EE431 Economics of Financial Markets and Institutions
Problem Set 4: Capital Asset Pricing Model (CAPM)

Solution

1. The percentage of a portfolio's total value invested in a particular asset is called that asset's ....portfolio weight...

2. Imagine you have a portfolio of two risky stocks which turns out to have no diversification. The reason you have no diversification is ....the returns move perfectly (in the same direction) with one another...

3. Company X has a beta of 1.45. The expected risk-free rate of interest is 2.5% and the expected return on the market as a whole is 10%. Using the CAPM, what is ABC's expected return?

   According to CAPM, $ER_i = R_f + \beta(E(R_m) - R_f)$

   $R_i = 2.5\% + 1.45\%(10\% - 2.5\%) = 13.375\%$

4. Suppose CAPM holds. Given the following data, calculate the betas and the expected rates of reuturns of the two securities.

	Expected rate of returns (%)	Standard deviation (%)	Correlation with the market portfolio
Security 1	ER_1?	20	0.9
Security 2	ER_2?	9	0.8
Market portfolio	12	12	$r_{mm} = \rho_{mm}$=? *
Risk free asset	5	0	0

   *Note that I made a mistake in the question sheet. I do appologize about that.

   The security market line (SML) is $ER_i = R_f + \beta_i(ER_m - R_f)$.

   $r_{mm} = 1$

   SML : $ER_i = 5\% + \beta_i(12\% - 5\%) = 5\% + 7\%\beta_i$

   Security 1. $\beta_1 = \dfrac{\sigma_{1m}}{\sigma_m^2} = \dfrac{r_{im}\sigma_1\sigma_m}{\sigma_m^2} = \dfrac{0.9 \times 20\% \times 12\%}{12\% \times 12\%} = 1.5$. $ER_1 = 5\% + 1.5 \times 7\% = 15.5\%$.

   Security 2. $\beta_2 = \dfrac{\sigma_{2m}}{\sigma_m^2} = \dfrac{r_{im}\sigma_2\sigma_m}{\sigma_m^2} = \dfrac{0.8 \times 9\% \times 12\%}{12\% \times 12\%} = 0.6$. $ER_1 = 5\% + 0.6 \times 7\% = 9.2\%$.

5. Consider an economy with just two assets. The details of these are given below.

Stock	Number of shares	Price ($)	Expected return (%)	Standard deviation (%)
A	100	1.5	15	15
B	150	2	12	9

   The correlation coefficient between the returns on the two assets is $\frac{1}{3}$ and there is also a risk free asset. Assume the CAPM model is satisfied.

(a) What is the expected rate of return on the market portfolio?

Total Value of Stock A $= 100 \times \$1.5 = \$150$

Total Value of Stock B $= 150 \times \$2 = \$300$

Total Value of Market $= \$150 + \$300 = \$450$

Weight of stock A in the market portfolio is equal to $w_A = \frac{150}{450} = \frac{1}{3}$.

Weight of stock B in the market portfolio is equal to $w_B = \frac{300}{450} = \frac{2}{3}$.

Hence, the return on the market portfolio is equal to

$$\begin{aligned} ER_m &= w_A ER_A + w_B ER_B, \\ &= \frac{1}{3} \times 15\% + \frac{2}{3} \times 12\%, \\ &= 13\%. \end{aligned}$$

(b) What is the standard deviation of the market portfolio?

$$\begin{aligned} \sigma_m^2 &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B r_{AB} \sigma_A \sigma_B, \\ &= \left(\frac{1}{3}\right)^2 \times (15\%)^2 + \left(\frac{2}{3}\right)^2 \times (9\%)^2 + 2 \times \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(15\%)(9\%) \\ &= 81 \\ \sigma_m &= 9\% \end{aligned}$$

(c) What is the beta of stock A?

By definition, $\beta_A = \frac{r_{A,m}\sigma_A\sigma_m}{\sigma_m^2} = \frac{COV(R_A, R_m)}{\sigma_m^2}$.

