \sigma^2 = \dfrac{5564(31.1836)}{30(5564)} = \dfrac{173{,}505.5504}{166{,}920} = 1.0395$ #

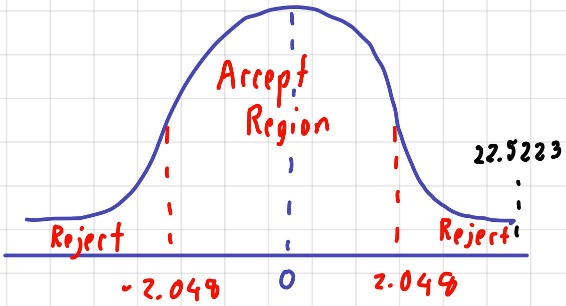
where $\sigma^2 = \dfrac{\Sigma \hat{u}_i^2}{n-k}$

$Var(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{31.1836}{5564} = 0.0056$ #

$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{22.9637}{1.0196} = 22.5223$ #

lower bound : -2.048
Upper bound : 2.049

$-2.048 < 2.048 < 22.5223$
t_{cal} fall in reject region #



$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

Hence, we reject H_0, B_1 are not 0 #

CI Lower : $\hat{\beta}_1 - t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_1} = 22.9637 - 2.088$
$= 20.8757$

CI Upper : $\hat{\beta}_1 + t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_1} = 22.9637 + 2.088$
$= 25.0517$

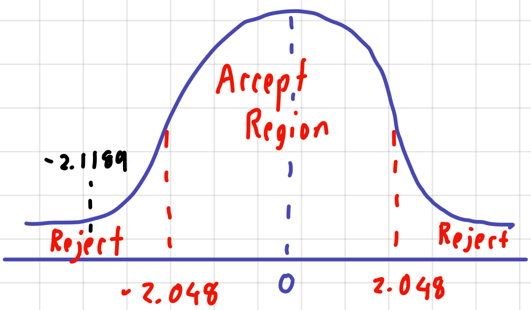
$0 < 20.8757 < 25.0517$

0 doesn't fall in CI interval so we reject $H_0 : \beta_1 = 0$ #

$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{-0.1585}{0.0748} = -2.1189$ #

lower bound : -2.048
Upper bound : 2.048

$-2.1189 < -2.048 < 2.048$
t_{cal} fall in reject region #



$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

Hence, we reject H_0, B_2 are not 0 #

CI Lower : $\hat{\beta}_2 - t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_2} = -0.1585 - 0.1532$
$= -0.3117$

CI Upper : $\hat{\beta}_2 + t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_2} = -0.1585 + 0.1532$
$= -0.0053$

$-0.3117 < -0.0053 < 0$

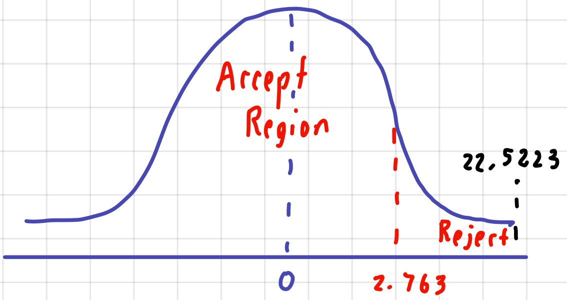
0 doesn't fall in CI interval so we reject $H_0 : \beta_2 = 0$ #

f)

$t_{cal} = \dfrac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \dfrac{22.9637}{1.0196} = 22.5223$ #

$2.763 < 22.5223$

Upper bound : 2.763   t cal fall in reject region #



$H_0 : \beta_1 < 0$

$H_1 : \beta_1 \neq 0$

CI Lower : $\hat{\beta}_1 - t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_1} = 22.9637 - 2.8172$
$= 20.1465$

CI Upper : $\hat{\beta}_1 + t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_1} = 22.9637 + 2.8172$
$= 25.7809$

$(x<0) < 20.1465 < 25.7809$

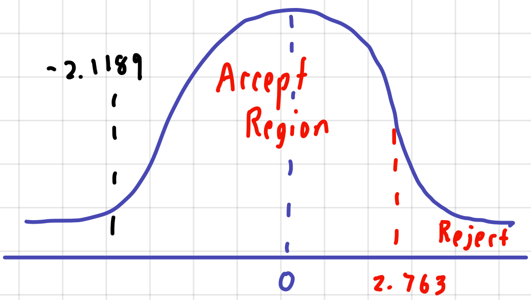
$(x<0)$ doesn't fall in CI interval so we reject $H_0 : \beta_1 < 0$ #

Hence, we reject H_0, B_1 are not less than 0 #

$t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \dfrac{-0.1585}{0.0748} = -2.1189$ #

$-2.1189 < 2.763$

Upper bound : 2.763   t cal fall in accept region #



$H_0 : \beta_2 < 0$

$H_1 : \beta_2 \neq 0$

CI Lower : $\hat{\beta}_2 - t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_2} = -0.1585 - 0.2067$
$= -0.3652$

CI Upper : $\hat{\beta}_2 + t_{\alpha/2} \cdot \hat{\sigma}_{\hat{\beta}_2} = -0.1585 + 0.2067$
$= 0.0482$

$-0.3652 < (x<0) < 0.0482$

$(x<0)$ fall in CI interval so we fail to reject $H_0 : \beta_2 < 0$ #

Hence, we fail to reject H_0, B_2 are less than 0 #

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$ 74.5

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) Yes, since when car age increase the value of the car would dcrease because the model are being obsolete, higher repair cost, harder to find parts for repair. #

b) $\hat{Y}_i = 7{,}836 - 502.4(5) = 7{,}836 - 2{,}512 = 5{,}324$

$$Var(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right] = 212{,}877 \left[\frac{1}{11} + \frac{(5-7.45)^2}{78.73} \right] = 35{,}582.5345$$

$$\sigma_{\hat{Y}_0} = \sqrt{Var(\hat{Y}_0)} = \sqrt{35{,}582.5345} = 188.6333$$

95% CI for $E(Y|X_0 = 5)$

upper: $\hat{Y}_0 + (t_{\alpha/2} \cdot \sigma_{\hat{Y}_0}) = 5{,}324 + (2.262 \cdot 188.6333) = 5{,}750.6885$

lower: $\hat{Y}_0 - (t_{\alpha/2} \cdot \sigma_{\hat{Y}_0}) = 5{,}324 - (2.262 \cdot 188.6333) = 4{,}897.3115$

We are sure that 95% of the time value will be between 4,897.3115 - 5,750.6885. #

c) SRF when X(10) : $\hat{Y} = \underset{(52)}{7836} - \underset{(4118)}{5024}(X)$

d) $\frac{d\hat{Y}}{dX} = -5024$; when $X = 10$, $\hat{Y} = 2{,}812$

$$\frac{d\hat{Y}}{dX} \cdot \frac{X}{Y} = -5024 \frac{X}{Y} = -5024 \frac{10}{2{,}812} = -1.7866$$