Application of integration in economics

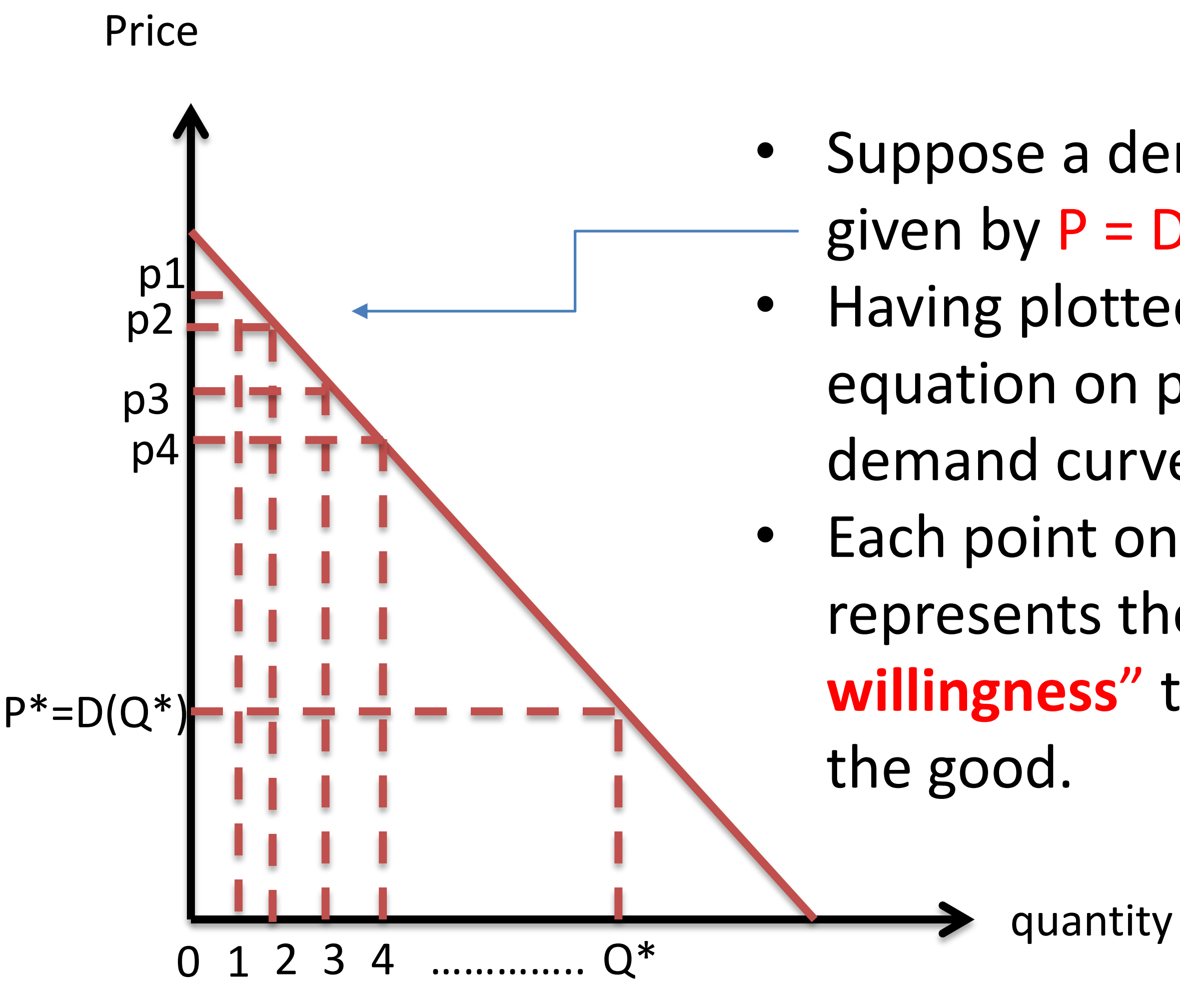
Total and Marginal

- Total ➔ Marginal concept, using derivative method.
  - Integration a reverse-engineering process
  - Allowing us to recover the total function that is corresponded to the given marginal function.
- I will skip this topic. You're responsible to study this topic yourself.
- See the example yourself in the handout.

Welfare analysis

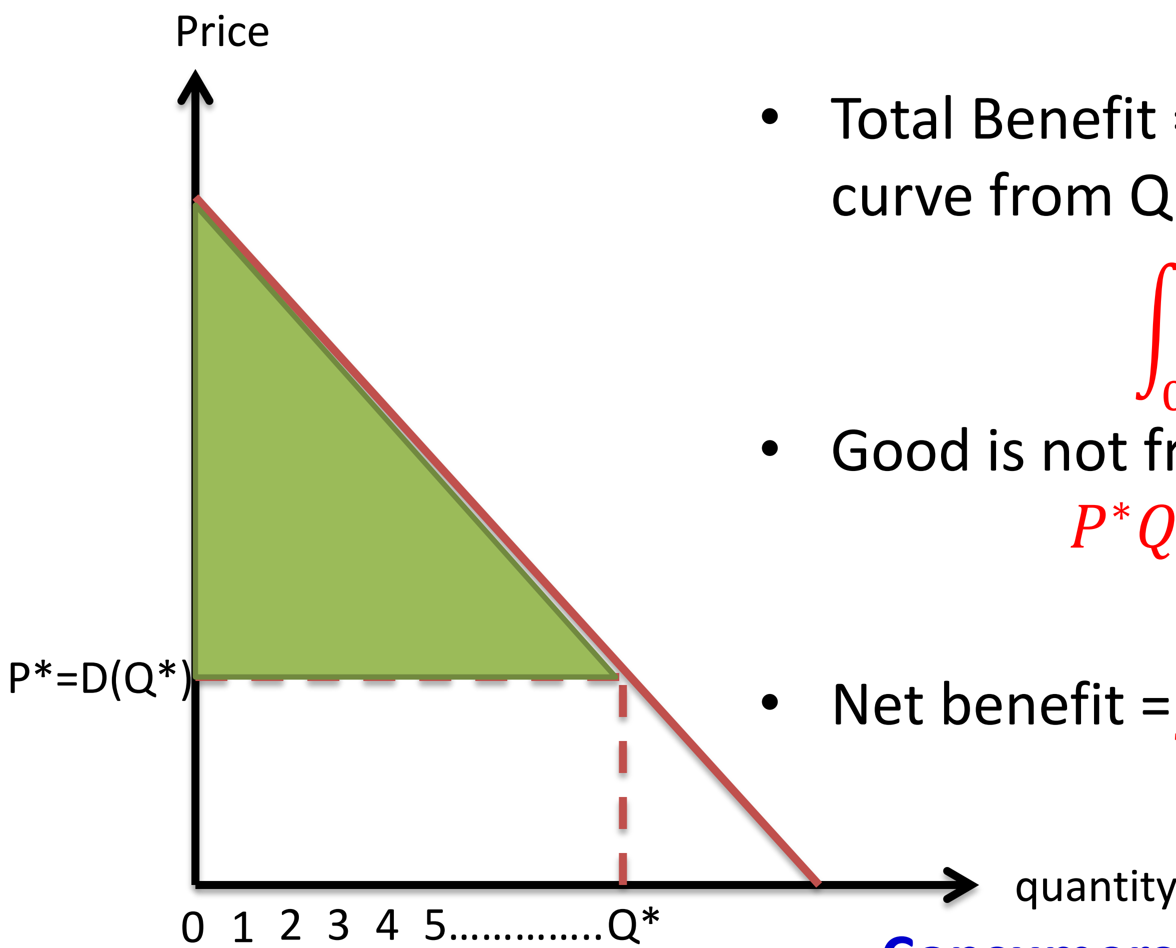
- Several welfare measurements
- Consumers' and producers' surplus (CS/PS) are the most popularized concept that most economists use.
  - Given information about demand and supply curve, we can measure CS and PS by applying the (definite) integration as the tool for measurement.
- What are they?

