

Macroeconomics

Lecture 9

Solow Growth Model

- Production function is
- $Y(t) = F\{K(t), A(t).L(t)\}$
- Where
- $A(t)$ is effectiveness of labor.
- Production function is assumed to be constant returns to scale.

Solow Growth Model

- $F(cK, cAL) = cF(K, AL)$, for all $c > 0$
- If $c = (1/AL)$, then
- $F((K/AL), 1) = (1/AL)F(K, AL)$
- $k = K/AL$ is the amount of capital per unit of effective labor
- $y = F(K, AL)/AL$ is output per unit of labor

Solow Growth Model

- Hence, $y = f(k)$ (1)
- Assuming that
- $f(0)=0$, $f'(k)>0$, $f''(k)<0$
- $f'(k)$ is the marginal product of capital.

$$F(K, AL) = AL \cdot f\left(\frac{K}{AL}\right),$$
$$\frac{\partial F(K, AL)}{\partial K} = AL \cdot f'\left(\frac{K}{AL}\right) \left(\frac{1}{AL}\right) = f'(k)$$

Solow Growth Model

- Assuming that

$$\lim_{k \rightarrow 0} f'(k) = \infty, \quad \lim_{k \rightarrow \infty} f'(k) = 0$$

- The initial levels of capital, labor and knowledge are taken as given.
- Labor and knowledge grow at constant rates:

$$\begin{aligned} \dot{L}(t) &= nL(t), & \dot{A}(t) &= gA(t), \\ \text{or } L(t) &= L(0)e^{nt}, & A(t) &= A(0)e^{gt} \end{aligned}$$

Solow Growth Model

- Existing capital depreciates at rate δ .
Thus,

$$\dot{K}(t) = sY(t) - \delta K(t) \quad (2)$$

The dynamics of k

$$\begin{aligned}
 \therefore k(t) &= \frac{K(t)}{A(t)L(t)} \\
 \therefore \dot{k}(t) &= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{[A(t)L(t)]^2} [A(t)\dot{L}(t) + L(t)\dot{A}(t)] \\
 &= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)} \left(\frac{\dot{L}(t)}{L(t)} \right) - \frac{K(t)}{A(t)L(t)} \left(\frac{\dot{A}(t)}{A(t)} \right) \\
 &= \frac{\dot{K}(t)}{A(t)L(t)} - k(t)n - k(t)g \tag{3}
 \end{aligned}$$