Find $COV(R_A, R_m)$.

$$\begin{aligned} COV(R_A, R_m) &= E([(R_A - E(R_A)(R_m - E(R_m))] \\ &= E([(R_A - E(R_A)(w_A R_A + w_B R_B - w_A ER_A - w_B ER_B)] \\ &= E([(R_A - E(R_A)\{(w_A R_A - w_A ER_A) + (w_B R_B - w_B ER_B)\}] \\ &= E\left[w_A (R_A - E(R_A)^2 + w_B (R_A - ER_A)(R_B - ER_B)\right] \\ &= w_A \sigma_A^2 + w_B COV(R_A, R_B) \\ &= w_A \sigma_A^2 + w_B r_{AB} \sigma_A \sigma_B \\ &= \left(\frac{1}{3}\right)(15\%)^2 + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)(15\%)\,(9\%) \\ &= 105 \\ \beta_A &= \frac{COV(R_A, R_m)}{\sigma_m^2} \\ &= \frac{105}{9^2} \\ &= 1.2963 \end{aligned}$$

(d) What is the risk free rate of return?

The risk-free return is derived from the the security market line. The security market line gives

$$\begin{aligned}
ER_A &= R_f + \beta_A(ER_m - R_f), \\
15\% &= R_f + 1.2963(13\% - R_f), \\
15\% &= (1 - 1.2963)R_f + (1.2963 \times 13\%), \\
0.2963R_f &= 16.8519\% - 15\%, \\
0.2963R_f &= 1.8519, \\
R_f &= \frac{1.8519}{0.2963}, \\
&= 6.25\%.
\end{aligned}$$

(e) Construct the capital market line and the security market line.

The capital market line

$$\begin{aligned}
ER_p &= R_f + \left(\frac{ER_m - R_f}{\sigma_m}\right)\sigma_p, \\
&= 6.25\% + \left(\frac{13\% - 6.25\%}{9\%}\right)\sigma_p, \\
&= 6.25\% + 0.75\sigma_p.
\end{aligned}$$

The security market line

$$\begin{aligned}
ER_i &= R_f + \beta_i(ER_m - R_f),, \\
&= 6.25\% + (13\% - 6.25\%)\,\beta_i, \\
&= 6.25\% + 6.75\beta_i.
\end{aligned}$$

6. Let the expected rate of return on the market portfolio M be equal to 0.10. The standard deviation of the market portfolio M is equal to 0.20. The risk-free interest rate is equal to 0.02.

(a) Find the equation for the CML and interpret its meaning.

Solutions.

$$\begin{aligned}
E(R_p) &= R_f + \left(\frac{E(R_M) - R_f}{\sigma_m}\right)\sigma_p \\
&= 0.02 + \left(\frac{0.10 - 0.02}{0.20}\right)\sigma_p \\
&= 0.02 + 0.4\sigma_p
\end{aligned}$$

CML shows the relationship between market required rate of return and risk (measured by standard deviation) for the effiecient portfolios (portfolios consisting of the risk-free asset and the market portfolio). As the level of risk (σ_p) of the efficient portfolio increases, the market requires a higher rate of return in order to hold the portfolio.

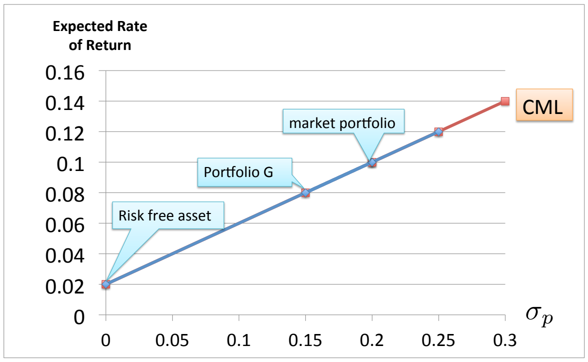
(b) Let G be a portfolio. The expected rate of return on the portfolio G is equal to 0.08 and the variance of portfolio G is equal to 0.0225. Does the portfolio G lie on CML? Graphically illustrate and explain. If the portfolio G lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio G.

Solutions.

On the CML, a portfolio with $\sigma_p = \sqrt{0.0225} = 0.15$ must have the expected rate of return equal to $E(R_p) = 0.02 + 0.4 \times 0.15 = 0.08$.

Portfolio G has 0.08 expected rate of return and 0.0225 variance. Therefore, portfolio G must lie on the CML.