Consumer surplus



- Suppose a demand function is given by P = D(Q).
- Having plotted the demand equation on p-q axis, we get a demand curve.
- Each point on the demand curve represents the “**maximum willingness**” to pay for each unit of the good.

Consumer surplus



- Total Benefit = Area under the demand curve from Q = 0 upto Q = Q*

$$\int_0^{Q^*} D(Q)dQ$$

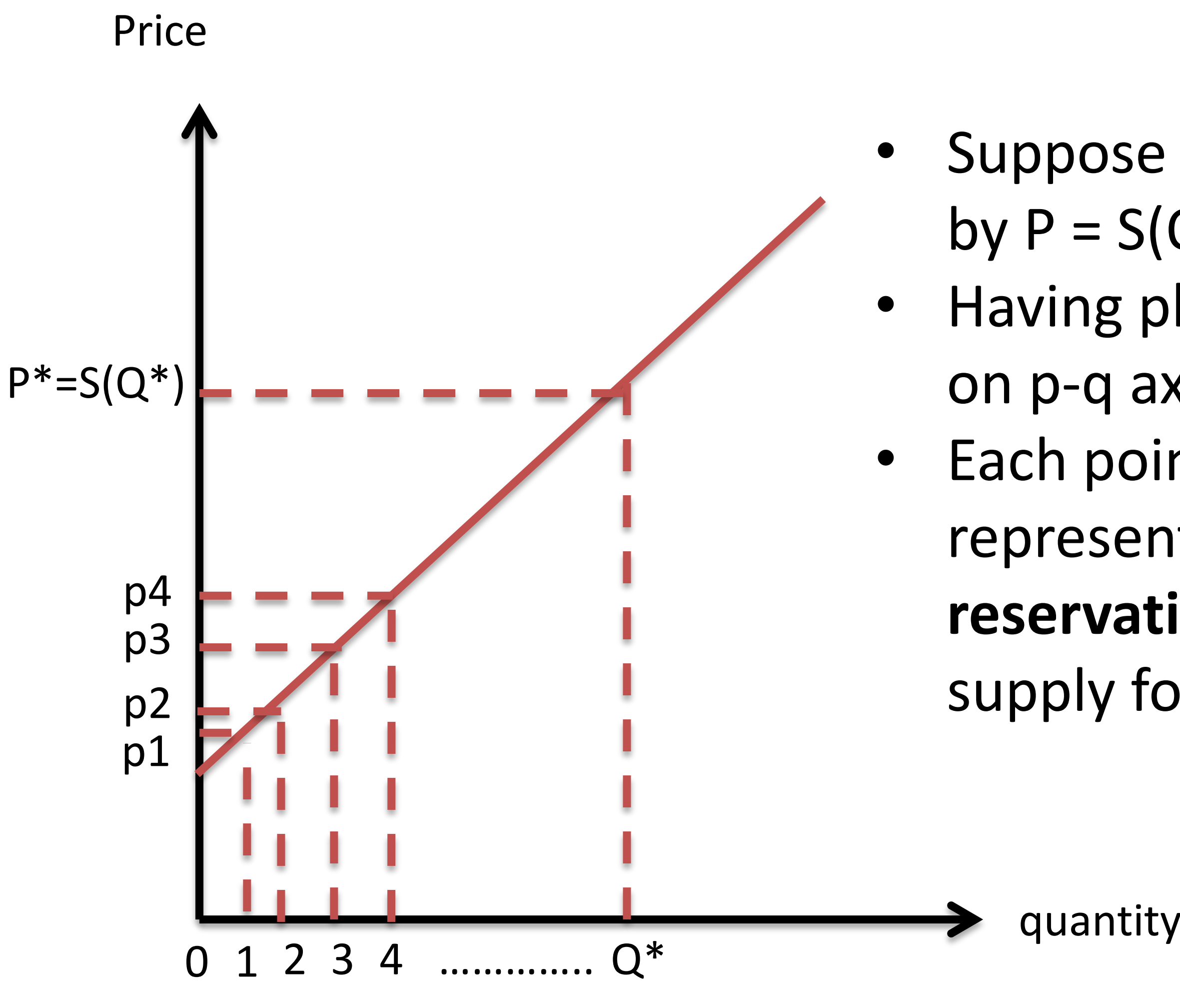
- Good is not free!

