EE431 Economics of Financial Markets and Institutions, 2/2014

Problem Sets 9 : Bank Run, Systemic Risk and Deposit Insurance

Please submit at the BE office, 5th floor department of Economics building.

1. The Diamond-Dybvig model of bank runs

   Consider an economy where

   - $U(C) = 1 - \frac{1}{C}$; U(C) is the utility function and C is consumption.
   - There are 3 dates, date 0, date 1 and date 2.
   - "Primary investment" costs 1 in date 0, and yields 1 if cashed in date 1, 1.44 if cashed at date 2.
   - "Type 1" investors are those who need to consume at 1.
   - "Type 2" investors are those who can choose to consumer at 2.
   - At date 0, an investor does not know when he/she needs to consume, but each investors has a probability of 0.2 of being type 1 and 0.8 of being type 2.
   - Total number of investors is 100. Suppose that the number of investors is large enough for the law of large number to hold.
   - (The other settings are the same as The Diamond-Dybvig (2007), which are as follows. Each investor begins with 1 to invest on date 0. Investors face an uncertain horizon to hold the asset. Each will need to consume either at date 1 or 2. As of date 0, an investor does not know at which date he/she will need to consume. Each investor's type is his/her own private information.)

   Answer the following questions. Calculate to four decimal places.

(a) The bank offers a deposit contract in which 1 unit invested at date 0, it will worth 1.386 at date 2, but only 1.15 at date 1.

i. Why do investors would prefer the deposit contract to the primary investment? Explain economic reasons and show numerically.

ii. Show numerically that the bank is able to offer the deposit contract (liquid asset), even though the bank invests in the primary investment (illiquid asset).

ANSWER

(i) Investors are facing with liquidity risk. They do not know which type they will be at date 0. Therefore, investors face an uncertain horizon to hold the asset. At date 0, they need to make their investment decision based on their expected utility.

$$\begin{aligned} EU = & [\text{probability of being type } 1 \times \text{utility gained from being type } 1] \\ & + [\text{probability of being type } 2 \times \text{utility gained from being type } 2] \end{aligned}$$

If an investor decide to invest in **the primary investment**, then the investor will have 1 to consume at date 1 if he/she is type 1 (with probability 0.2) and the investor will have 1.44 to consume at date 2 if he/she is type 2 at date 2. Therefore, the investor's expected utility is as follows.

$$\begin{aligned} EU_{\text{primary investment}} &= \text{prob.being type } 1 \times U(C_1) \\ &\quad + \text{prob.being type } 2 \times U(C_2) \\ &= 0.2 \times \left[1 - \frac{1}{1}\right] + 0.8 \times \left[1 - \frac{1}{1.44}\right] \\ &= 0.2444 \end{aligned}$$

If an investor decide to deposit into the bank or invest in **the deposit contract**, then the investor will have 1.15 to consume at date 1 if he/she is type 1 (with probability 0.2) and the investor will have 1.386 to consume at date 2 if he/she is type 2 at date 2. Therefore, the investor's expected utility is as follows.

$$\begin{aligned} EU_{\text{deposit contract}} &= \text{prob.being type } 1 \times U(C_1) \\ &\quad + \text{prob.being type } 2 \times U(C_2) \\ &= 0.2 \times \left[1 - \frac{1}{1.15}\right] + 0.8 \times \left[1 - \frac{1}{1.386}\right] \\ &= 0.2489 \end{aligned}$$

Expected utility gained from deposit contract is **higher than** expected utility gained from primary investment. Therefore the investor would prefer deposit contract to primary investment.

$$\text{Expected Payoff} = \sum_i p_i R_i,$$

where p_i is probability of situation i and R_i is payoff of situation i.

Expected payoff on the primary investment and on the deposit contract are as follows.

$$\begin{aligned} \text{Expected Payoff}_{\text{Primary Investment}} &= (0.2 \times 1) + (0.8 \times 1.44) \\ &= 1.3520 \end{aligned}$$

$$\begin{aligned} \text{Expected Payoff}_{\text{Primary Investment}} &= (0.2 \times 1.15) + (0.8 \times 1.386) \\ &= 1.3010 \end{aligned}$$

"Each investor prefers the more liquid asset. A **risk-averse** investor prefers this smoother pattern of returns; holding the illiquid asset is risky because it delivers a low amount when liquidated early, on date 1. Note that **if investors were not risk averse** and had constant marginal utility of consumption, **they would not prefer this particular liquid asset**."

