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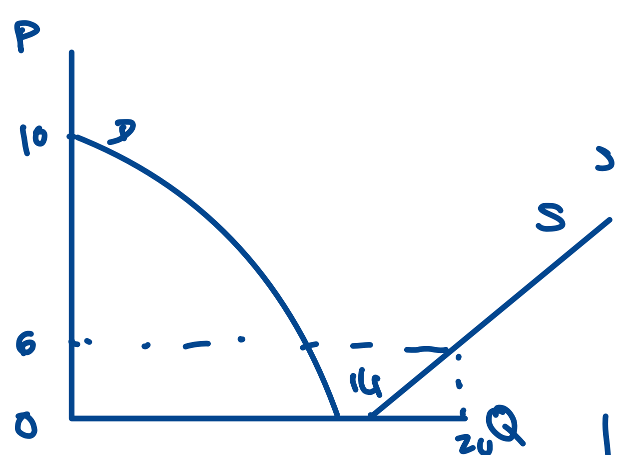
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1.

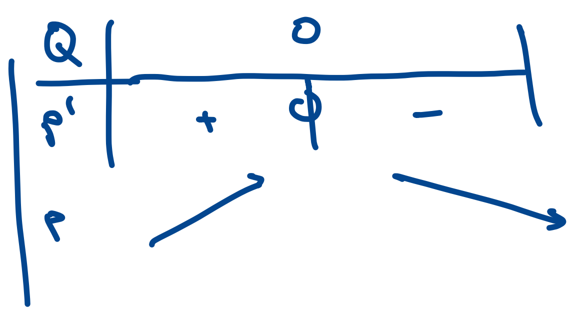


$D: P = 10 - Q^2$

$S: Q = a + P \Rightarrow P = -a + Q$

$a = -14$

$\Rightarrow S: Q = -14 + P$



Graph $D: P = 10 - Q^2$

$P' = -2Q$

$Q = 0 \Rightarrow P = 10$

Graph $S: P = -14 + Q$

$Q = 0 \Leftrightarrow P = -14$

$Q = 14 \Leftrightarrow P = 0$

$Q = 20 \Leftrightarrow P = 6$

1.2. Solve for Q^* and P^*

Market in equilibrium when $Q_D = Q_S$

Since: $D: P = 10 - Q^2 \Rightarrow Q_D = \sqrt{10-P}$

$S: Q_S = -14 + P$

$$\Leftrightarrow \sqrt{10-P} = -14 + P$$

$$10 - P = (P-14)^2$$

$$= P^2 - 28P + 196$$

$$P^2 - 27P + P + 196 - 10 = 0$$

$$P^2 - 27P + 186 = 0$$

$$P = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta = b^2 - 4ac = (-27)^2 - 4(1)(186)$$

$$= 729 - 744 = -15$$

Thus, there is no solution for $Q_D = Q_S$.

$\Rightarrow$ No equilibrium.

1.3. if $a = -12$

$$\Rightarrow Q_D = Q_S \Leftrightarrow \sqrt{10-P} = -12 + P$$

$$10 - P = P^2 - 24P + 144$$

$$P^2 - 23P + 134 = 0$$

$$P = \frac{-b \pm \sqrt{\Delta}}{2a} \; ; \; \Delta = b^2 - 4ac = (-23)^2 - 4(1)(134)$$

$$= 529 - 556 = -27$$

No solution $\Rightarrow$ no equilibrium.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right) \quad ; Q \geq 0$$

$$R'(Q) = \left[\ln(Q^2+1)\right]' + \left[3\left(\frac{Q}{Q+1}\right)\right]'$$

but, $\left[\ln(Q^2+1)\right]' = (Q^2+1)' \frac{1}{Q^2+1} = \frac{2Q}{Q^2+1}$

+ $\left[3\left(\frac{Q}{Q+1}\right)\right]' = 3\left(\frac{Q}{Q+1}\right)' = 3\left[\frac{Q'(Q+1) - Q(Q+1)'}{(Q+1)^2}\right]$

$$= 3\left[\frac{(Q+1) - Q}{(Q+1)^2}\right] = \frac{3}{(Q+1)^2}$$

$$R'(Q) = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

since $Q \geq 0 \Rightarrow R'(Q) > 0$

Thus $R(Q)$ is an increasing function.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$$

$$\pi'(Q) = -Q^2 - 2Q + 8$$

Find critical values

$\pi'(Q)$ is differentiable on $\mathbb{R}$

$$\pi'(Q) = 0 \iff -Q^2 - 2Q + 8 = 0$$

$$Q = \frac{-b \pm \sqrt{\Delta}}{2a} \; ; \; \Delta = b^2 - 4ac = (-2)^2 - 4(-1)(8) = 4 + 32 = 36$$

$$Q = \frac{-(-2) \pm \sqrt{36}}{2(-1)} = \frac{4 \pm 6}{-2}$$

$$Q_1 = \frac{10}{-2} = -5$$

$$Q_2 = \frac{-2}{-2} = 1$$

Q		-5		1	
$\pi'(Q)$	$-$	0	$+$	0	$-$
$\pi(Q)$	$\searrow$		$\nearrow$		$\searrow$

$\pi(Q)$ max is at $\pi(1) = -\frac{1}{3}\cdot 1 - 1 + 8 - 1$

$$= -\frac{4}{3} + 8 - 1$$

$$= \frac{24}{3} - \frac{4}{3} - \frac{3}{3} = \frac{17}{3}$$

$$\pi''(Q) = -2Q - 2$$

$$\pi''(1) = -2(1) - 2 = -4 < 0$$

$\pi'(1) = 0$ and $\pi''(1) < 0 \Rightarrow \pi(1)$ is max.

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

4.2 A*B

4.3 det(A)

4.4 det(B)

4.5 det(C)

5. Suppose that $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

calculate $\frac{\partial u}{\partial x}$

$$\frac{\partial u}{\partial x} = a y^b x^{a-1} + \frac{x - x}{(x+y)^2} \cdot \frac{1}{\left[\frac{x}{x+y}\right]} = a y^b x^{a-1}$$

calculate $\frac{\partial U}{\partial y}$

$$\frac{\partial u}{\partial y} = b x^a y^{b-1} + \frac{0 - 1}{(x+y)^2} \cdot \frac{1}{\left[\frac{x}{x+y}\right]} = b x^a y^{b-1} + \frac{1}{\frac{x(x+y)^2}{x+y}}$$