$$P^*Q^* = D(Q^*)Q^*$$

- Net benefit = $\int_0^{Q^*} D(Q)dQ - D(Q^*)Q^*$

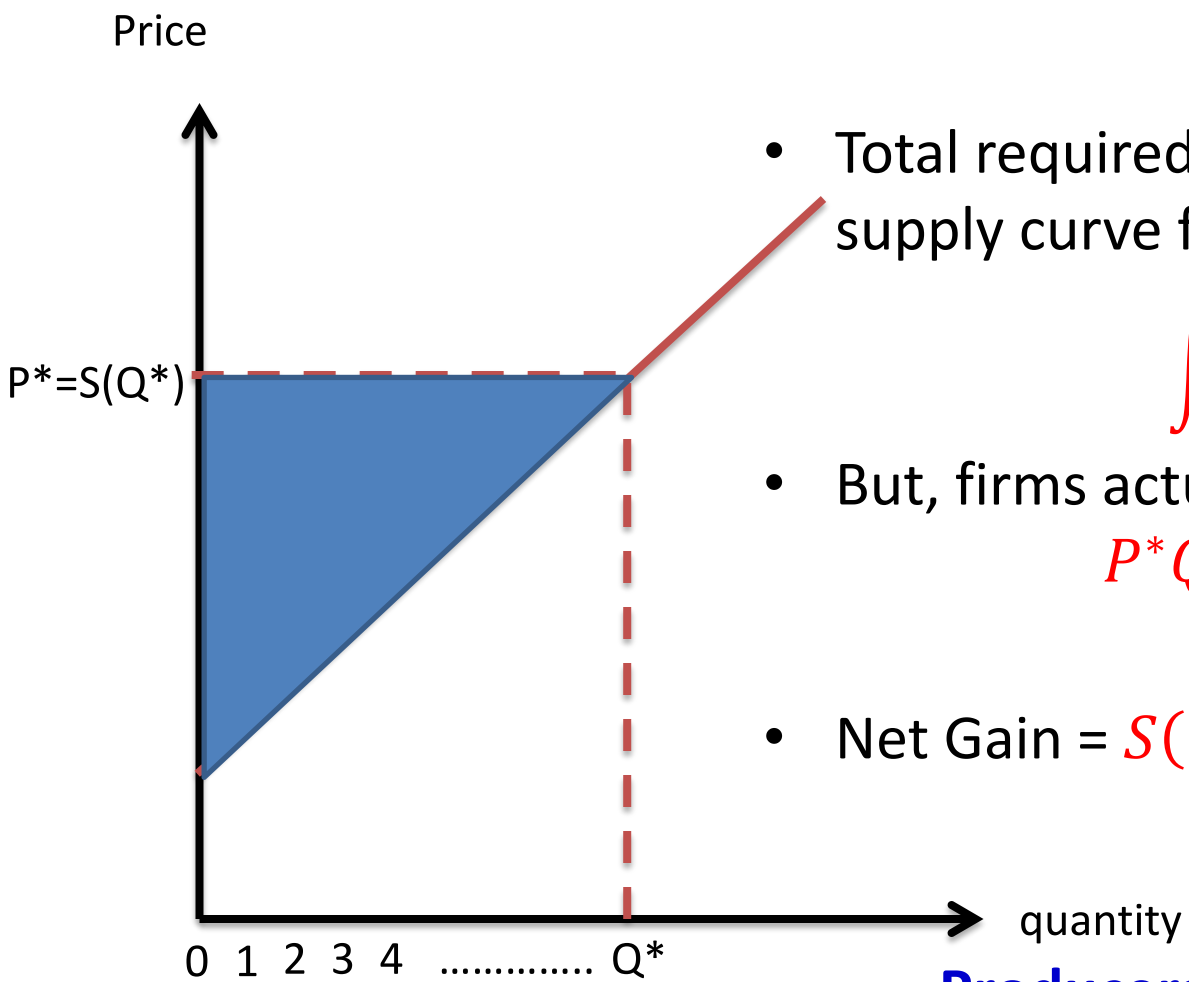
Consumers' surplus =
Area under demand, but above price

Producer surplus



- Suppose a supply function is given by P = S(Q).
- Having plotted the supply equation on p-q axis, we get a supply curve.
- Each point on the supply curve represents the "**minimum reservation price**" required to supply for each unit of the good.