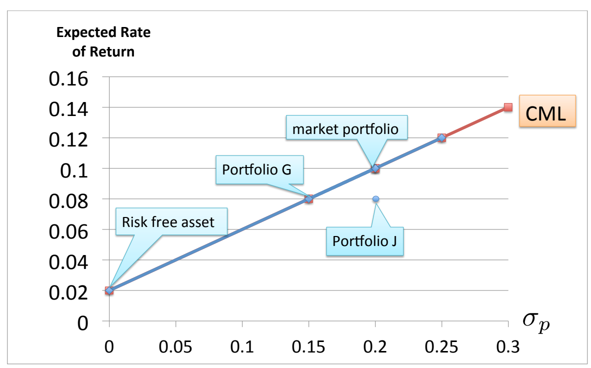
$R_G = aR_f + (1-a)R_m = 0.08$. $0.08 = 0.02a + 0.10 - 0.10a$. $0.08a = 0.02$. Therefore, $a = 0.25$. Portfolio G consists of 0.25 of risk-free asset and 0.75 of the market portfolio.



(c) Let J be a portfolio. The expected rate of return on the portfolio J is equal to 0.08 and the variance of portfolio J is equal to 0.04. Does the portfolio J lie on CML? Graphically illustrate and explain. If the portfolio J lie on the CML, derive the weight of the risk-free asset and the weight of the market portfolio in the portfolio J.

Solutions.

On the CML, a portfolio with $\sigma_p = \sqrt{0.04} = 0.2$ must have the expected rate of return equal to $E(R_p) = 0.02 + 0.4 \times 0.2 = 0.10$. On the CML, the portfolio with $\sigma_p = \sigma_m$ must have the expected rate of return equal to the market portfolio. Portfolio J has the same variance as the market portfolio but the expected rate of return is different. Therefore, portfolio J



does not lie on the CML.

(d) According to James Tobin's investment decision process, which portfolio(s) investors might choose to hold? Describe James Tobin's investment decision process. Indentify the portfolio(s) investors might choose to hold. Explain the reason.

Solutions.

According to James Tobin's investment decision process, each investor will have a utility-maximising portfolio that is a combination of the risk-free asset and a portfolio (or fund) of risky assets that is determined by the line drawn from the risk-free rate of return tangent to the investor's effiecient set of risky assets. If investors have homogenous beliefs, the they all have the same linear efficient set called the Capital Market Line (CML). Therefore, they will try to hold some combination of the risk-free aseet, R_f and the tangency porfolio M. At

equilibrium, all assets must be held. All prices will be adjusted so that the demand is equal to supply. The tangency portfolio must be the market portfolio, M. [If V_i is the market value of the ith asset, then the percentage of wealth in each asset is equal to the ratio of the market value of the asset to the market value of all assets. $w_i = \frac{V_i}{\sum_{i=1}^{N} V_i}$.]
Investors will hold portfolios along the CML. Therefore, investors might hold portfolio M and G. No investor will choose to hold portfolio J.

(e) According to James Tobin's investment decision process and the CML in (a), what are the market required rates of return on the portfolio M, G and J? Is the given information sufficient to determine the market required rate of returns of the three portfolios?
Solutions.
The given information is sufficient to determine the market required rate of return for portfolio M and G. Portfolio M and G lie on the CML. They are efficient. Their market required rate of returns are determined by the relationship between the market required rate of return and standard deviation along the CML. Therefore, the market required rate of return on portfolio M and G are equal to 0.10 and 0.08, respectively. The given information is insufficient to determine the market requied rate of return for portfolio J. This is because portfolio J does not lie on the CML. Portfolio J is inefficient. Eventhough we know the variance and the rate of return on portfolio J, we cannot be sure at what rate of return that the market will require from asset J because it is not on the CML. Therefore, we cannot use our knowledge of the mean and the variance of asset J to determine the rate of return that the market will require from asset J in order to hold it in equilibrium. The variance may not be the correct measure of riskiness for an individual asset. Only the portion of total variance that is correlated with the economy(covariance risk) is relevant. Any portion of total risk that is not correlated with the economy is irrelevant and can be avoided at zero cost through diversification. We need the information on how the rate of return on asset J is correlated with the economy(covariance risk, systematic risk) to determine the market required rate of return on asset J.

7. The following information is provided for a stock market.

	ER_i	σ_i	r_{im}
Asset 1	0.07	0.30	0.2
Asset 2	0.08	0.20	0.4
Asset 3	0.10	0.15	0.8
Market Portfolio (M)	0.09	0.10	1

ER_i is the expected rate of return on asset i. σ_i is the standard deviation of the rate of return on asset i. r_{im} is **the correlation coefficient** between the rate of return on asset i and the rate of return on the market portfolio M. At equilibrium, the expected rate of return on the market portfolio is equal to 0.09 and the standard deviation of the market portfolio is equal to 0.10. The risk free interest rate is 4%. (Hint: $\sigma_{xy} = r_{xy} \times \sigma_x \times \sigma_y$, $r_{xy} = \frac{COV(X,Y)}{\sigma_x \sigma_y}$)

(a) In the context of CAPM, find the beta-coefficient for each asset.
Solutions.

