1 (a)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 10 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 4R_1 \to R_2$, $R_3 - 7R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -11 & -7 & 0 & 1 \end{array}\right]$$

$\frac{R_2}{-3} \to R_2$, $\frac{R_3}{-6} \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{11}{6} & \frac{7}{6} & 0 & -\frac{1}{6} \end{array}\right]$$

$R_3 - R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{6} & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \end{array}\right]$$

$R_3 \times (-6) \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$R_2 - 2R_3 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{2}{3} & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$R_1 - 2R_2 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{7}{3} & -22/3 & 4 \\ 0 & 1 & 0 & -2/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$R_1 - 3R_3 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & -4/3 & 1 \\ 0 & 1 & 0 & -2/3 & 11/3 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} -2/3 & -4/3 & 1 \\ -2/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

1 (a)

$$Ax = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$A^{-1}Ax = A^{-1}\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$x = \underbrace{\begin{bmatrix} -2/3 & -4/3 & 1 \\ -2/3 & 11/3 & -2 \\ 1 & -2 & 1 \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}}_{3\times 1}$$

3×1

$$= \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

---

1 (b)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 4R_1 \to R_2$
$R_3 - 7R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -12 & -7 & 0 & 1 \end{array}\right]$$

$\frac{R_2}{-3} \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & -6 & -12 & -7 & 0 & 1 \end{array}\right]$$

$R_3 + 6R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4/3 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right] \implies \text{No inverse}$$

$\therefore \det \underline{A} = 0$

1 (c) MA217 Mock Ex Soln 3

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{array}\right]$$

$R_2 - 4R_1 \to R_2$

$R_3 - 7R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0 & -6 & -12 & 6 \end{array}\right]$$

$\frac{R_2}{(-3)} \to R_2$

~~$R_3 + 6R_2 \to R_3$~~

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -6 & -12 & 6 \end{array}\right]$$

$R_3 + 6R_2 \to R_3$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 0 \\ 0 & \boxed{1} & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ x_3 is a free variable $= C$

$x_2 + 2x_3 = -1$

$x_2 = -1 - 2x_3 = -1 - 2C$ —— (1)

$x_1 + 2x_2 + 3x_3 = 0$

$x_1 = -2x_2 - 3x_3$

$= -2(-1-2C) - 3C$

$= 2 + 4C - 3C$

$x_1 = 2 + C$ —— (2)

$$\therefore \underline{x} = \begin{bmatrix} 2 + C \\ -1 + (-2)C \\ 0 + C \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + C\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \text{ where } C \in \mathbb{R}$$

1 (d)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & D \end{array}\right] \underset{\substack{R_2-4R_1 \\ R_3-7R_1}}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0 & -6 & -12 & D \end{array}\right]$$

$$\frac{R_2}{(-3)} \to R_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -6 & -12 & D \end{array}\right] \underset{\substack{R_3+6R_2 \\ \to R_3}}{\Rightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & D-6 \end{array}\right]$$

The 3rd row is zero row so there will be a solution only if $D-6=0$

consistent when $D=6$

inconsistent when $D \neq 6$

---

2 x_1 = no of portfolio A

x_2 = " B

x_3 = " C

$$6x_1 + x_2 + 3x_3 = 35$$
$$3x_1 + 2x_2 + 3x_3 = 22$$
$$x_1 + 5x_2 + 3x_3 = 18$$

(a)

$$\left[\begin{array}{ccc|c} 6 & 1 & 3 & 35 \\ 3 & 2 & 3 & 22 \\ 1 & 5 & 3 & 18 \end{array}\right]$$

MA217 Mock Solution

5

(b)

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 3 & 2 & 3 & 22 \\ 6 & 1 & 3 & 35 \end{array}\right]$$

$R_2 - 3R_1 \to R_2$

$R_3 - 6R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & -13 & -6 & -32 \\ 0 & -29 & -15 & -73 \end{array}\right]$$

$\frac{R_2}{-13} \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & -29 & -15 & -73 \end{array}\right]$$

$R_3 + 29R_2 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & 0 & -15 + \frac{174}{13} & -73 + 928/13 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 18 \\ 0 & 1 & 6/13 & 32/13 \\ 0 & 0 & \frac{-21}{13} & \frac{-21}{13} \end{array}\right]$$

$\therefore$ A = 5

B = 2

C = 1

$$-\frac{21}{13} x_3 = -\frac{21}{13} \Rightarrow x_3 = 1$$

$$x_2 + \frac{6}{13}(x_3) = \frac{32}{13}$$

$$x_2 = \frac{32}{13} - \frac{6}{13}(1) = \frac{26}{13} = 2$$

$$x_1 + 5x_2 + 3x_3 = 18$$

$$x_1 = 18 - 5(2) - 3(1) = 5$$

3 $|B| = |B^T|$

$$= \begin{vmatrix} 2a & b & c & 0 \\ c & a+d & 0 & c \\ b & 0 & a+d & b \\ 0 & b & c & 2d \end{vmatrix}$$

want to have more zeros

$$R_1 - R_4 \to R_1 \Rightarrow \begin{vmatrix} 2a & 0 & 0 & -2d \\ c & a+d & 0 & c \\ b & 0 & a+d & b \\ 0 & b & c & 2d \end{vmatrix}$$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ -2d & c & b & 2d \end{vmatrix}$$

only do replacement operation which does not affect the value of determinant in this case.

