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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$ $= (a+bx_1)+(a+bx_2)+(a+bx_3)+(a+bx_4)+(a+bx_5)$

   $= 5a+bx_1+bx_2+bx_3+bx_4+bx_5$

   $= 5a+b(x_1+x_2+x_3+x_4+x_5)$ #

   b. $\sum_{y=0}^{5}f(x+y)$ $= (x+0)+(x+1)+(x+2)+(x+3)+(x+4)+(x+5)$

   $= 6x+15$ #

   c. $\sum_{i=1}^{10}i^2$ $= 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

   $= 385$ #

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= (2(1)+2)+(2(2)+3)$

   $= 11$ #

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $0.5b+b+2.25b+2b+1.5b+0.5b+0.25b=1$

   $8b=1$

   $b=1/8=0.125$ #

   b. Find the answer for $P(X \le 2)$

   $P(X\le 2) = 1-P(X>2)$

   $= 1-P(X=3)-P(X=4)$

   $= 1-0.5(0.125)-0.25(0.125)$

   $= 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $P(-2\le X\le 3) = 1-P(X>3)$

   $= 1-P(X=4)$

   $= 1-0.25(0.125) = 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $P(X\ge 1) = 1-P(X<1)$

   $= 1-P(X=0)-P(X=-1)-P(X=-2)$

   $= 1-(2.25)(0.125)-(0.125)-0.5(0.125) = 0.53125$

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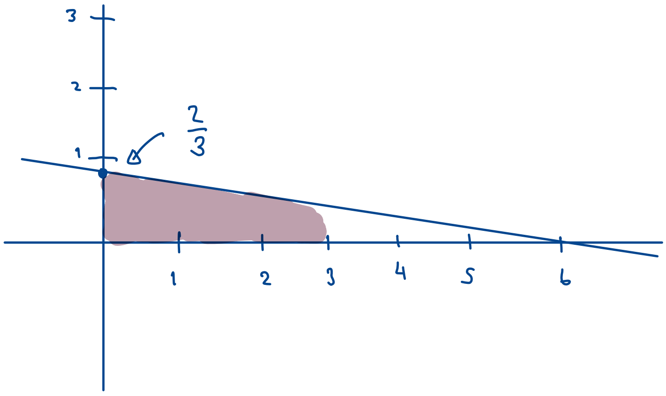
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$(x, y) = (6, \frac{2}{3})$

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \le x \le 3) = \int_1^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_1^3$$

$$= \left(-\frac{(3)^2}{18} + \frac{6}{9}(3)\right) - \left(\frac{-(1)^2}{18} + \frac{6(1)}{9}\right) = -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9}$$

$$= -\frac{8}{18} + \frac{12}{9} = \frac{16}{18}$$ #

c. Find the answer for $P(X \ge 2)$

$$P(x \geq 2) = \int_2^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_2^3$$

$$= \left[\frac{-(3)^2}{18} + \frac{6(3)}{9}\right] - \left[\frac{-(2)^2}{18} + \frac{6(2)}{9}\right] = -\frac{9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9} = \frac{7}{18}$$ #

d. Find the expected value of X

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$= \frac{-x^3}{27} + \frac{6x^2}{18} \Big|_0^3$$

$$= -\frac{(3)^3}{27} + \frac{6(3)^2}{18}$$

$$= -\frac{27}{27} + \frac{6 \cdot 9}{18}$$

$$= -1 + 3$$

$$= 2$$ #

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

y \ x	1	2	3	4	5	6
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

   b. Find the marginal probability distribution function (PDF) of X

X	1	2	3	4	5	6
f(x)	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$

   c. Find the marginal probability distribution function (PDF) of Y

y	0	1
f(y)	$\frac{1}{2}$	$\frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$\frac{1/12}{1/2} = \frac{1}{6}$

$$f(x \mid y=1) = \frac{f(x,y)}{f_y(1)} = \frac{f(x,1)}{1/2}$$

X	1	2	3	4	5	6
f(x|y=1)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

   e. Find the expected value of X given Y is equal to 1

$$E(X \mid y=1) = \Sigma x_i P(X=x_i \mid y=1) = \frac{\Sigma x_i P(X=x_i, y=1)}{P(y=1)} = \frac{1}{P(y=1)} \Sigma x_i P(X=x_i, y=1)$$

