NAME: VIRAKROTH ONG
ID 6204820028

Assignment 4

1) $\ln C_i = 4.3 - 1.34 \ln P_i + 0.17 \ln Y_i$

We have $\hat{\beta}_1 = 4.30$, $\hat{\beta}_2 = -1.34$, $\hat{\beta}_3 = 0.17$, $n = 46$, $K = 3$

1.1)

- We can say that the estimation results follow the law of demand since we have $\hat{\beta}_2$ = elasticity of demand for cigarettes equal to (-1.34) mean that if the prize of cigarettes increase by 1%. we will have quantities demand of cigarette will drop by 1.34%.

1.2). elasticity of demand for cigarettes with respect to price $\hat{\beta}_2 = -1.34$.

+ test whether it is statistically different from 1

1) state hypothesis

$H_0 : \beta_2 = 1$

$H_a : \beta_2 \neq 1$

2) Calculate test statistic

$$t_{cal}(\beta_2) = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} \quad \begin{cases} \hat{\beta}_2 = -1.34 \\ \beta_2 = 1 \end{cases}$$

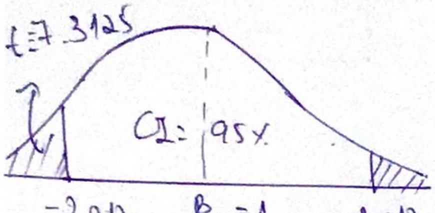
$\sigma_{\hat{\beta}_2} = 0.32$, $d.f = n-k = 46-3 = 43$

$$t_{cal}(\beta_2) = \frac{-1.34-1}{0.32} = -7.3125$$

3) state level of significance : pick $\alpha = 0.05$

$t_{lower} = -2.017$

$t_{upper} = 2.017$



4) conclussion = since the value lie beyond confidence interval. we can be 95% sure that β_2 is different from 1. β_2 is statistically significant.

1.3) the income elasticity of demand for cigarette is $\hat{\beta}_3 = 0.17$.

+ test β_3

1) State hypothesis

$H_0: \beta_3 = 0$

$H_a: \beta_3 \neq 0$

2) Calculate t-stat.

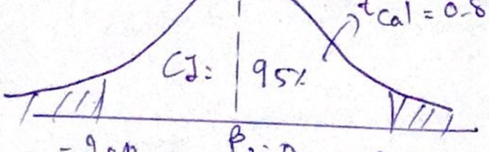
$$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{\sigma_{\hat{\beta}_3}} \quad \begin{cases} \hat{\beta}_3 = 0.17 \\ \beta_3 = 0 \\ \sigma_{\hat{\beta}_3} = 0.2 \end{cases}, \; df = 43$$

$$\Rightarrow t_{cal}(\beta_3) = \frac{0.17 - 0}{0.2} = 0.85_{df.43}$$

3) State level of significant : $\alpha = 0.05$

$t_{lower} = -2.017$

$t_{upper} = 2.017$



4) Conclusion: the t value lie within the (CI). we can not be sure that 95 out of 100 time β_3 equal to zero. fail to reject. β_3 is statistically insignificant.

- We can say that even if people receive lower income. Still there will be cigarette consumption demand since the elasticity of demand for cigarettes with respect to price is lower. As well as TI does not affect people decision much in smoking.

2). 2.1) Test coefficient $\beta_3 < 1$ or not in the first model.

1) State hypothesis

$H_0: \beta_3 \geq 1$

$H_a: \beta_3 < 1$

2) Calculate t-state

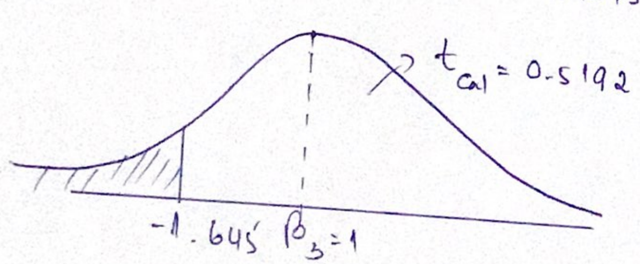
$$t_{cal}(\beta_3) = \frac{\hat{\beta}_3 - \beta_3}{\sigma_{\hat{\beta}_3}} \quad \begin{cases} \hat{\beta}_3 = 1.0307 \\ \beta_3 = 1 \\ \sigma_{\hat{\beta}_3} = 0.05912 \end{cases}$$

$$t_{cal}(\beta_3) = \frac{1.0307 - 1}{0.05912} = 0.5192$$

d.f. 9274

3) State level of significant $\alpha = 0.05$

$t_{lower} = t_{left\ tail} = -1.645$



4) Conclusion: t-value lies within the CI. therefore, we can not say 95% of the time $\beta_3 < 1$. fail to reject. β_3 is not statistically significant.