Producer surplus



- Total required revenue = Area under the supply curve from Q = 0 upto Q = Q*

$$\int_0^{Q^*} S(Q)dQ$$

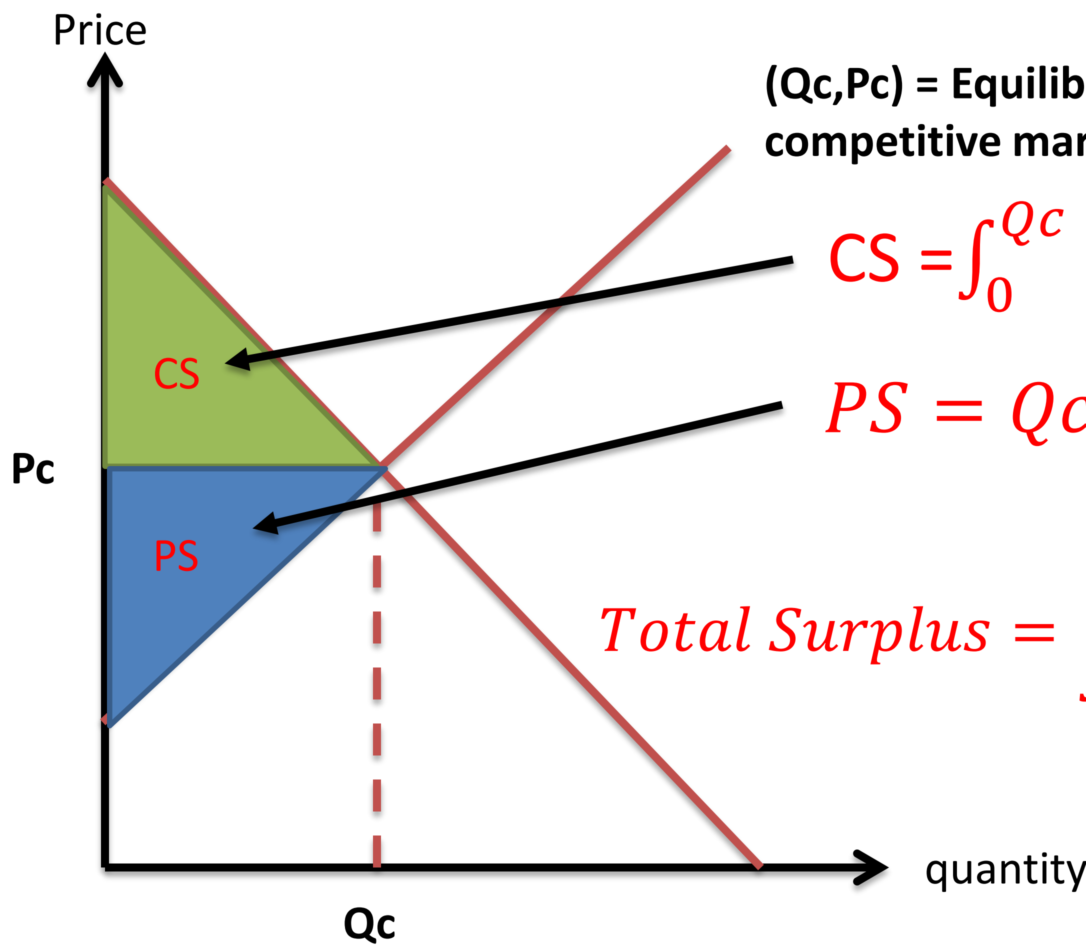
- But, firms actually get

$$P^*Q^* = S(Q^*)Q^*$$

- Net Gain = $S(Q^*)Q^* \; - \int_0^{Q^*} S(Q)dQ$

Producers' surplus =
area above supply, but under price

Welfare under perfectly competitive equilibrium



(Qc,Pc) = Equilibrium quantity/price under perfectly competitive market

$$CS = \int_0^{Qc} D(Q)dQ - Qc * Pc$$

$$PS = Qc * Pc - \int_0^{Qc} S(Q)dQ$$

$$Total\ Surplus = \int_0^{Qc} [D(Q) - S(Q)]dQ$$

Example

- Suppose the market demand equation is $\mathrm{Q} = \frac{1}{2}(25 - P)$ and the market supply equation is $\mathrm{Q} = \frac{1}{2}(P - 1)$.
  - Evaluate the welfare at the market equilibrium price and quantity.

Market equilibrium price and quantity?

- First solve for equilibrium: $(P^*, Q^*) = (6, 13)$.
- Given equations are not the equation for demand/supply curve; you need to rewire everything in P-equal form.
- Demand curve: $P = 25 - 2Q$
- Supply curve: $P = 1 + 2Q$

Welfare

- Consumers' surplus =

Area under demand curve – Total expenditure

$$\int_0^{Q^*=6} 25\text{-}2Q\ dQ\ - (13) * (6) = 36$$

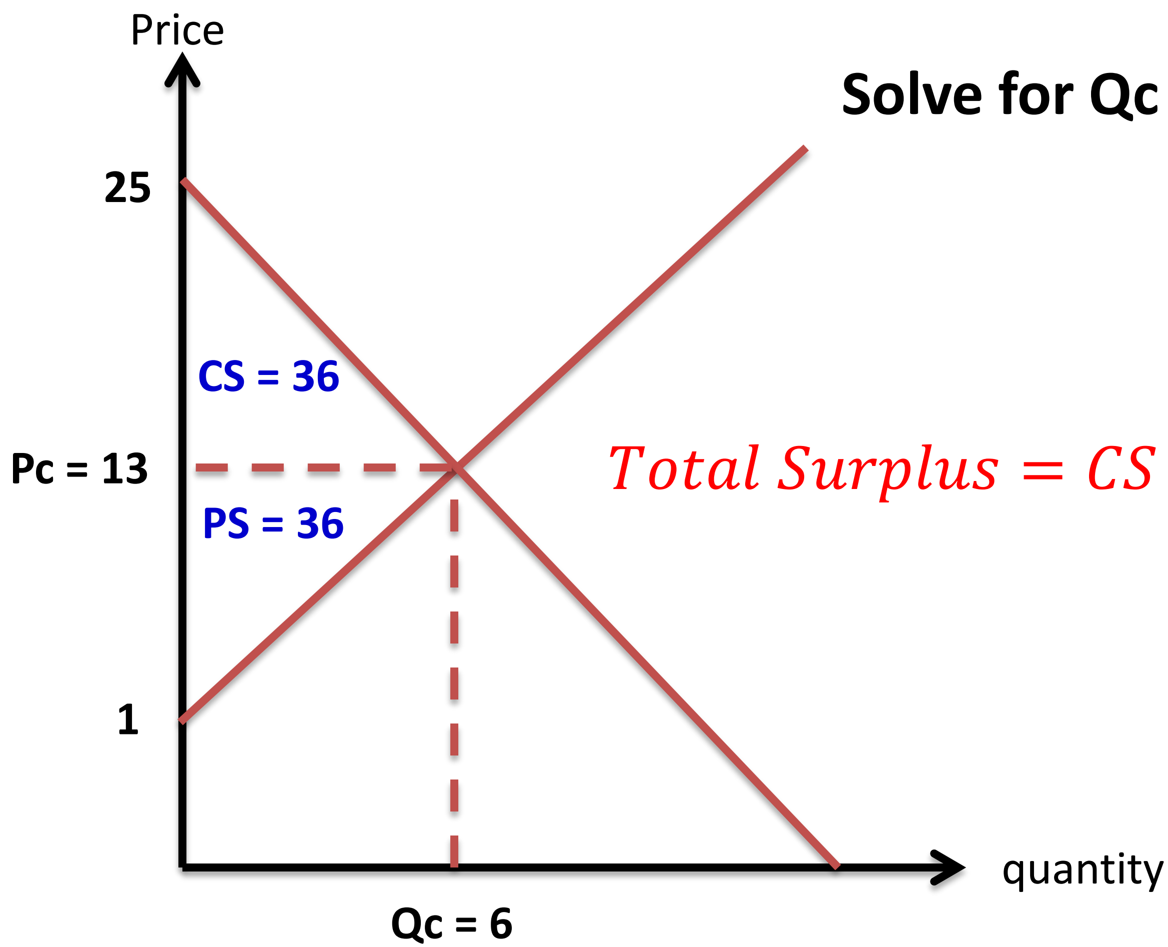
- Producers' surplus

Total revenue – Area under supply curve

$$(13) * (6) - \int_0^{Q^*=6} 1+2Q\ dQ\ = 36$$

Welfare under perfectly competitive

Solve for Qc and Pc ➔ (Qc,Pc) = (6, 13)



$$Total\ Surplus = CS + PS = 36 + 36 = 72$$

First-best equilibrium

- Economist typically refers to the "perfectly competitive" equilibrium/allocation as the benchmark for the first-best equilibrium.
  - Proof should be given in Micro311, for the part that discusses about General Equilibrium. Rigorous proof can be found in Debreu (1959).
- In what sense?
  - In the sense that the allocation generates the biggest total surplus of the two (consumers/producers) combined.

First-best equilibrium and Deadweight loss

- Implications:
  - Any interventions, such as taxes/subsidies/regulations, would do harm, rather than doing good, e.g. yielding us lowered welfare
  - Any markets with the structure deviating from the assumptions imposed under perfectly competitive market would yield lower welfare.
- The lowered welfare, as compared to the welfare attained under the first-best, is called the "deadweight loss".

