

15. First let $u = (\ln x)^2$, $dv = dx \Rightarrow du = 2 \ln x \cdot \frac{1}{x} dx$, $v = x$. Then by Equation 2,

$$I = \int (\ln x)^2 dx = x(\ln x)^2 - 2 \int x \ln x \cdot \frac{1}{x} dx = x(\ln x)^2 - 2 \int \ln x dx. \text{ Next let } U = \ln x, dV = dx \Rightarrow dU = 1/x dx, V = x \text{ to get } \int \ln x dx = x \ln x - \int x \cdot (1/x) dx = x \ln x - \int dx = x \ln x - x + C_1. \text{ Thus, } I = x(\ln x)^2 - 2(x \ln x - x + C_1) = x(\ln x)^2 - 2x \ln x + 2x + C, \text{ where } C = -2C_1.$$

19. First let $u = z^3$, $dv = e^z dz \Rightarrow du = 3z^2 dz$, $v = e^z$. Then $I_1 = \int z^3 e^z dz = z^3 e^z - 3 \int z^2 e^z dz$. Next let $u_1 = z^2$, $dv_1 = e^z dz \Rightarrow du_1 = 2z dz$, $v_1 = e^z$. Then $I_2 = \int z^2 e^z dz = z^2 e^z - 2 \int z e^z dz$. Finally, let $u_2 = z$, $dv_2 = e^z dz \Rightarrow du_2 = dz$, $v_2 = e^z$. Then $\int z e^z dz = z e^z - \int e^z dz = z e^z - e^z + C_1$. Substituting in the expression for I_2 , we get $I_2 = z^2 e^z - 2(z e^z - e^z + C_1) = z^2 e^z - 2z e^z + 2e^z - 2C_1$. Substituting the last expression for I_2 into I_1 gives $I_1 = z^3 e^z - 3(z^2 e^z - 2z e^z + 2e^z - 2C_1) = z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z + C$, where $C = 6C_1$.

32. Let $u = (\ln x)^2$, $dv = x^{-3} dx \Rightarrow du = \frac{2 \ln x}{x} dx$, $v = -\frac{1}{2} x^{-2}$. Then

$$I = \int_1^2 \frac{(\ln x)^2}{x^3} dx = \left[-\frac{(\ln x)^2}{2x^2} \right]_1^2 + \int_1^2 \frac{\ln x}{x^3} dx. \text{ Now let } U = \ln x, dV = x^{-3} dx \Rightarrow dU = \frac{1}{x} dx, V = -\frac{1}{2} x^{-2}.$$

Then

$$\int_1^2 \frac{\ln x}{x^3} dx = \left[-\frac{\ln x}{2x^2} \right]_1^2 + \frac{1}{2} \int_1^2 x^{-3} dx = -\frac{1}{8} \ln 2 + 0 + \frac{1}{2} \left[-\frac{1}{2x^2} \right]_1^2 = -\frac{1}{8} \ln 2 + \frac{1}{2} \left(-\frac{1}{8} + \frac{1}{2} \right) = \frac{3}{16} - \frac{1}{8} \ln 2.$$

$$\text{Thus } I = \left(-\frac{1}{8} (\ln 2)^2 + 0 \right) + \left(\frac{3}{16} - \frac{1}{8} \ln 2 \right) = -\frac{1}{8} (\ln 2)^2 - \frac{1}{8} \ln 2 + \frac{3}{16}.$$

37. Let $y = \sqrt{x}$, so that $dy = \frac{1}{2} x^{-1/2} dx = \frac{1}{2\sqrt{x}} dx = \frac{1}{2y} dx$. Thus, $\int \cos \sqrt{x} dx = \int \cos y (2y dy) = 2 \int y \cos y dy$. Now

use parts with $u = y$, $dv = \cos y dy$, $du = dy$, $v = \sin y$ to get $\int y \cos y dy = y \sin y - \int \sin y dy = y \sin y + \cos y + C_1$,

$$\text{so } \int \cos \sqrt{x} dx = 2y \sin y + 2 \cos y + C = 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C.$$