2.2 test whether we should include age^2 or not.

1) State hypothesis

$H_0 : \beta_4 = 0$

$H_a : \beta_4 \neq 0$

2) Calculate t-stat and F-test

$$t_{cal}(\beta_4) = \frac{\hat{\beta}_4 - \beta_4}{\hat{\sigma}_{\hat{\beta}_4}}$$

$\hat{\beta}_4 = 0.0377221$

$\beta_4 = 0$

$\hat{\sigma}_{\hat{\beta}_4} = 0.0056214$, d.f. 9,274

$$t_{cal}(\beta_4) = \frac{0.0377221 - 0}{0.0056214}$$

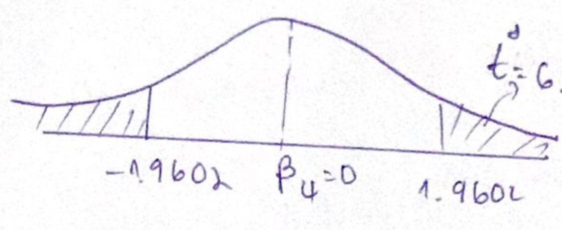
$t_{cal} = 6.71$ d.f. 9274

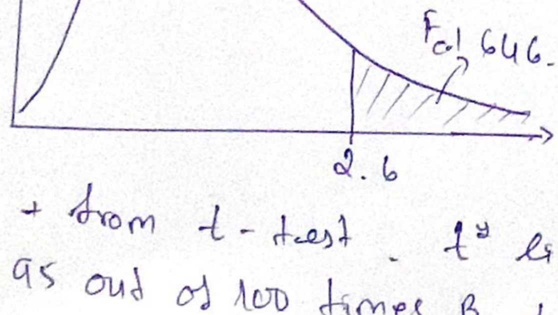
$$F_{cal} = \frac{ESS / d.f}{RSS / d.f} = 646.8 \sim F(3, 9271)$$

3) State level of significant. $\alpha = 0.05$

$t_{lower} = -1.9602$

$t_{upper} = 1.9602$





+ from t-test. t^* lie beyond CI. we can say that 95 out of 100 times $\beta_4 \neq 0$ (reject H_0) $F_{cal} > F_{table}$ β_4 is statistically significant.

+ from F-dist., the $F_{cal} > F_{table}$. We can say that 95% of the time $\beta_4 \neq 0$ (reject H_0). therefore, β_4 is significandly significant

* from both tests (t-test, F test) we should add the age^2 to the model as it is statistically significant and fit to the model.

We have $\hat{\beta}_4 = 0.03772$ mean that the higher level of age of respondent is, the highest weath the individual has. it means that the older they get, the more wealthier they are.

3). 3.1) interpret coefficient estimates

+ $\beta_1 = 11.08$ mean the price of the house even other factors all equal to zero

+ $\beta_2 = -0.9535$ mean that if the level of nitrous oxide in the air of community increase by one percent therefore, median house price will fall by 0.9535%.

+ $\beta_3 = -0.1343$ mean that if the weighted distance of the community increase 1 percent from municipal area. therefore, the median House price will fall by 0.1343%.

+ $\beta_4 = 0.2545$ mean that if the average number of room per house in community increase by 1 unit. therefore, the median house price will be increase by 0.2545%.

+ $\beta_5 = -0.0524$ means that if the average student ratio of school in community increase by 1 unit. therefore the median housing price in community i will be decreased by 0.05245%

3.2) test coefficient estimates

① test β_2

1) Hypothesis test

$H_0: \beta_2 = 0$

$H_a: H_2 \neq 0$

2) test statistic

$t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}}$ $\begin{cases} \hat{\beta}_2 = 0. \\ \beta_2 = 0 \\ \sigma_{\hat{\beta}_2} = 0.1167 \end{cases}$

