Math Precourse

SUBJECT: HW 2 - Solutions NO: 1. DATE: / /

p. 550 - 551.

③ The system is:

$$\begin{aligned} x_2 + x_3 + x_4 &= b_1 \\ x_1 + x_3 + x_4 &= b_2 \\ x_1 + x_2 + x_4 &= b_3 \\ x_1 + x_2 + x_3 &= b_4. \end{aligned}$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = \frac{1}{3}(b_1 + b_2 + b_3 + b_4)$$

$$\Rightarrow \quad x_1 = -\frac{2}{3}b + \frac{1}{3}(b_2 + b_3 + b_4) \;;\quad x_2 = -\frac{2}{3}b_2 + \frac{1}{3}(b_1 + b_3 + b_4)$$

$$x_3 = -\frac{2}{3}b_3 + \frac{1}{3}(b_1 + b_2 + b_4) \;;\quad x_4 = -\frac{2}{3}b_4 + \frac{1}{3}(b_1 + b_2 + b_3).$$

⑦ $x = 93.53$; $y \approx 482.11$

$s \approx 49.73$; $c \approx 438.31$.

p.554

② $u = 3$ and $v = -2$.

p.559.

②(i) $\begin{pmatrix} -1 & 15 \\ -6 & -13 \end{pmatrix}$ 2(ii) $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ 2(iii) $C(AB) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

③ $A + B = \begin{pmatrix} 4 & 1 & -7 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{pmatrix}$; $A - B = \begin{pmatrix} -2 & 3 & -5 \\ 1 & -2 & -3 \\ -1 & -1 & -2 \end{pmatrix}$

$AB = \begin{pmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{pmatrix}$; $BA = \begin{pmatrix} 0 & 4 & -9 \\ 19 & 3 & -3 \\ 5 & 1 & -3 \end{pmatrix}$

$(AB)C = A(BC) = \begin{pmatrix} 23 & 8 & 25 \\ 92 & -28 & 76 \\ 4 & -8 & -4 \end{pmatrix}$

p. 559.

4 (a) $\begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$; (b) $\begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

p. 565

3 $(AB)C = A(BC)$

$$\left.\begin{array}{l} (AB)C \\ A(BC) \end{array}\right\} = \begin{pmatrix} a_{11}b_{11}c_{11} + a_{11}b_{12}c_{21} + a_{12}b_{21}c_{11} + a_{12}b_{22}c_{21} & a_{11}b_{11}c_{12} + a_{11}b_{12}c_{22} + a_{12}b_{21}c_{12} + a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11} + a_{21}b_{12}c_{21} + a_{22}b_{21}c_{11} + a_{22}b_{22}c_{22} & a_{21}b_{11}c_{12} + a_{21}b_{12}c_{22} + a_{22}b_{21}c_{12} + a_{22}b_{22}c_{22} \end{pmatrix}$$

5 (a) $\begin{pmatrix} 5 & 3 & 1 \\ 2 & 0 & 9 \\ 1 & 3 & 3 \end{pmatrix}$; (b) $(1, 2, -3)$.

7 (a) Direct verification.

$$A^2 = (a+d)A - (ad-bc)I_2 = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}.$$

(b) $A^2 = 0$ if $a+d=0$ and $ad = bc$.

For example, $A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

p. 567

1 $A' = \begin{pmatrix} 3 & -1 \\ 5 & 2 \\ 8 & 6 \\ 3 & 2 \end{pmatrix}$; $B' = (0, 1, -1, 2)$; $C' = \begin{pmatrix} 1 \\ 5 \\ 0 \\ 1 \end{pmatrix}$

SUBJECT: NO: 2 DATE: / /

p.567

(2) $A' = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$; $B' = \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}$; $(A+B)' = \begin{pmatrix} 3 & 1 \\ 4 & 7 \end{pmatrix}$;

$\alpha A' = \begin{pmatrix} -6 & 2 \\ -4 & -10 \end{pmatrix}$; $AB = \begin{pmatrix} 4 & 10 \\ 10 & 8 \end{pmatrix}$; $(AB)' = B'A' = \begin{pmatrix} 4 & 10 \\ 10 & 8 \end{pmatrix}$

$A'B' = \begin{pmatrix} -2 & 4 \\ 10 & 14 \end{pmatrix}$

(3) Show that $A = A'$ and $B = B'$.

(7) (a) Verify that $\begin{pmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{pmatrix}' \begin{pmatrix} \lambda & 0 & \lambda \\ \lambda & 0 & -\lambda \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} p & q \\ -q & p \end{pmatrix} \begin{pmatrix} p & -q \\ q & p \end{pmatrix} = \begin{pmatrix} p^2+q^2 & 0 \\ 0 & p^2+q^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ iff $p^2+q^2 = 1$.

p.577

(7) $a \cdot a = 5$; $a \cdot b = 2$; $a \cdot (a+b) = 7$.