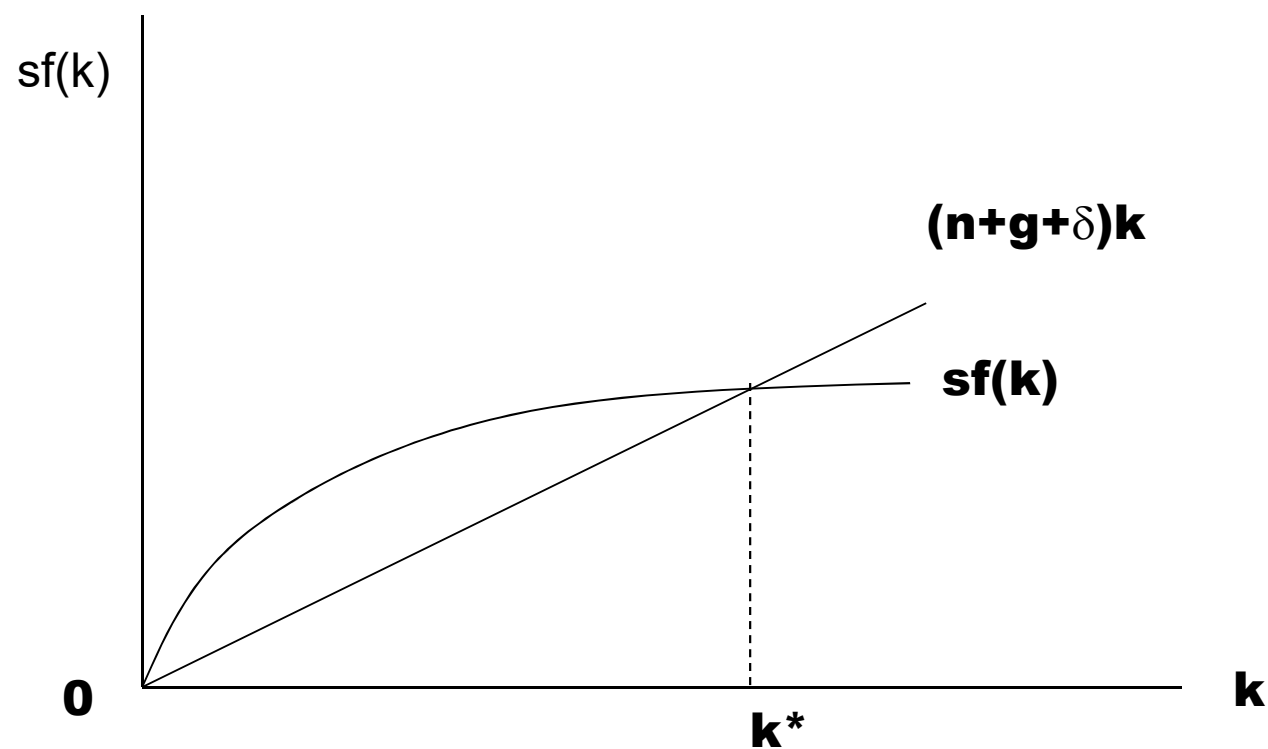
From eq.(2) and (3), we have

$$\begin{aligned}
 \dot{k}(t) &= \frac{sY(t) - \delta K(t)}{A(t)L(t)} - k(t)n - k(t)g \\
 &= sf(k(t)) - (n + g + \delta)k(t) \tag{4}
 \end{aligned}$$

