

Question 1 (20 points)

(a) (5 points) What is the LIBOR rate and how is it determined? Are there any concerns with using it as a reference rate?

The London Interbank offered rate is the rate at which large banks in London lend money among themselves

It is obtained based on a survey of estimated borrowing rates of selected banks

Banks have been charged for collusion and manipulating submissions ie they understate the rates they think they can borrow to look stronger and therefore there are some concerns in using it as a reference rate.

(b) (5 points) What are Treasury Inflation Protected Securities (TIPs)? Why might the difference between the TIPs and T-bill yield not accurately reflect market inflation expectations?

TIPs pay a real rate of a return since its principal is adjusted for inflation. That is, the rate of return is higher when inflation is higher and vice versa. According to the Fisher relation, the difference between the yield of TIPs and the T-bill of same maturity should reflect market expectations but in reality it may differ since T-bills and TIPs have different characteristics. For example, TIPs are more illiquid than T-bills so the difference between their yields may reflect the fact that investors demand a higher premium for holding TIPs as an illiquid asset.

c) (5 points) Draw the empirical distribution of stock returns in comparison to a normal distribution and describe its properties. If you only care about downside risk, should you use the standard deviation as a measure of risk? If not, describe an alternative measure of risk.

Stock returns are leptokurtic and are left-skewed. Therefore, using standard deviation as a measure of risk especially if you care only about downside risk may not be an accurate measure of risk. Measures such as VaR and expected shortfall can be used instead to assess a portfolio's exposure to these adverse events.

(d) (5 points) Historically, there has been a positive relationship between the volatility and average returns for large portfolios, but not for individual stocks. Explain why such is the case.

For large portfolios, there has been a positive relationship between volatility and average returns, but not so for individual stocks. This is because for large portfolios independent risks are averaged out and diversified away. For this reason, there is a clear cut relationship between the remaining systematic risk and returns of the portfolio.

Question 2 (40 points)

Your employer offers two funds for your pension plan, a money market fund and an S&P 500 index fund. The money market fund holds 3-month Treasury bills, which currently offer a 3% safe return per year. The S&P 500 index fund offers an expected return of 10% per year with a standard deviation of 20%.

(a) (5 points) Consider portfolio A, which according to your level of risk aversion, requires an expected return of 8% per year. What should be the composition of Treasury bills and the S&P 500 index fund in this portfolio? What is the standard deviation of its returns?

Linearity of expected returns require

$$w * 10\% + (1-w) * 3\% = 8\%$$

$$0.07w + 0.03 = 0.08$$

$$w = 0.714$$

meaning 71.4% should be invested in the S&P 500 index and 28.6% in the risk free asset.

The standard deviation of returns is

$$0.714 * 0.20 = 0.1428 \text{ or } 14.28\%$$

(b) (5 points) Solve for your level of risk aversion. What does it mean to be risk averse compared to being risk loving?

to solve for risk aversion level A

$$y^* = E(r_p) - r_f = \frac{0.1 - 0.03}{A \cdot 0.2^2} = 0.714$$

$$\Rightarrow 0.07 = \frac{0.03 - 0.03}{A \cdot 0.2^2} \quad (\text{not very risk averse})$$

$$A = \frac{0.07}{0.02856} = 2.45$$

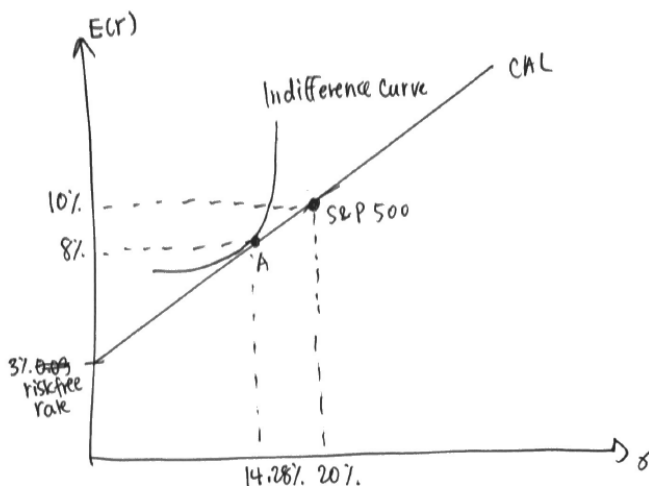
To be risk averse means that your utility declines as there is more risk in the portfolio whereas the utility of a risk loving investor increases as there is more risk.