10. $\frac{1}{(t+4)(t-1)} = \frac{A}{t+4} + \frac{B}{t-1} \Rightarrow 1 = A(t-1) + B(t+4).$

$$t = 1 \Rightarrow 1 = 5B \Rightarrow B = \frac{1}{5}. t = -4 \Rightarrow 1 = -5A \Rightarrow A = -\frac{1}{5}. \text{ Thus,}$$

$$\int \frac{1}{(t+4)(t-1)} dt = \int \left(\frac{-1/5}{t+4} + \frac{1/5}{t-1} \right) dt = -\frac{1}{5} \ln |t+4| + \frac{1}{5} \ln |t-1| + C \quad \text{or} \quad \frac{1}{5} \ln \left| \frac{t-1}{t+4} \right| + C$$

15. $\frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} = 1 + \frac{-4}{x^2(x-2)}$. Write $\frac{-4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$. Multiplying both sides by $x^2(x-2)$ gives

$-4 = Ax(x-2) + B(x-2) + Cx^2$. Substituting 0 for x gives $-4 = -2B \Leftrightarrow B = 2$. Substituting 2 for x gives

$-4 = 4C \Leftrightarrow C = -1$. Equating coefficients of x^2 , we get $0 = A + C$, so $A = 1$. Thus,

$$\begin{aligned}\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} dx &= \int_3^4 \left(1 + \frac{1}{x} + \frac{2}{x^2} - \frac{1}{x-2} \right) dx = \left[x + \ln|x| - \frac{2}{x} - \ln|x-2| \right]_3^4 \\ &= \left[\left(4 + \ln 4 - \frac{1}{2} - \ln 2 \right) - \left(3 + \ln 3 - \frac{2}{3} - 0 \right) \right] = \frac{7}{6} + \ln \frac{2}{3}\end{aligned}$$

24. $\frac{x^2 - x + 6}{x^3 + 3x} = \frac{x^2 - x + 6}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$. Multiply by $x(x^2 + 3)$ to get $x^2 - x + 6 = A(x^2 + 3) + (Bx + C)x$.

Substituting 0 for x gives $6 = 3A \Leftrightarrow A = 2$. The coefficients of the x^2 -terms must be equal, so $1 = A + B \Rightarrow$

$B = 1 - 2 = -1$. The coefficients of the x -terms must be equal, so $-1 = C$. Thus,

$$\begin{aligned}\int \frac{x^2 - x + 6}{x^3 + 3x} dx &= \int \left(\frac{2}{x} + \frac{-x-1}{x^2+3} \right) dx = \int \left(\frac{2}{x} - \frac{x}{x^2+3} - \frac{1}{x^2+3} \right) dx \\ &= 2 \ln|x| - \frac{1}{2} \ln(x^2 + 3) - \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C\end{aligned}$$

$$38. \frac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2} = \frac{Ax + B}{x^2 + 2x + 2} + \frac{Cx + D}{(x^2 + 2x + 2)^2} \Rightarrow$$

$$x^3 + 2x^2 + 3x - 2 = (Ax + B)(x^2 + 2x + 2) + Cx + D \Rightarrow$$

$$x^3 + 2x^2 + 3x - 2 = Ax^3 + (2A + B)x^2 + (2A + 2B + C)x + 2B + D.$$

$$\text{So } A = 1, 2A + B = 2 \Rightarrow B = 0, 2A + 2B + C = 3 \Rightarrow C = 1, \text{ and } 2B + D = -2 \Rightarrow D = -2. \text{ Thus,}$$

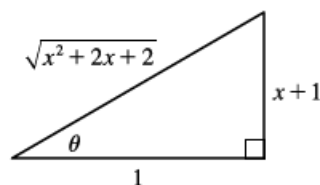
$$\begin{aligned} I &= \int \frac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2} dx = \int \left(\frac{x}{x^2 + 2x + 2} + \frac{x - 2}{(x^2 + 2x + 2)^2} \right) dx \\ &= \int \frac{x + 1}{x^2 + 2x + 2} dx + \int \frac{-1}{x^2 + 2x + 2} dx + \int \frac{x + 1}{(x^2 + 2x + 2)^2} dx + \int \frac{-3}{(x^2 + 2x + 2)^2} dx \\ &= I_1 + I_2 + I_3 + I_4. \end{aligned}$$

$$I_1 = \int \frac{x + 1}{x^2 + 2x + 2} dx = \int \frac{1}{u} \left(\frac{1}{2} du \right) \quad \left[\begin{array}{l} u = x^2 + 2x + 2, \\ du = 2(x + 1) dx \end{array} \right] = \frac{1}{2} \ln |x^2 + 2x + 2| + C_1$$