Example: tax and deadweight loss

- Having recycled the same market demand/supply equation as used before, calculate the total surplus if the government taxes consumers for $8 per each unit of purchased quantity.

$$Pd = 25 - 2Q$$

$$Ps = 1 + 2Q$$

Example: tax and deadweight loss

- With \$8 tax, Pd = Ps + 8.

$Pd = 25 - 2Q$ == Demand

$$Ps + 8 \ = 25 - 2Q$$

$$Ps = 1 + 2Q$$

Q= ___4______, $Ps =$ ____9_____, $Pd =$ _17____

Example: tax and deadweight loss

- CS = Area under demand – Total payment

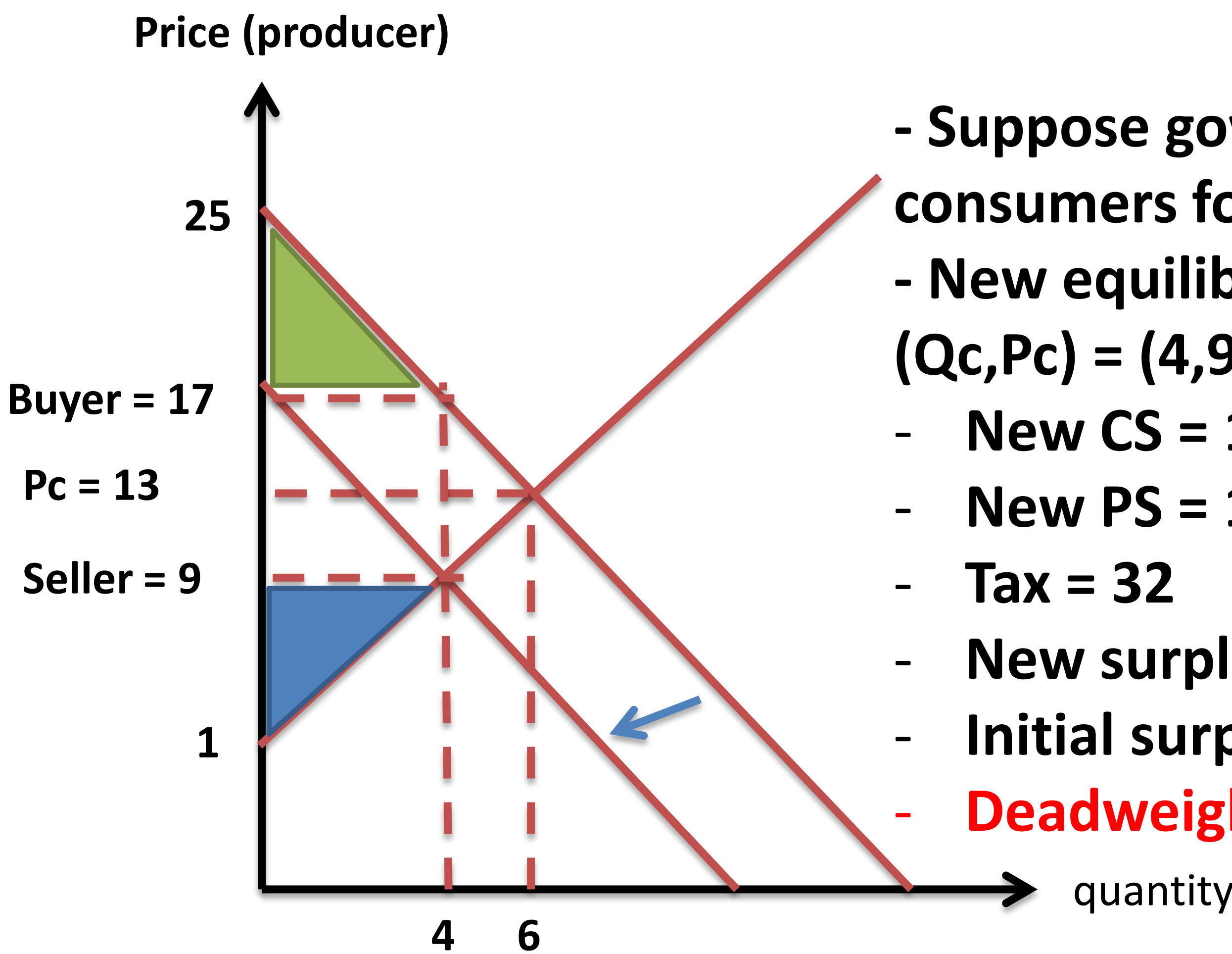
$$\int_0^{Q^*=4\rightarrow new\ Q} 25–2Q\ dQ\ - (17) * (4) = 16$$

- PS = Total revenue – Area under supply

$$(9) * (4) - \int_0^{Q^*=4} 1+2Q\ dQ\ = 16$$

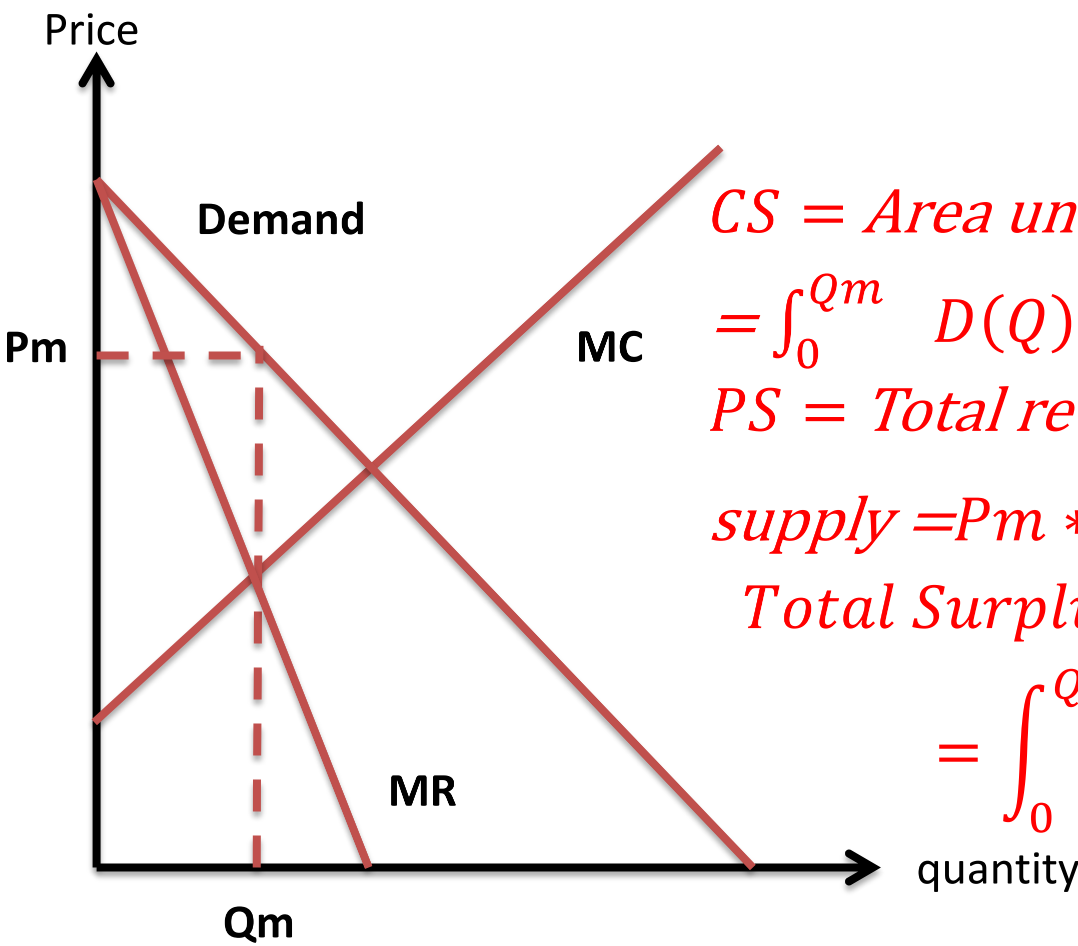
NOTE: What firms get is not equal to what consumers pay.