(c) (5 points) Solve for the Sharpe ratio of Portfolio A. What does it represent? How is it related to the capital allocation line?

The slope of the CAL is the Sharpe ratio which measures the excess return being earned per unit of volatility

$$\text{Sharpe ratio} = \frac{E(r_A) - r_f}{\sigma_A} = \frac{0.08 - 0.03}{0.1428} = 0.35$$

(d) (7 points) Illustrate Portfolio A on a graph where you also plot the risk-free asset, the S&P 500 index fund, the capital allocation line, and your indifference curve. Are you a relatively risk averse investor? Explain.



You are not a relatively risk averse investor since a larger proportion of your wealth is invested in the risky portfolio

(e) (5 points) Now your employer adds an emerging-market fund to the two existing funds. The emerging-market fund offers an expected return of 10% per year, the same as the S&P 500 index fund, but with a standard deviation of 30%, higher than the S&P 500 index fund. Its correlation with the S&P500 index fund is 0. Would you consider including the emerging-market fund as part of your portfolio? Explain.

Even though the emerging markets fund has the same return and higher standard deviation its correlation is < 1 and may tend to achieve lower levels of risk for the portfolio as a whole. This is because we will be able to diversify away some risk. Therefore we should add the emerging fund.

(f) (7 points) Now consider portfolio B, which consists of 80% in the S&P index fund and 20% in the emerging-market fund. Calculate portfolio B's expected return and standard deviation. If you mix portfolio B with the money-market fund to achieve an expected return of 8%, is it better than Portfolio A? Explain.

(g) (6 points) Is portfolio B the global minimum variance portfolio? If not, solve for the global minimum variance portfolio.

$$E(R_B) = 0.8 * 10\% + 0.2 * 10\% = 10\%$$

$$\sigma_B^2 = 0.8^2 * (20\%)^2 + 0.2^2 * (30\%)^2 = 0.0292$$

$$\sigma_B = \sqrt{\sigma_B^2} = 17.09\%$$

The weights will be the same as in part a) ie 28.6% in the risk free asset and 71.4% in portfolio B. The standard deviation of this new portfolio will be $0.7143 * 17.09\% = 12.2\%$ which is lower than for the Portfolio in part a. We are now better off as we can achieve the same return for lower risk.

Question 3 (35 points)

In 1997, the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the capital asset pricing model (CAPM):

(a) (6 points) What is the expected rate of return on the tangency portfolio? What about the expected rate of return on a stock with $\beta = 0$? If a stock has $\beta = 0$ does this mean that it has no risk?

The tangency portfolio is the market portfolio with $\beta = 1$ therefore its expected rate of return is 12%

$\beta = 0$ means no systematic risk and therefore the return is the risk free rate ie 5%

If $\beta = 0$ it does not mean there is no risk, it just means the systematic risk is zero but could still have positive total risk.

(b) (6 points) The stock risk has been evaluated by $\beta = -0.5$. What does it mean for a stock to have negative beta? What type of other assets do you know of that might have negative beta?

This means that the stock moves in opposite direction with the market.
Assets that tend to have negative beta are those that are used for hedging such as gold, Swiss franc, Japanese yen.

(c) (6 points) Write out the equilibrium relation underlying CAPM. Describe in terms of the reward to risk ratio the intuition behind this equilibrium relationship. Why is it considered to be an equilibrium relationship?

$$\frac{E(r_i) - r_f}{\text{Cov}(r_i, r_m)} = \frac{E(r_m) - r_f}{\sigma_m^2}$$

This is the equilibrium condition underlying CAPM saying that the reward to risk ratio for all investments should be equal.

If one was better the other, investors would rearrange portfolios by putting more in the one with better trade off. They will keep rebalancing portfolios until all reward to

the risk ratios are equalized. When no further changes are made, this becomes the equilibrium situation.

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(d) (5 points) According to CAPM, what type of risk is compensated and which type of risk is not? Explain.

According to the CAPM only systemic risk is compensated since idiosyncratic risk can be diversified away. If idiosyncratic risk were to be compensated, there would be an arbitrage opportunity which cannot exist in equilibrium.

(e) (12 points) What is the difference between the Capital Market Line (CML) and the Security Market Line (SML)? Draw graphs to illustrate your answer and explain what the intercepts, slopes and what portfolios on the CML and SML represent.

(see notes for answer)