Quiz 2

1. Let $D = \{-1, 0, 1\}$ and $E = \{0, 1, 2\}$. Consider the statement

$$\exists y \in E, \forall x \in D \text{ such that } (xy \geq 0) \rightarrow (x \geq 0) \wedge (y > 0).$$

   (a) Write the negation for the above statement (without using negation symbol "∼" in the final answer).

   (b) Determine the truth value of the above statement. Explain your answer.

Answer First, using the order of the logical operations: we have $p \rightarrow q \wedge r \equiv p \rightarrow (q \wedge r)$.
(a) Negation:

$$
\begin{aligned}
& \sim (\exists y \in E, \forall x \in D, \text{ such that } (xy \geq 0) \rightarrow (x \geq 0) \wedge (y > 0)) \\
\equiv\ & \forall y \in E, \sim (\forall x \in D \text{ such that } (xy \geq 0) \rightarrow (x \geq 0) \wedge (y > 0)) \\
\equiv\ & \forall y \in E, , \exists x \in D \text{ such that } \sim ((xy \geq 0) \rightarrow (x \geq 0) \wedge (y > 0)) \\
\equiv\ & \forall y \in E, , \exists x \in D \text{ such that } (xy \geq 0) \wedge \sim ((x \geq 0) \wedge (y > 0)) \\
\equiv\ & \forall y \in E, , \exists x \in D \text{ such that } (xy \geq 0) \wedge (\sim (x \geq 0) \vee \sim (y > 0)) \\
\equiv\ & \forall y \in E, , \exists x \in D \text{ such that } (xy \geq 0) \wedge ((x < 0) \vee (y \leq 0))
\end{aligned}
$$

(b) This statement is **true** because there is $y \in E$ such that the statement is false for all $x \in D$. We can show that this is true by using the following case(I) or case(II). NOTE: Using **only one** of these would be enough.

(I) When $y = 1$, the statement is **always true** for all $x \in D$.

- For $x = -1$: $(xy \geq 0)$ is false and $(x \geq 0) \wedge (y > 0)$ is false, i.e.$F \rightarrow F$.
- For $x = 0$: $(xy \geq 0)$ is true and $(x \geq 0) \wedge (y > 0)$ is true, i.e. $T \rightarrow T$.
- For $x = 1$: $(xy \geq 0)$ is true and $(x \geq 0) \wedge (y > 0)$ is true, i.e. $T \rightarrow T$.

(II) When $y = 2$, the statement is **always true** for all $x \in D$.

- For $x = -1$: $(xy \geq 0)$ is false and $(x \geq 0) \wedge (y > 0)$ is false, i.e.$F \rightarrow F$.
- For $x = 0$: $(xy \geq 0)$ is true and $(x \geq 0) \wedge (y > 0)$ is true, i.e. $T \rightarrow T$.
- For $x = 1$: $(xy \geq 0)$ is true and $(x \geq 0) \wedge (y > 0)$ is true, i.e. $T \rightarrow T$.

By using (I) or (II), there exists $y \in D$ ($y = 1$ or 2), such that all values of $x = -1, 0, 1$ make the statement true. Hence the truth value is true.

[Note that when $y = 0$, the statement is false for all $x = -1, 0, 1$ because the hypothesis$(xy \geq 0)$ is always true and the conclusion $(x \geq 0) \wedge (y > 0)$ is always false.]