$$= \frac{1}{0.5}\left[\left(1 \cdot \frac{1}{12}\right) + \left(2 \cdot \frac{1}{12}\right) + \left(3 \cdot \frac{1}{12}\right) \ldots + \left(6 \cdot \frac{1}{12}\right)\right] = \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$V(X \mid y=1) = \Sigma (X - E(X \mid y=1))^2 \cdot P(X \mid y=1)$$

$$= \left[\left(\left(1-\frac{7}{2}\right)^2 \cdot \frac{1}{6}\right) + \left(\left(2-\frac{7}{2}\right)^2 \cdot \frac{1}{6}\right) + \ldots + \left(\left(6-\frac{7}{2}\right)^2 \cdot \frac{1}{6}\right)\right]$$

$$= \left(\frac{25}{4} \cdot \frac{1}{6}\right) + \left(\frac{9}{4} \cdot \frac{1}{6}\right) + \left(\frac{1}{4} \cdot \frac{1}{6}\right) + \left(\frac{1}{4} \cdot \frac{1}{6}\right) + \left(\frac{9}{4} \cdot \frac{1}{6}\right) + \left(\frac{25}{4} \cdot \frac{1}{6}\right)$$

$$= \frac{120}{24} \rightarrow \frac{10}{3} \#$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{N}\sum_{i=1}^{3} X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3)$$

$$= \frac{1}{N}\left[E(X_1) + E(X_2) + E(X_3)\right]$$

$$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{3} \cdot 3\mu_x = \mu_x \quad \#$$

$$Var(\bar{X}) = var\left(\frac{1}{N}\sum_{i=1}^{3} X_i\right)$$

$$= \frac{1}{N^2} var(X_1 + X_2 + X_3)$$

$$= \frac{1}{N^2}\left[var(X_1) + var(X_2) + var(X_3) + 2\,Cov(X_1, X_2) + 2\,Cov(X_1, X_3) + 2\,Cov(X_2, X_3)\right]$$

$$= \frac{1}{3^2}\left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma_x^2\right]$$

$$= \frac{1}{3^2} \cdot \frac{9}{4}\sigma_x^2 = \frac{\sigma_x^2}{4} \quad \#$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{N}\left[E(X_1) + E(X_2) + E(X_3) + E(X_4)\right]$$

$$= \frac{1}{4}\left[\mu_x + \mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x \quad \#$$

$$Var(\bar{X}) = var\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{N^2} Var(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{N^2}\left[Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4)\right]$$

$$= \frac{1}{4^2}\left[\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2\right]$$

4

$$= \frac{1}{16} \cdot 4\sigma_x^2 = \frac{\sigma_x^2}{4} = 0.25\sigma_x^2 \quad \#$$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$\tilde{x} = \frac{1}{4}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$E(\tilde{X}) = E(\frac{1}{N}\sum_{i=1}^{4} X_i)$

$= \frac{1}{4}E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$= \frac{1}{4}[E(0.5X_1) + E(X_2) + E(0.5X_3) + E(2X_4)]$

$= \frac{1}{4}[0.5E(X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)]$

$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x$

$= \frac{1}{4} \cdot 4\mu_x = \mu_x$

$\tilde{x}$ is unbiased estimator of μ

$var(\tilde{X}) = Var(\frac{1}{N}\sum_{i=1}^{4} x_i)$

$= \frac{1}{4^2} Var(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$= \frac{1}{4^2}[var(0.5X_1) + var(X_2) + Var(0.5X_3) + Var(2X_4)]$

$= \frac{1}{4^2}[0.25 Var(X_1) + Var(X_2) + 0.25 Var(X_3) + Var(X_4)]$

$= \frac{1}{4^2}[0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2]$

$= \frac{5.5\sigma_x^2}{16} = 0.34\sigma_x^2$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{x}$ is more efficient of μ_x than $\tilde{x}$ due to smaller variance.