Tax and deadweight loss



- Suppose government taxes on consumers for $8 per each unit of Q.
- New equilibrium would be (Qc,Pc) = (4,9)
  - New CS = 16
  - New PS = 16
  - Tax = 32
  - New surplus = 16 + 16 + 32 = 64
  - Initial surplus = 72
  - Deadweight loss = 8

Monopoly and Deadweight loss: monopoly output is where MR = MC.



$$CS = Area\ under\ demand - Total\ pay$$

$$= \int_0^{Qm} D(Q)dQ - Pm * Qm$$

$$PS = Total\ revenue - Area\ under$$

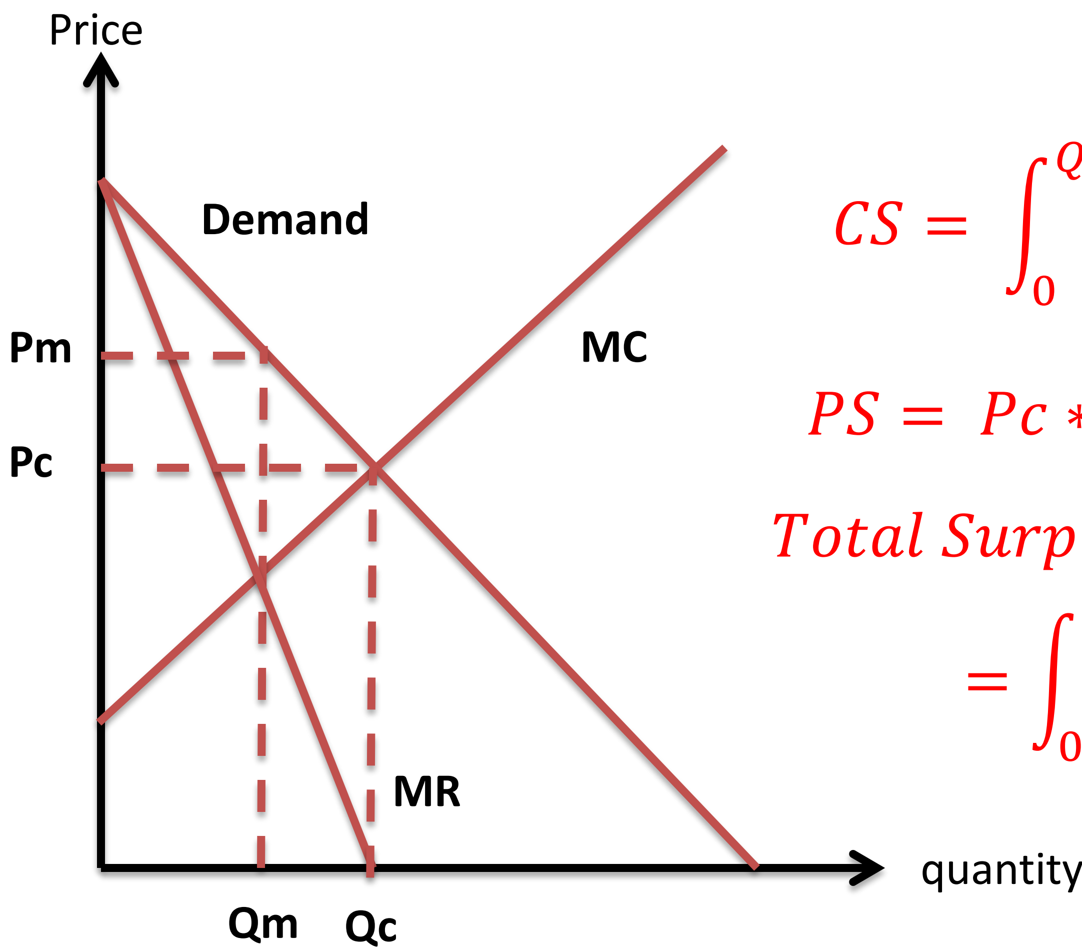
$$supply = Pm * Qm - \int_0^{Qm} MC(Q)dQ$$

$$Total\ Surplus = CS + PS$$

$$= \int_0^{Qm} [D(Q) - MC(Q)]dQ$$

Having solved for Qm by setting MR = MC, we can evaluate the welfare of monopoly as shown above, using CS and PS concept.

Monopoly and Deadweight loss: benchmark first-best/efficient output is Qc.