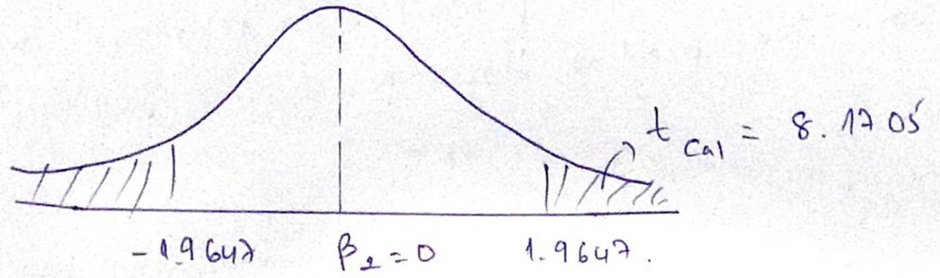
$t_{cal} = \dfrac{0.9535 - 0}{0.1167} = 8.1705$ d.f. 501

3) stat level of significant

$\alpha = 0.05$

$\rightarrow t_{lower} = -1.9647$

$t_{upper} = 1.9647$



4) conclusion. t_{cal} lies beyond the CI. we can reject the null hypothesis. we can say that 95% of the time $\beta_2 \neq 0$. β_2 is statistically significant.

② test β_4

1). test hypothesis

$H_0 : \beta_4 = 0$

$H_a : \beta_4 \neq 0$

2) test statistic

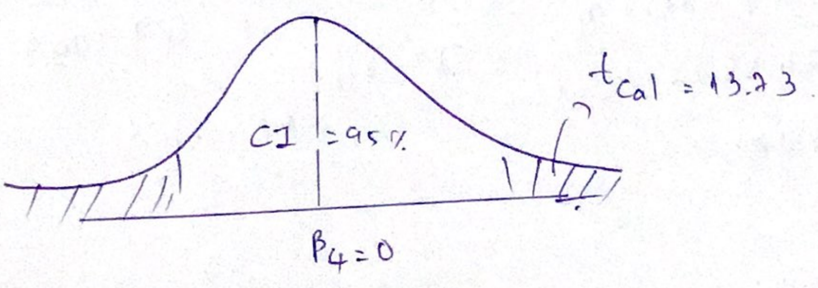
$$t_{cal} = \frac{\hat{\beta}_4 - \beta_4}{\sigma_{\hat{\beta}_4}} \quad \begin{cases} \hat{\beta}_4 = 0.2545 \\ \sigma_{\hat{\beta}_4} = 0.01853 \\ d.f = 501 \end{cases}$$

$$t_{cal} = \frac{0.2545}{0.01853} = \underline{13.73 \quad d.f.501}$$

3) State rule decision. $\alpha = 0.05$

$t_{lower} = -1.9647$

$t_{upper} = 1.9647$



4) conclusion: t_{cal} lies beyond the boundary of CI. we can reject the null hypothesis. we can say that 95% of the time. $\beta_4 \neq 0$. β_4 is statistically significant.

3.3) find R^2 and $\bar{R}^2$

+ $R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{35.1835}{84.5822} = \underline{0.584}$

+ $\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-k} = 1 - (1 - (0.584))\frac{(505)}{(501)}$

$= \underline{-0.5966}$

$$\boxed{\begin{aligned} R^2 &= 0.584 \\ \bar{R}^2 &= -0.5966 \end{aligned}}$$

+ test the joint significance of all the slope

1) state hypothesis

$H_0 : \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_a : otherwise

2) Calculate test statistic

$F_{cal} = \frac{ESS/df}{TSS/df} = \frac{(49.3987)/4}{84.5822/501} = 73.149$, $ESS = TSS - RSS = 84.5822 - 35.1835$

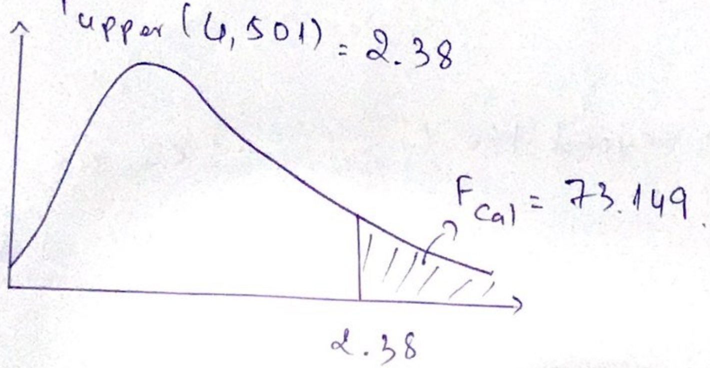
$= 49.3987$

$\sim F(4, 501)$

3) state decision rule

$\alpha = 0.05$

$F_{upper}(4, 501) = 2.38$



4) conclusion : F_{cal} lies beyond the aceptance region. We can reject the null hypothesis. we can 95% sure that all slope are all Joint significance.