$$R_4 + \frac{d}{a} R_1 \to R_4 = \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & \frac{ca+dc}{a} & \frac{ab+db}{a} & 2d \end{vmatrix}$$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & \frac{c}{a}(a+d) & \frac{b}{a}(a+d) & 2d \end{vmatrix}$$

3

$R_4 - \frac{c}{a}R_2 \to R_4$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & 0 & \frac{b}{a}(a+d) & \frac{2ad-cb}{a} \end{vmatrix}$$

$R_4 - \frac{b}{a}R_3$

$$= \begin{vmatrix} 2a & c & b & 0 \\ 0 & a+d & 0 & b \\ 0 & 0 & a+d & c \\ 0 & 0 & 0 & \frac{2ad-2bc}{a} \end{vmatrix} \qquad \begin{bmatrix} + & - & + & - \end{bmatrix}$$

$$|B| = (2a) \begin{vmatrix} a+d & 0 & b \\ 0 & a+d & c \\ 0 & 0 & \frac{2ad-2bc}{a} \end{vmatrix}$$

$$= (2a)(a+d) \begin{vmatrix} a+d & c \\ 0 & \frac{2(ad-bc)}{a} \end{vmatrix}$$

$$= (2a)(a+d)\left[(a+d)\,\frac{2(ad-bc)}{a} - c(0)\right]$$

$$|B| = (2a)(a+d)(a+d)(2)\frac{(ad-bc)}{a}$$

$$= 4(a+d)^2(ad-bc)$$

$$|C| = \begin{vmatrix} c & a+b & 0 & d \\ 0 & b & a & c+d \\ 0 & 0 & a+c & 2a \\ 0 & 0 & 0 & d+b \end{vmatrix}$$

$$= c\begin{vmatrix} b & a & c+d \\ 0 & a+c & 2a \\ 0 & 0 & d+b \end{vmatrix}$$

$$= cb\begin{vmatrix} a+c & 2a \\ 0 & d+b \end{vmatrix} =$$

$$= cb\left[(a+c)(d+b) - 2a(0)\right]$$

$$= cb(a+c)(d+b)$$

$|D|$ is undefined as det is a property of a square matrix

3 (b) $\det B = 0$

$$4(a+d)^2(ad-bc) = 0$$

$\therefore$ either $a+d=0$ or $ad-bc=0$

$a=-d$ or $ad=bc$

(c) $a=1,\ b=2,\ c=1,\ d=0$

$$|B| = 4(1+0)^2(0-(2)(1)) = -8$$

$$\det(2B^2) = 2^4|B||B| = 16(-8)(-8) = 1024$$

$$|C| = (1)(2)(1+1)(0+2) = 2(2)(2) = 8$$

(d) $$|AC^{-1}|^T = |AC^{-1}| = |A||C^{-1}| = \frac{|A|}{|C|}$$

$$|C^{\frac{1}{3}}B^{-1}| = \frac{|C|^{1/3}}{|B|} = \frac{2}{(-8)} = \frac{|A|}{8}$$

$$|A| = -2$$

Mock

MA 217 Mock

19:54

19:57

10

4.

$$\int_{y=2}^{y=3}\int_{x=1}^{x=2} 2xy \, dx \, dy$$

$$= \int_{y=2}^{y=3} \left[2y\frac{x^2}{2} \right]_{x=1}^{x=2} dy$$

$$= \int_{y=2}^{y=3} \left[yx^2 \right]_{x=1}^{x=2} dy$$

$$= \int_{y=2}^{y=3} y\underbrace{(4-1)}_{} \, dy \quad \text{// } 3y$$

$$= \left[\frac{3y^2}{2} \right]_{y=2}^{y=3}$$

$$= 1.5(9-4) = 7.5 \text{ unit}^3$$

5 Average output $= \dfrac{1}{(12-10)(3200-2800)} \iint 50K^{3/5} L^{2/5}\, dA$

$$= \frac{1}{800} \int_{2800}^{3200} \int_{10}^{12} 50K^{3/5} L^{2/5}\, dK\, dL$$

$$= \frac{1}{800} \int_{2800}^{3200} 50 L^{2/5} \left[\frac{5}{8} K^{8/5} \right]_{K=10}^{K=12} dL$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right) \int_{2800}^{3200} L^{2/5} \left(12^{8/5} - 10^{8/5}\right) dL$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right)\left(12^{8/5} - 10^{8/5}\right) \left[\frac{5}{7} L^{7/5} \right]_{K=2800}^{L=3200}$$

$$= \frac{1}{800} (50)\left(\frac{5}{8}\right)\left(\frac{5}{7}\right)\left(12^{8/5} - 10^{8/5}\right)\left[(3200)^{7/5} - (2800)^{7/5}\right]$$

≈ 5181 units