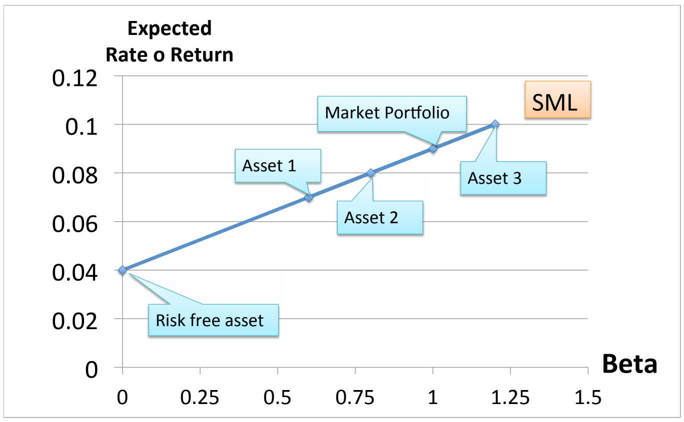
	σ_i	r_{im}	$\beta_i = \frac{r_{im} \times \sigma_i \times \sigma_m}{\sigma_m^2}$
Asset 1	0.30	0.2	0.6
Asset 2	0.20	0.4	0.8
Asset 3	0.15	0.8	1.2
Market Portfolio (M)	0.10	1	1.0

(b) Construct SML from the given information and interpret its meaning. Graphically illustrate SML and show the points where each asset lie in the graph.

Solutions.

$E(R_i) = R_f + \beta_i(E(R_M - R_f))$.

$E(R_i) = 0.04 + 0.05\beta_i$.



(c) In the context of CAPM, detemine whether each asset is overpriced, underpriced.or correctly priced. Explain the price adjustment process that might happen (if any).

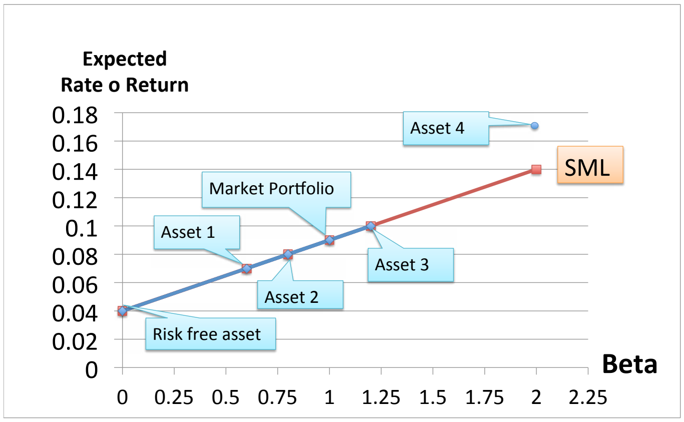
Solutions. All assets are correctly priced. They lie on the SML.

(d) You are informed that the fourth asset, with the beta-coefficient is equal to 2 is available. Empirical evidence reveals that its expected rate of return is 17%. Determine whehter this asset is overpriced, underpriced or correctly priced. In the context of CAPM, explain the price adjustment process that might happen(if any).

Solutions.

Asset 4 lies above the SML. Asset 4 is underpriced. Investors will buy asset 4. The price of asset 4 will go up and the rate of return on asset 4 will go down. At equilibrium asset 4 must lie on the SML.

Another implication is that we have found an empirical evidence against CAPM.



(e) A portfolio K consists of 0.10 of asset 1 and 0.90 of asset 2. What is the beta-coefficient for the portfolio K? In the context of CAPM, what is the equilibrium market required rate of return on the portfolio K?

Solutions.

$\beta_K = 0.1\beta_1 + 0.90\beta_2 = 0.1(0.6) + 0.90(0.80) = 0.78.$

$E(R_K) = R_f + \beta_K(E(R_m) - R_f) = 0.04 + 0.78(0.05) = 0.079 = 7.9\%$