3.4) Write restricted model and perform test.

$\ln(P_i) = \beta_1 + \beta_2 \ln(NOX_i) + \beta_3 \ln(Dist_i) + \beta_4 ROOM_i + \beta_5 Strat_i$

given $\beta_2 = \beta_3$

$\Rightarrow \ln(P_i) = \beta_1 + \beta_2 \ln(NOX_i) + \beta_2 \ln(Dist_i) + \beta_4 ROOM_i + \beta_5 Stratd_i$

Restricted model.

$\ln(P_i) = \beta_1 + \beta_2 [\ln(NOX_i \times Dist_i)] + \beta_4 ROOM_i + \beta_5 Stratd_i$

⊕ perform test

1) State hypothesis

$H_0: \beta_2 = \beta_3$ or $\beta_2 - \beta_3 = 0$

$H_a: \beta_2 \neq \beta_3$ or $\beta_2 - \beta_3 \neq 0$

2) Calculate test stat

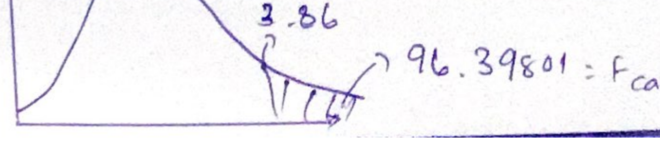
$$F_{cal} = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/n-k} = \frac{TSS_{UR} - ESS_R / m}{RSS/n-k}$$

$$= \frac{41.9532 - 35.1835}{35.1835/501} = 96.39801 \sim F(1,501)$$

3) State rule decision

$\alpha = 0.05$

$F_{upper} = 3.86$



4) conclusion = F_{cal} lie beyond the aceptance region. therefore we can reject the null hypothesis.

4) 4.1) Interpret the coefficient of independent variable in model 1 and 2.

⊕ model 1: $\ln Y_t = 18.27 + 0.536 \ln L_t + 0.024 \ln K_t$

$\hat{\beta}_2 = 0.536$, $\hat{\beta}_3 = 0.024$.

+ $\hat{\beta}_2 = 0.536$ mean that if labour factor is to increase by one percent, therefore, output will be increase by 0.536%.

+ $\hat{\beta}_3 = 0.024$ mean that if Capital factor is to increased by 1 percent. therefore, output will be increase by 0.024%

⊕ model 2: $\ln \left(\frac{Y}{L}\right)_t = 2.13 + 1.12 \ln \left(\frac{K}{L}\right)_t$

+ $\hat{\beta}_2 = 1.12$ mean that if we increase Capital per Labour $\left(\frac{K}{L}\right)$ for 1 more percent. therefore the output per Labour will be increased by 1.12%.

4.2) test hypothesis

1) State hypothesis.

$H_0: \beta_K + \beta_L = 1$

$H_a: \beta_K + \beta_L \neq 1$

2) test statistic

$$F_{cal} = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n - k_{UR})}$$

$RSS_R = 0.0153$

$RSS_{UR} = 0.0124$

$m = 1$

$n - k_{UR} = 30 - 3 = 27$

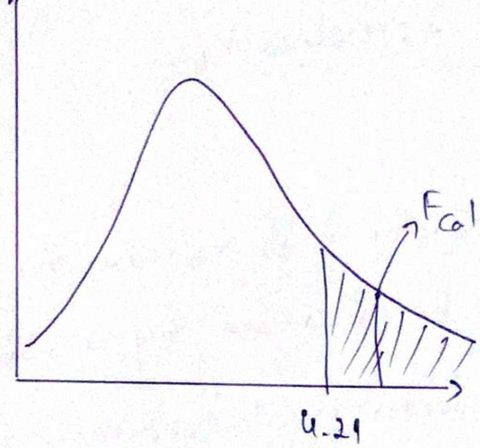
$$= \frac{0.0153 - 0.0124}{0.0124/27}$$

$F_{cal} = 6.31$

3) Pick α and state decision Rules

$\alpha = 0.05$

$F_{upper,\alpha}(m, n-k_{UR}) = F(1,27) = 4.21$



4) conclude the result.

- F_{cal} value lie beyond the confidence interval. we Can reject the null hypothesis at significance level of 95% we can make sure that 95 percent of all times $\beta_L + \beta_K \neq 1$

+ the industrial production function is not characterized by Constant return to scale.

+ the restricted imposed is not Valid $\beta_K + \beta_L \neq 1$

4.3) we Cannot Compare the R^2 Value between the two regression model because. the Variable on the left hand side in both model is not the same.