

# EE320 (1/2016)

## INTRODUCTORY MATHEMATICAL ECONOMICS

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MATHEMATICS AND ECONOMIC RELATIONS

# Recap

- Last time, we started talking about the structure of a mathematical economic model, which generally consists of a **systems of equations**.
- Also, to analyze an economic problem, we need to first determine what are **dependent** or **independent variables** to be included in the model.
- So, today's lecture will start with a review of **set theory**, the concepts of **relation and functions**, and discuss **possible forms of functions** that can be used to characterize the *behavioral* equations.

# Topics

- Review of set theory
- Relations and functions
  - Ordered pairs and Cartesian products
  - Relations and Functions
  - Inverse Function
- Types of functions
  - Constant functions
  - Polynomial functions
  - Rational functions
  - Non-algebraic functions
  - Functions of two or more independent variables

# Set Notation

- A *set* is a collection of distinct objects (e.g. numbers, persons, food items, etc.).
- The objects in a set are called the *elements* of the set.
- Two ways of writing a set:
  - By enumeration:  
 $A = (7, 11, 13, 17, 19)$   
 $S = \{\text{apple, blackberry, computer}\}$
  - By description:  
 $I = \{x \mid x \text{ is a positive integer}\}$   
 $J = \{x \mid 5 < x < 20\}$

# Set Notation (cont'd)

- A *finite* set: A set with a finite number of elements
- An *infinite* set: A set with infinite number of elements

## Examples:

$$B = \{3, 6, 9\}$$

$$C = \{x \mid 10 < x < 29\}$$

$$D = \{x \mid x \text{ is a rational number}\}$$

Questions: Which of the sets above are finite or infinite?

→ B is a finite set. C and D are infinite sets.

- $\in$  indicates membership in a set:

E.g.  $9 \in B$        $9 \notin C$

# Relationships between Sets

- **Set equality**
  - e.g. If  $A = \{2, 5, r, t\}$  and  $B = \{2, r, 5, t\}$ , then  $A = B$ .
- **Subset:**  $T \subset S$  iff  $x \in T$  implies  $x \in S$ .
  - e.g. If  $S = \{2, 3, 4, 6\}$  and  $T = \{3, 4\}$ , then  $T \subset S$ .
  - A **proper subset** of  $S$  is any subset that does not contain *all* the elements of  $S$ .
  - Every set is a subset of itself.
- **Null set or empty set:**  $\{ \}$  or  $\Phi$ .
  - Every set contains the null set.
- **Disjoint sets:**
  - e.g. Sets  $A$  and  $T$  are disjoint.

# Operations on Sets

- **Intersection:**  $C = A \cap B$

$$C = \{a \mid a \in A \wedge a \in B\}$$

- **Union:**  $C = A \cup B$

$$C = \{a \mid a \in A \vee a \in B\}$$

- **Complement of a set:**  $\tilde{A}$

$$\tilde{A} = \{x \mid x \in U \wedge x \notin A\}$$

- **Example:** Let  $U = \{1,2,3,4,5,6,7,8\}$ ,  $A = \{3,4,5,6\}$ , and  $B = \{5,6,7,8\}$ .

$$A \cap B = \{5,6\}$$

$$A \cup B = \{3,4,5,6,7,8\}$$

$$\tilde{A} = \{1,2,7,8\}$$

# Laws of Set Operations

- Commutative law:

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

- Associative law:

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive law:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

# Ordered Pairs and Cartesian Products

- Ordered pairs
  - Unordered pair :  $\{a, b\} = \{b, a\} \rightarrow$  Ordering does not matter.
  - *Ordered pair*: The ordering of  $a$  and  $b$  matters.  
 $(a, b) \neq (b, a)$ , unless  $a = b$ .
- Example:  $(h, w) =$  (height in cm., weight in kg.) of students in class  
e.g.  $(h, w) = (165, 54)$ . Would writing  $(54, 165)$  make any senses?
- Given two sets  $x$  and  $y$ , the *Cartesian product* is the set of all the possible ordered pairs with the first element taken from set  $x$  and the second element taken from set  $y$ :

$$x \times y = \{(a, b) \mid a \in x \wedge b \in y\}$$

# Relations and Functions

- Any subset of the Cartesian product  $x \times y$  will constitute a *relation* between  $y$  and  $x$ . Given an  $x$  value, one or more  $y$  values will be specified by that relation.
- Example:  $\{(x, y) \mid y = 2x\}$  and  $\{(x, y) \mid y \geq x\}$

# Relations and Functions (cont'd)

- A function is a special case of a relation, in which for each  $x$  value there exists only *one* corresponding  $y$  value.
- Thus, a *function* is a set of ordered pairs with the property that any  $x$  value uniquely determines a  $y$  value.

$$y = f(x)$$

where  $x$  is the *argument* of the function (i.e. *independent* variable) and  $y$  is the *value* of the function (i.e. *dependent* variable).