$\Rightarrow a \cdot a + a \cdot b = 7 = a \cdot (a+b)$.

p.592

(1) (a) 18 ; (b) $= 0$; (c) $(a+b)^2 - (a-b)^2 = 4ab$

(d) $3^t 2^{t-1} - 3^{t-1} 2^t = 6^{t-1}$!

(3) (a) $x = \frac{11}{5}$, $y = \frac{-7}{5}$; (b) $x = 4$, $y = -1$.

(c) $x = \frac{a+2b}{a^2+b^2}$, $y = \frac{2a-b}{a^2+b^2}$, $(a^2+b^2 \neq 0)$.

(4) $|AB| = (a_{11}b_{11} + a_{12}b_{21})(a_{21}b_{12} + a_{22}b_{22}) - (a_{11}b_{12} + a_{12}b_{22})(a_{21}b_{11} + a_{22}b_{21})$

$|A||B| = (a_{11}a_{22} - a_{12}a_{21})(b_{11}b_{22} - b_{12}b_{21})$

Then, expand the above expressions to show that $|AB| = |A||B|$.

(6) $Y - C = I_0 + G_0$

$-bY + C = a$ $\Rightarrow \begin{pmatrix} 1 & -1 \\ -b & 1 \end{pmatrix}\begin{pmatrix} Y \\ C \end{pmatrix} = \begin{pmatrix} I_0 + G_0 \\ a \end{pmatrix}$

$\Rightarrow Y = \dfrac{a + I + G_0}{1-b}$; $C = \dfrac{a + b(I_0 + G_0)}{1-b}$.

p. 596

(1) (a) -2 ; (b) -2 ; (c) adf ; (d) $e(ad-bc)$

(3) (b) $x_1 = x_2 = x_3 = 0$.

p. 603

(1) (a) $AB = \begin{pmatrix} 13 & 16 \\ 29 & 36 \end{pmatrix}$; $BA = \begin{pmatrix} 15 & 22 \\ 23 & 34 \end{pmatrix}$; $A'B' = \begin{pmatrix} 15 & 23 \\ 22 & 34 \end{pmatrix}$; $B'A' = \begin{pmatrix} 13 & 29 \\ 16 & 36 \end{pmatrix}$

(b) $|A| = |A'| = -2$ and $|B| = |B'| = -2$.

$|AB| = 4 = |A| \cdot |B|$ and $|A'B'| = |A'||B'| = 4$

(3) (a) 0 ; b : 0

p. 604

(7) $X'X = \begin{pmatrix} 4 & 3 & 2 \\ 3 & 5 & 1 \\ 2 & 1 & 2 \end{pmatrix}$; $|X'X| = 10$.

p. 613

(2) Multiply the two matrices to get I_3.

p.613

(4) (a) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(7) (a) $AA' = \begin{pmatrix} 21 & 11 \\ 11 & 10 \end{pmatrix}$, $|AA'| = 89$, $(AA')^{-1} = \frac{1}{89}\begin{pmatrix} 10 & -11 \\ -11 & 21 \end{pmatrix}$.

(8) $A^2 = (PDP^{-1})(PDP^{-1}) = PD\overbrace{(P^{-1}P)}^{I}DP^{-1} = PD^2P^{-1}$.

p.614

(11) (a) Let $B = X(X'X)^{-1}X'$. Then, $A^2 = (I_m - B)(I_m - B) = I_m - B - B + B^2$

$B^2 = (X(X'X)^{-1}X')(X(X'X)^{-1}X') = X(X'X)^{-1}X' = B$. $\rightarrow$

$\therefore A^2 = I_m - B - B + B^2 = I_m - B - \cancel{B} + \cancel{B} = I_m - B = A$.

p.617.

(1) (a) $\begin{pmatrix} -5/2 & 3/2 \\ 2 & -1 \end{pmatrix}$; (b) $= \frac{1}{9}\begin{pmatrix} 1 & 4 & 2 \\ 2 & -1 & 4 \\ 4 & -2 & -1 \end{pmatrix}$; (c) The matrix has NO inverse.

p.621

(1) (a) $x = 1$, $y = -2$, $z = 2$.

(2) The determinant of the system $|A|$ $= \begin{vmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{vmatrix} = -10$.

$\therefore |A| \neq 0 \Leftrightarrow$ the solution is unique.

$x_1 = \frac{1}{2}b_1 - \frac{1}{10}b_2 - \frac{1}{5}b_3$

$x_2 = -\frac{1}{2}b_1 + \frac{3}{10}b_2 + \frac{3}{5}b_3$

$x_3 = -\frac{1}{2}b_1 + \frac{7}{10}b_2 + \frac{2}{5}b_3$.