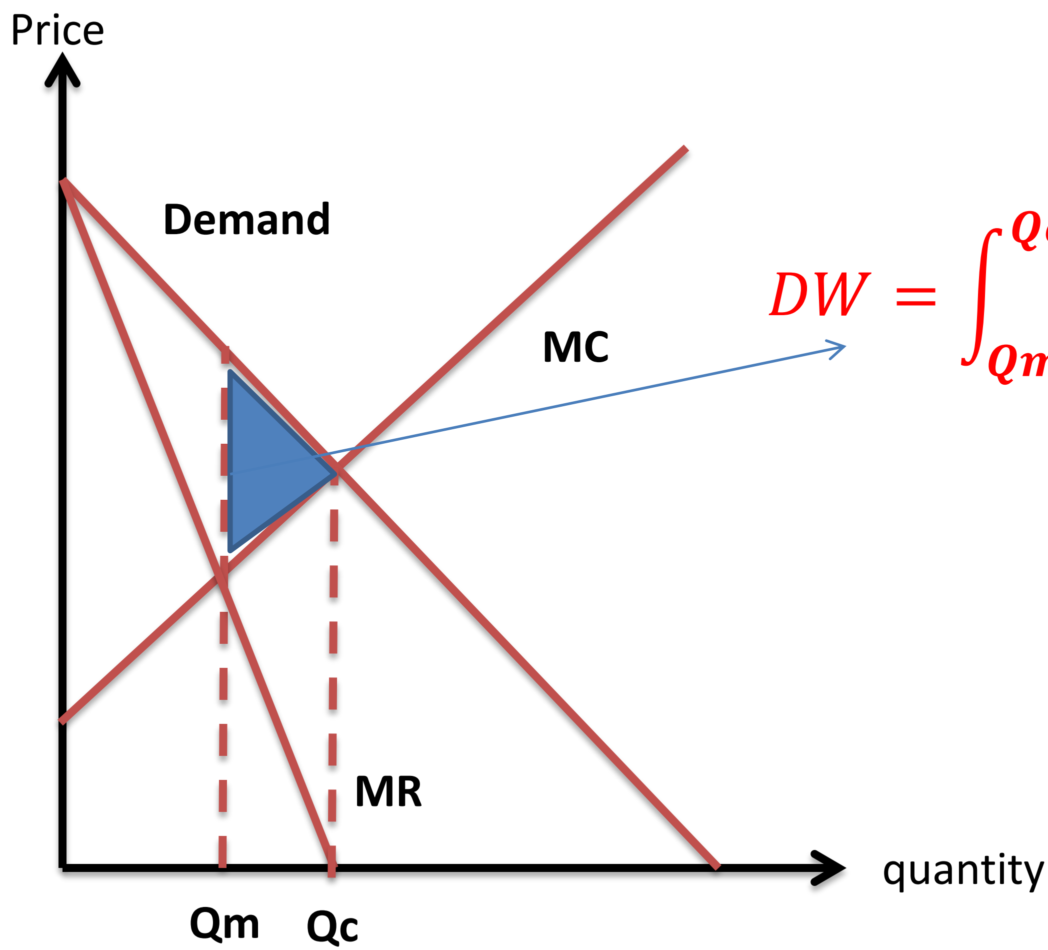
$$CS = \int_0^{Qc} D(Q)dQ - Pc * Qc$$

$$PS = Pc * Qc - \int_0^{Qc} MC(Q)dQ$$

$$Total\ Surplus$$

$$= \int_0^{Qc} [D(Q) - MC(Q)]dQ$$

Monopoly and Deadweight loss



$$DW = \int_{Qm}^{Qc} [D(Q) - MC(Q)]dQ$$

Example

- Market demand is given by P = 30 – Q and the cost function is given by $C(Q) = \frac{1}{2}Q^2$
  - Calculate the **<u>Deadweight loss</u>**

Example

- Need to know allocations (Q,P) under monopoly and first-best benchmark.
- Allocation under **the first-best benchmark** can be calculated by setting MR = 30-2Q and MC = Q equal to each other.
- So, Qm = 10 and Pm = 30 – Qm = 30 -10 = 20.

Example

- Allocation under the monopoly can be calculated by choosing Q that maximizes profit. This is equivalent to choose for Q such that $P = MC$.
- That is, we solve for $30 – Q = Q$.
- This yields us $Qc = 15$ and $P = 30 – Qc = 30 - 15$

