Exercise VI

1. Let $A=\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ Find det (A) by first row reducing A via elementary row operations.

   a) Is A invertible? If it is, what is $\det(A^{-1})$?

   b) Find $\det(A-\lambda I)$ by cofactors, where λ is an arbitrary number.

   c) For what value(s) of λ does $\det(A-\lambda I)=0$?

2. A 4x4 matrix A has eigenvalues $\lambda_1=-2, \lambda_2=1, \lambda_3=3, \lambda_4=4$.

   a) What is the characteristic polynomial of A?

   b) Compute det (A)

   c) Compute $\det(-2A)$

   d) Compute det $(A+2I_4)$.

   e) What are the eigenvalues of A^3?

   f) Compute $\det(A^3)$

   g) Is A invertible? Why and why not?

   h) Is A diagonalizable? Why and Why not?

3. Find the diagonalization $A=PDP^{-1}$ of $A=\begin{bmatrix} 0.5 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   What is the limit of A^k as $k\to\infty$?

4. Let A be the matrix $\begin{bmatrix} 7 & 5 \\ 3 & -7 \end{bmatrix}$

   a) Find matrices P and D such that A has a factorization of the form $A=PDP^{-1}$, where P is invertible and D is diagonal: $D=diag(\lambda_1,\lambda_2)$.

   b) Find a matrix B such that $B^3=A$

5. Find a 2×2 matrix A such that $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are eigenvectors of A, with eigenvalues –2 and 5 respectively.

6. Suppose A is 3×3 matrix with eigenvalues 0, 1 and 2. Find the following:

   a) the rank of A

   b) the determinant of A^TA

   c) the determinant of $(A+I)$.

7. Compute

$$\begin{vmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{vmatrix}$$

Exercise VII

1. If a 2 by 2 symmetric matrix passes the tests $a > 0, ac > b^2$, solve the quadratic equation $\det(A - \lambda I) = 0$ and show that the roots are positive.

2. For what range of numbers a and b are the matrices A and B positive definite?

$$A = \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{bmatrix}$$

3. let $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & 2 & 2 \end{bmatrix}$, $v_1 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Verify that v_1 and v_2 are eigenvectors of A. Then orthogonally diagonalize A.

4. Find the matrix of the quadratic from. Assume $\boldsymbol{x}$ is in R^3.

   a) $8x_1^2 + 7x_2^2 - 3x_3^2 - 6x_1x_2 + 4x_1x_3 - 2x_2x_3$

   b) $4x_1x_2 + 6x_1x_3 - 8x_2x_3$