$$I_2 = - \int \frac{1}{(x + 1)^2 + 1} dx = - \frac{1}{1} \tan^{-1} \left(\frac{x + 1}{1} \right) + C_2 = - \tan^{-1}(x + 1) + C_2$$

$$I_3 = \int \frac{x + 1}{(x^2 + 2x + 2)^2} dx = \int \frac{1}{u^2} \left(\frac{1}{2} du \right) = - \frac{1}{2u} + C_3 = - \frac{1}{2(x^2 + 2x + 2)} + C_3$$

$$\begin{aligned} I_4 &= -3 \int \frac{1}{[(x + 1)^2 + 1]^2} dx = -3 \int \frac{1}{(\tan^2 \theta + 1)^2} \sec^2 \theta d\theta \quad \left[\begin{array}{l} x + 1 = \tan \theta, \\ dx = \sec^2 \theta d\theta \end{array} \right] \\ &= -3 \int \frac{1}{\sec^2 \theta} d\theta = -3 \int \cos^2 \theta d\theta = -\frac{3}{2} \int (1 + \cos 2\theta) d\theta \\ &= -\frac{3}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) + C_4 = -\frac{3}{2} \theta - \frac{3}{2} \left(\frac{1}{2} \cdot 2 \sin \theta \cos \theta \right) + C_4 \\ &= -\frac{3}{2} \tan^{-1} \left(\frac{x + 1}{1} \right) - \frac{3}{2} \cdot \frac{x + 1}{\sqrt{x^2 + 2x + 2}} \cdot \frac{1}{\sqrt{x^2 + 2x + 2}} + C_4 \\ &= -\frac{3}{2} \tan^{-1}(x + 1) - \frac{3(x + 1)}{2(x^2 + 2x + 2)} + C_4 \end{aligned}$$



$$\text{So } I = I_1 + I_2 + I_3 + I_4 \quad [C = C_1 + C_2 + C_3 + C_4]$$

$$\begin{aligned} &= \frac{1}{2} \ln(x^2 + 2x + 2) - \tan^{-1}(x + 1) - \frac{1}{2(x^2 + 2x + 2)} - \frac{3}{2} \tan^{-1}(x + 1) - \frac{3(x + 1)}{2(x^2 + 2x + 2)} + C \\ &= \frac{1}{2} \ln(x^2 + 2x + 2) - \frac{5}{2} \tan^{-1}(x + 1) - \frac{3x + 4}{2(x^2 + 2x + 2)} + C \end{aligned}$$

$$8. \int_1^{\infty} \frac{1}{(2x + 1)^3} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{(2x + 1)^3} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{4(2x + 1)^2} \right]_1^t = \lim_{t \rightarrow \infty} \left[-\frac{1}{4(2t + 1)^2} + \frac{1}{36} \right] = 0 + \frac{1}{36}.$$

Convergent

12. $I = \int_{-\infty}^{\infty} (y^3 - 3y^2) dy = I_1 + I_2 = \int_{-\infty}^0 (y^3 - 3y^2) dy + \int_0^{\infty} (y^3 - 3y^2) dy$, but

$$I_1 = \lim_{t \rightarrow -\infty} \left[\frac{1}{4}y^4 - y^3 \right]_t^0 = \lim_{t \rightarrow -\infty} (t^3 - \frac{1}{4}t^4) = -\infty. \text{ Since } I_1 \text{ is divergent, } I \text{ is divergent,}$$

and there is no need to evaluate I_2 . Divergent

32. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} \int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} [\sin^{-1} x]_0^t = \lim_{t \rightarrow 1^-} \sin^{-1} t = \frac{\pi}{2}$. Convergent

34. There is an infinite discontinuity at $w = 2$.

$$\int_0^2 \frac{w}{w-2} dw = \lim_{t \rightarrow 2^-} \int_0^t \left(1 + \frac{2}{w-2} \right) dw = \lim_{t \rightarrow 2^-} \left[w + 2 \ln |w-2| \right]_0^t = \lim_{t \rightarrow 2^-} (t + 2 \ln |t-2| - 2 \ln 2) = -\infty, \text{ so}$$

$$\int_0^2 \frac{w}{w-2} dw \text{ diverges, and hence, } \int_0^5 \frac{w}{w-2} dw \text{ diverges.} \quad \text{Divergent}$$