(ii) Bank is able to offer the deposit contract (liquid asset), though the bank invests in the primary investment (illiquid asset).
If the bank receives $1 from each of the 100 investors, it receives $100 in deposits on date T = 0. If the bank invests in the illiquid asset, it will need to liquidate some of the illiquid asset at T = 1 to pay 1.15 to those who withdraw. At T = 1, the bank's entire portfolio isworth $100. Suppose 20 depositors withdraw 1.15 each, then 25(1.15) = 23 assets must be liquidated: 23 percent of the portfolio must be liquidated. If 23 assets are liquidated, then 77 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 80 depositors, each will receive

$$\begin{aligned} \frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} &= \frac{77 \times 1.44}{80} \\ &= 1.386 \end{aligned}$$

Depositors prefer the more liquid asset to the illiquid asset. A bank can provide the more liquid deposit which has a smaller loss from early liquidation than is available from holding the illiquid assets directly. This liquidity transformation service is one of the most important functions of banks. If the bank offers the more liquid deposits and invests in the illiquid assets, it can create liquidity. It is an equilibrium (a Nash equilibrium) for 20 depositors to withdraw at T = 1, because if all depositors expect 20 to withdraw at T = 1, only type 1 depositors will withdraw because the 80 type 2 depositors prefer the 1.386 available at T = 2 to the 1.15 available at T = 1.

(b) Explain and show numerically what action investors will take in the following situation,

i. Suppose that all depositors forecast that 30 depositors will withdraw at date 1.

ii. Suppose that all depositors forecast that 65 depositors will withdraw at date 1.
In each situation, would a bank run occur and how many depositors actually withdraw given their forecast? Explain.

ANSWER

i. Suppose that all depositors forecast that 30 depositors will withdraw at date 1. Then, if 30 depositors withdraw 1.15 each, then 30(1.15) = 34.5 assets must be liquidated: 34.5 percent of the portfolio must be liquidated. If 34.5 assets are liquidated, then 65.5 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 70 depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{65.5 \times 1.44}{70}$$

$$= 1.347$$

If all depositors expect 20 to withdraw at T = 1, only type 1 depositors will withdraw at date 1 because the 80 type 2 depositors prefer the 1.347 available at T = 2 to the 1.15 available at T = 1. Therefore, a bank run would not occur.

ii. Suppose that all depositors forecast that 65 depositors will withdraw at date 1. Then, if 65 depositors withdraw 1.15 each, then 65(1.15) = 74.75 assets must be liquidated: 74.75 percent of the portfolio must be liquidated. If 74.75 assets are liquidated, then 25.25 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 35 depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{25.25 \times 1.44}{35}$$

$$= 1.039$$

If all depositors expect 65 to withdraw at T = 1, all depositors will withdraw at date 1 because the 80 type 2 depositors prefer the 1.15 available at T = 1 to the 1.039 available at T = 2 to . Therefore, a bank run would occur at T = 1.

(c) Suppose that all depositors forecast that w depositors will withdraw. What is the minimum value of w, for which a bank run occurs?

ANSWER.

Consider how much is left to pay depositors whowait until date 2 to withdrawif a fraction w of initial depositors withdraw at date 1. [In stead of w, you may use $\hat{f}$.] If all depositors forecast that fraction w depositors will withdraw at date 1, they will be indifferent between to withdraw at date 1 and to witdraw at date 2. This means that they would receive the same amount regardless of timing of withdrawal.

Suppose that all depositors forecast that fraction w depositors will withdraw at date 1. Then, if fraction w depositors withdraw 1.15 each, then 1.15(100w) assets must be liquidated: 1.15(100w) percent of the portfolio must be liquidated. If fraction of assets are liquidated, then (100-1.15(100w)) percent of asset will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain (1-w100) depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{(100 - 1.15(100w) \times 1.44}{100 - w(100)}$$

$$1.15 = \frac{(100 - 1.15(100w) \times 1.44}{100 - w(100)}$$

$$w = 0.5731$$

A bank run would occur if all depositors forecast that more than 57.31 depositors withdraw at date 1. A bank run would not occur if all depositors forecast that less than 57.31 depositors withdraw at date 1.