- Alternative names of a function: a *mapping* or *transformation*:

$$f: x \rightarrow y$$

- **Domain**: The set of all permissible values that  $x$  can take.
- **Range**: The set of all images ( $y$  value into which an  $x$  value is mapped)

# Example: Domain and Range

- The total cost  $C$  of a firm is a function of its daily output  $Q$ :  
 $C = 200 + 3Q$ .
- Suppose that there is a capacity limit of 300 units of output per day.
- What are the domain and range of the cost function?

Domain =

Range:

# Inverse Function

- Let  $y = f(x)$  be an invertible function, and  $f$  maps  $X$  to  $Y$ .  
→ The **inverse** function  $f^{-1}(x) = g(y)$  maps  $Y$  to  $X$ .
- Example: Given  $y = f(x) = 3x + 2$ , what is  $f^{-1}$ ?  
→ The inverse function:  $x = g(y) =$
- Example:  
Suppose a demand function is given by  $Q_d = 100 - 0.2P$ . Derive the **inverse demand function**.  
→

# Constant Function

- A **constant function** is a function whose range consists of only one element:

$$y = f(x) = y_0, \quad \text{where } y_0 \text{ is a constant.}$$

- Example: In a cost function of a firm, the total cost (TC) includes only a fixed cost (F), say \$50. Thus, the cost function can be written as:  $TC = F = 50$ .
- Graph

# Polynomial Functions

- General form of a polynomial function of a variable  $x$ :

$$y = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where  $a_0 \dots a_m$  are the coefficients and the value of  $n$  is the degree of the polynomial function.

- Subclass of polynomial functions:

- $n=0$ :  $y = a_0$  [constant function]
- $n=1$ :  $y = a_0 + a_1x$  [linear function]
- $n=2$ :  $y = a_0 + a_1x + a_2x^2$  [quadratic function]
- $n=3$ :  $y = a_0 + a_1x + a_2x^2 + a_3x^3$  [cubic function]

# Linear Function: $y = a_0 + a_1x$

- Example:  $TC = FC + VC = a + bQ$

Quadratic Function:  $y = a_0 + a_1x + a_2x^2$

- Case of  $a_2 < 0$
- Case of  $a_2 > 0$

Cubic Function:  $y = a_0 + a_1x + a_2x^2 + a_3x^3$

- Case of  $a_3 > 0$
- Case of  $a_3 < 0$

# Rational Functions

- A *rational function* is a function expressed as a ratio of two polynomials in the variable  $x$ :  $y = f(x)/g(x)$ , where  $g(x) \neq 0$ .

- Special case: A *rectangular hyperbola*:

$$y = a/x \quad \text{or} \quad xy = a.$$

- Applications:

- A demand curve  $Q = f(P)$  with constant total expenditure  $PQ$ , where the price elasticity is unitary.
- An average fixed cost (AFC) curve:  $AFC = f(Q) = c/Q$ , where  $c$  is a constant. Here, total fixed cost is a constant because  $TFC = c$ .

# Example: Rectangular hyperbola

- Suppose  $TFC = 50$ .

# Nonalgebraic Function

- A *nonalgebraic* function is a function that *cannot* be expressed in terms of polynomials and/or roots of polynomials.
  - **Exponential function:**  $y = b^x$
  - **Logarithm function:**  $y = \log_b x$
- Graphs

# Example: Exponential Function

# Functions of Multiple Independent Variables

- Given two independent variables  $x$  and  $y$ , a function of uniquely determined (dependent) variable  $z$  is:

$$z = g(x, y),$$

where the domain of the function will be some subset of the points in the  $xy$  plane.

- The association between the three variables are summarized by the ordered triple  $(x, y, z)$ .
- Examples:
  - Production function:  $Q = Q(K, L)$
  - Utility function:  $U = U(x_1, x_2)$

# Example: Production function in a 3-dimension space