Midterm Examination: 1/2013 MA217

IMPORTANT NOTE: The same objective function $f(x, y) = x^2y - 2xy^2 + 3xy + 4$ will be used for all questions. In addition, the constraint equations are also very similar. Hence, there are parts that you can take from the questions that you have done previously. State clearly from where you take the repeated information. **Repeating the same calculation will waste your time.** You do not need to do the question sequentially but be aware that there are certain repeated calculations.

1. (a) For a function $f(x, y) = x^2y - 2xy^2 + 3xy + 4$,

   Fide possible critical point(s) and classify them.

   (b) Use the **<u>Envelope theorem</u>** to approximate the relative minimum value when the function changes to $f(x, y) = x^2y - 2xy^2 + 3.1xy + 4$. (You cannot do this.)

(11 marks)

Ans: (0,0) saddle; (-3, 0) saddle; (0, 3/2) saddle and (-1, ½) relative minimum

2. Use information from Question 1(a) if possible and use **<u>Extreme Value theorem</u>** to optimise

   the function $f(x, y) = x^2y - 2xy^2 + 3xy + 4$

   subject to the constraint $y + 4x \le 1$; $y \ge 1$ and $x \ge -2$.

(15 marks)

Ans: *f*(-2, 9) = 310 absolute maximum, *f*(-1/2, 1) = 3.75 absolute minimum, other critical points in the domain (0, 1), (-2, 1), (-0.06, 1.24)

3. (a) Use **<u>Lagrange Multiplier method</u>** to optimise

   the function $f(x, y) = x^2y - 2xy^2 + 3xy + 4$

   subject to the constraint $y + 4x = 1$.

   (b) Estimate the minimum value if the constraint changes to $y + 4x = 0.9$.

(12 marks)

Ans: (a) *f*(-0.06, 1.23) = 3.9458 absolute minimum, *f*(0.15, 0.4) = 4.141 absolute maximum

(b) f^*_{new} = 3.9337

4. Use **Lagrange Multiplier method** to optimise

the function $f(x,y) = x^2 y - 2xy^2 + 3xy + 4$

subject to the constraint $y = 1$ and $y + 4x \le 1$.

(7 marks)

Ans: (a) *f*(0, 1) = 4 absolute maximum, *f*(-1/2, 1) = 3.75 absolute minimum

5. Use information from Question 1 if possible and optimise

the function $f(x,y) = x^2 y - 2xy^2 + 3xy + 4$

subject to the constraint $y \ge 1$ and $x \ge -2$.

(15 marks)

Ans: *f*(0, 3/2) = 4 only 1 critical point satisfy all conditions.

Formulas

We assume that u is a differentiable function of x.

$\frac{d}{dx}(c) = 0$	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$	$\frac{d}{dx}\left(u^n\right) = n\,u^{n-1}\frac{du}{dx}$
$\frac{d}{dx}\left[c\ f(x)\right] = c\,f'(x)$	$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$
$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$	$\frac{d}{dx}\left(e^u\right) = e^u \frac{du}{dx}$
$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(\log_b u) = \frac{1}{(\ln b)u} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$	$\frac{d}{dx}\left(a^u\right) = a^u(\ln a)\frac{du}{dx}$