Example

- $DW = \int_{Qm}^{Qc} [D(Q) - MC(Q)]dQ$

$$= \int_{Q=10}^{Q=15} [30 - Q - Q]dQ$$

$$= \int_{Q=10}^{Q=15} [30 - 2Q]dQ$$

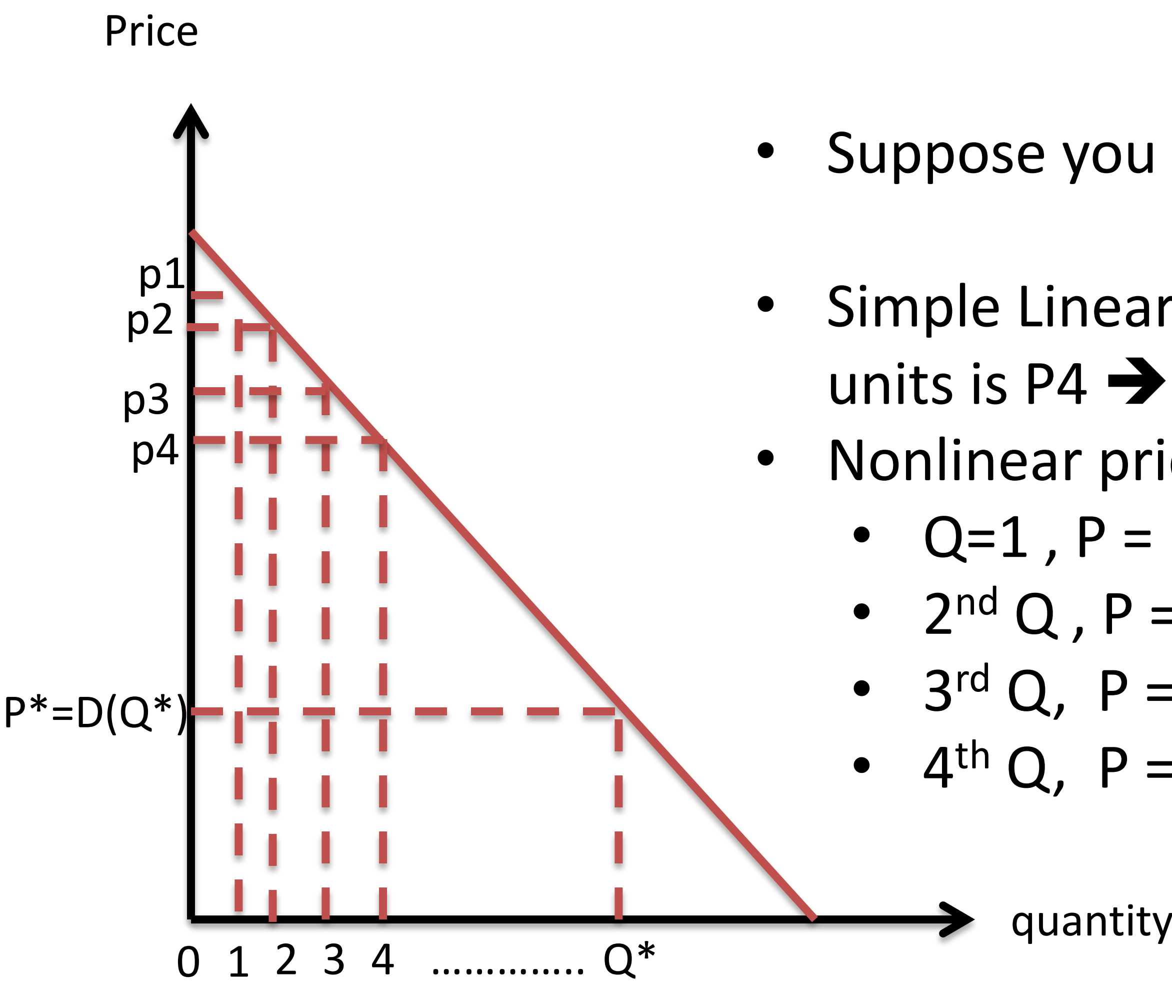
$$= 30(15 - 10) - (15^2 - 10^2) = 25$$

DW = 25

Does the monopoly always generate the deadweight loss?

- Answers depend…
  – Yes, if monopoly can only use linear pricing strategy.
    - Single price for each of the unit of Q that firms sell.
  – No, if monopoly can do the **first-degree price discrimination**.
    - Stronger version of price discrimination.
    - Setting price for "each unit of Q" EQUAL to maximum willingness to pay of consumers, and varied prices with respect to each unit Q.

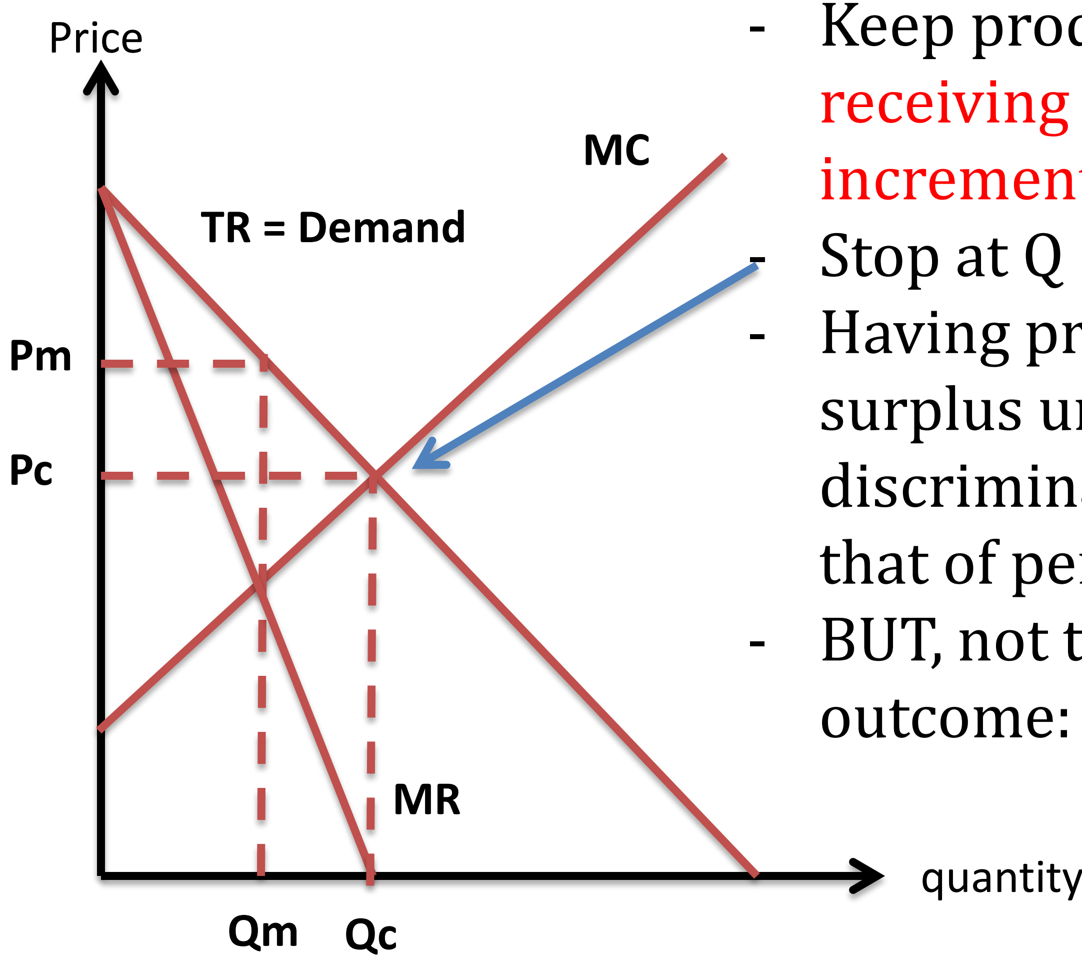
Linear v.s. Non-linear (first-degree)



- Suppose you buy Q = 4.

- Simple Linear price for Buying 4 units is P4 → You pay = 4*P4
- Nonlinear pricing (first-degree)
  - Q=1 , P = P1
  - 2nd Q , P = P2
  - 3rd Q, P = P3 ,
  - 4th Q, P = P4 .... etc..

First-price discrimination and Efficient outcome



- Keep producing as long as receiving price covers incremental cost (MC).
- Stop at Q = Qc (demand = MC)
- Having produced here, total surplus under first-degree price discrimination must be equal to that of perfect competition.
- BUT, not the same distributional outcome: NO consumer surplus

First-price discrimination and Efficient outcome

- Having recycled the example for linear-pricing monopoly problem, monopolist will instead produce at Q = 15 under the first-degree price discrimination, and set price for each Q along the market demand. (decline price as selling more)
- Total surplus would be

$$= \int_{0}^{Q=15} [30 - Q - Q]dQ$$

$$= 30(15) - (15)^2 = 450 - 225 = 175$$

Final exam

- Collects 50%
  - *Likely* to have **4 BIG questions**, allocate time wisely.
  - 3-hour exam. Dec,14th.
- Topics, applications.
  - Multivariate calculus (Easy)
  - Unconstrained optimization
  - Constrained optimization
  - Integration (Moderate)