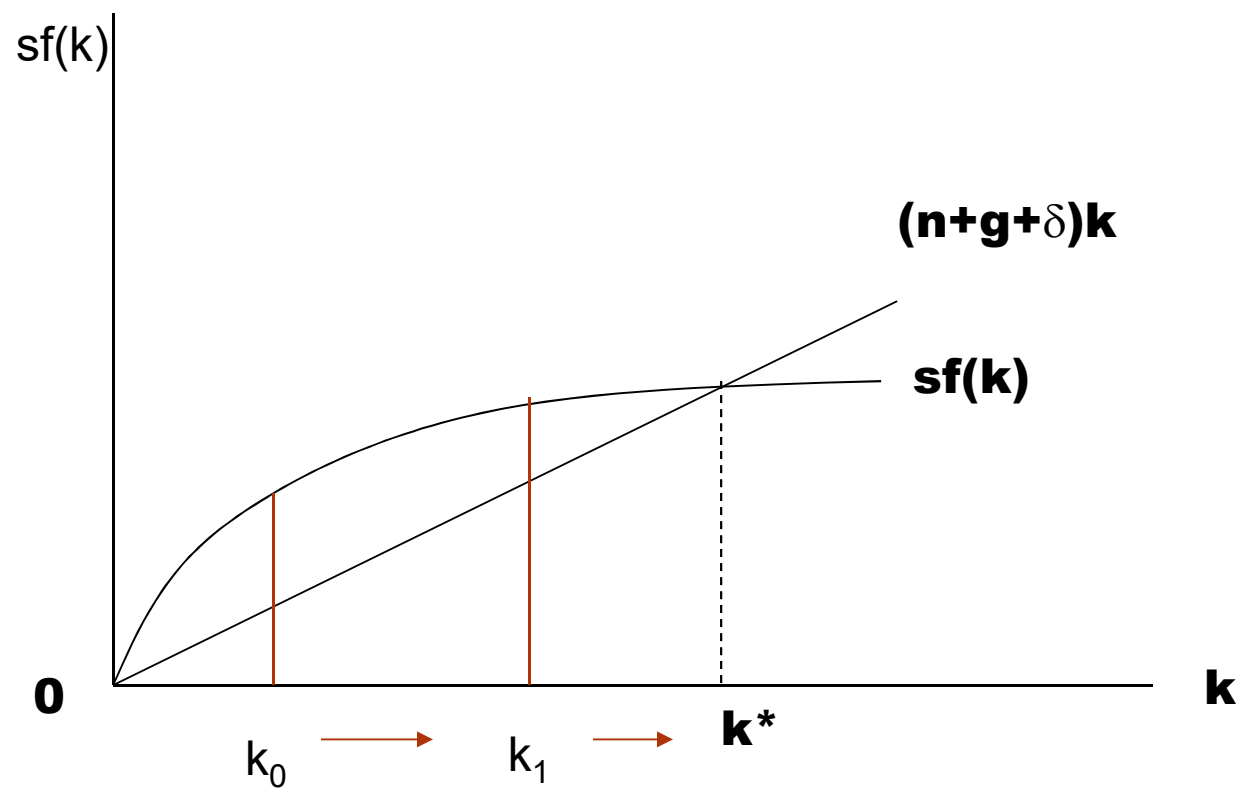
The dynamics of k

- Eq. (4) states that the rate of change of the capital stock per unit of effective labor is the difference between 2 terms. The first term is the actual investment per unit of effective labor. The second term is break-even investment, the amount of investment that must be done to keep k at its existing level.

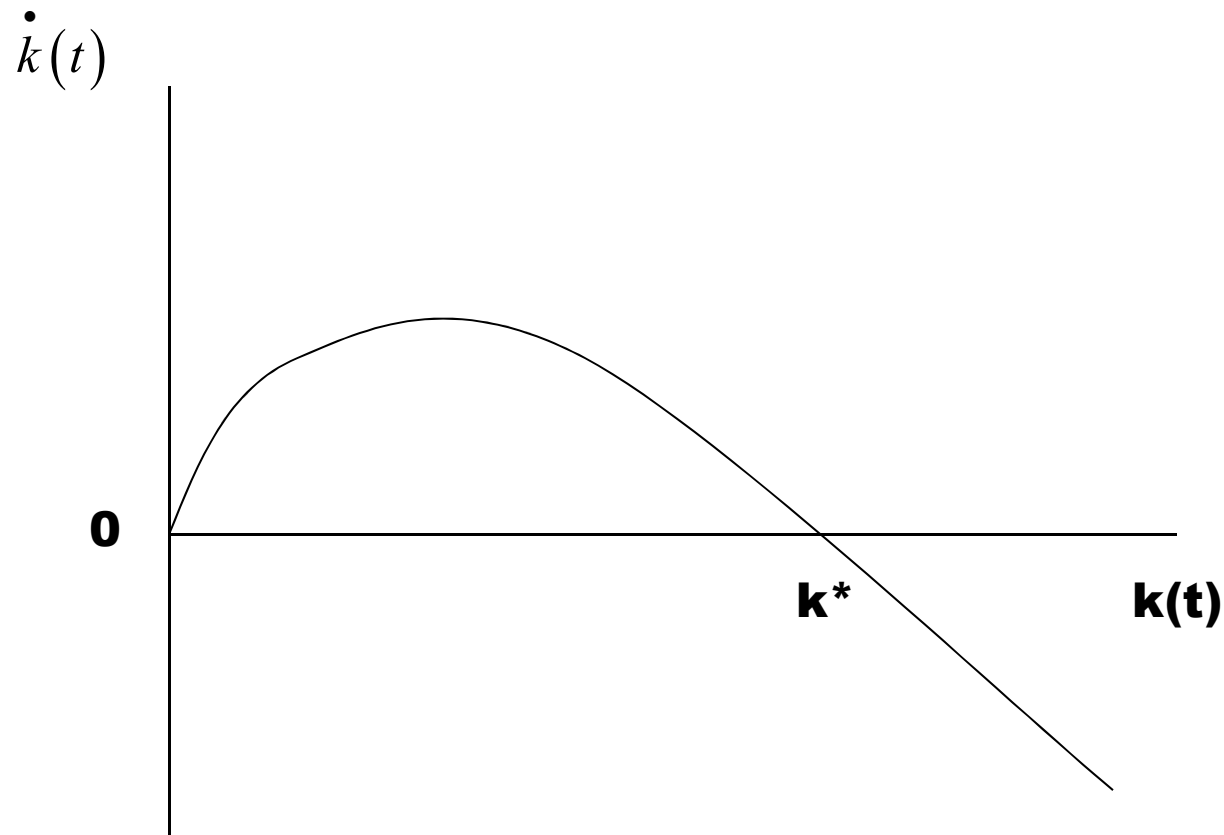
The dynamics of k



The dynamics of k



The dynamics of k



Home work

- At the steady state k^* , please find:
- (1) growth rate of y ,
- (2) growth rate of k ,
- (3) growth rate of c .