EE 462 Development Macroeconomics (1 / 2012)

Lecture 6 Endogenous Growth Theory

Read Chapter 5 in Charles Jones

Endogenous Growth Theory or New Growth Theory

- We focus on understanding the economic forces underlying technological progress.
- We recognize that technological progress occurs as profit-maximizing firms or inventors seek out newer and better ways to produce.
- Improvements in technology and the process of economic growth are understood as an endogenous outcome of the economy.

The Romer Model

- Paul Romer (1990) "Endogenous Technological Change" based on Jones (1995)
- Try to explain why and how the advanced countries of the world exhibit sustained growth.
- We first think advanced countries of the world as a whole. Technology is driven by R&D in the advanced world. We will explore the process of technology transfer and why different economies have different levels of technology later.

- Model elements

  (1) production function

  (2) set of equations describing how inputs evolve over time (differential equations)
- Aggregate Production function

$$Y=K^{\alpha}(AL_Y)^{1-\alpha}$$

- Y outputs
- K physical capital
- A stock of ideas
- L_Y labor
- $0<\alpha<1$

- Given A constant, K and L_Y are adjustable. We have constant returns to scale:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}$$

$$Y = A^{1-\alpha}K^{\alpha}L_Y^{1-\alpha}$$

$$aY = A^{1-\alpha}(aK)^{\alpha}(aL_Y)^{1-\alpha}$$

$$aY = A^{1-\alpha}a^{\alpha+1-\alpha}K^{\alpha}L_Y^{1-\alpha}$$

$$aY = a\left[K^{\alpha}(AL_Y)^{1-\alpha}\right]$$

$$aY = aY$$

- When A are also an input, we have increasing returns to scale:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}$$

$$Y = K^{\alpha}A^{1-\alpha}L_Y^{1-\alpha}$$

$$(aK)^{\alpha}(aA)^{1-\alpha}(aL_Y)^{1-\alpha}$$

$$= a^{\alpha+1-\alpha+1-\alpha}K^{\alpha}A^{1-\alpha}L_Y^{1-\alpha}$$

$$= a^{2-\alpha}\left[K^{\alpha}(AL_Y)^{1-\alpha}\right]$$

$$= a^{2-\alpha}Y$$

Doubling all inputs, you will have more than double outputs. Since you need only to double K and L, to produce double outputs.

- Capital accumulation is $\overset{\circ}{K} = s_K Y - dK$
- where s_K is saving = investment, and constant
- d depreciation rate
- Labor or population grows exponentially at some exogenous rate n:

$$\frac{\overset{\circ}{L}}{L} = n$$

- To produce ideas or knowledge A, we need researchers. So, Adot is the number of new ideas produced at any given point in time.
- A(t) is the stock of knowledge or numbers of ideas that have been invented over the course of history up until time t.

- This simplest version assumes $\overset{\circ}{A} = \bar{\delta} L_A$
- Where $\bar{\delta}$ is the rate of which they discover new idea ;
- And L_A is the number of people attempting to discover new ideas.
- Discovery rate can be constant. Or we can imagine that it depends on the stock of ideas that have already been invented.
- In this case, rate of discovery would be an increasing function of A.
- However, if you think that later ideas are more difficult to discover, then it would be a decreasing function of A.
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- Discovery rate ($\bar{\delta}$) can be written as

$$\bar{\delta}=\delta A^{\phi}$$

- where δ and ϕ are constants
- If $\phi>0$, this indicates that the productivity of research increases with the stock of ideas that have already been discovered. (standing on shoulders)
- if $\phi<0$, this indicates that the productivity of research declines with A (fishing out case)
- If $\phi=0$, this indicates that the productivity of research is independent with A
- A^{ϕ} is treated as external to the investor. For $\phi>0$, reflecting a positive externalities in research.

- The simplest case, we have

$$(1)\quad \bar{\delta} = \delta A^{\phi}$$

$$(2)\quad \overset{\circ}{A} = \bar{\delta} L_A = \delta A^{\phi} L_A \text{ or } \delta L_A A^{\phi}$$

$$(3)\quad \overset{\circ}{A} = \bar{\delta} L_A^{\lambda} = \delta A^{\phi} L_A^{\lambda} \text{ or } \delta L_A^{\lambda} A^{\phi}$$

- In (3), we generalize (2) and assume that Adot depends on the number of people searching for new ideas, L_A^{λ} with $0<\lambda<1$, (possibly of duplication of effort when more persons engaged in research) and $\phi<1$.
- For each investor, they are small relative to the whole economy, take $\bar{\delta}$ as given and see constant returns to research.

- Next, we need to discuss how resources are allocated in this economy. There are tow key allocations.
- First, a constant fraction of output is invested in capital K.
- Second, we he to decide how much labor works to product output (L_Y) and how much to produce ideas (L_A). These two activities employ all of the labor in the economy:
- $L_A + L_Y = L$
- We simplify the model by assuming that the allocation of labor is constant (instead of letting agents choose to maximize their utility).
- Assume that labor force engages in R&D as $S_R = L_A/L$

Growth in the Romer model

- Per capita output is

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}$$

$$\frac{Y}{L_Y} = \frac{K^{\alpha}(AL_Y)^{1-\alpha}}{L_Y}$$

$$\frac{Y}{L_Y} = \frac{K^{\alpha}}{L_Y^{\alpha}}A^{1-\alpha}$$

$$y = A^{1-\alpha}k^{\alpha}$$

- Growths of y และ k along balanced growth path

$$y = A^{1-\alpha}k^{\alpha}$$

$$\ln y = (1-\alpha)\ln A + \alpha \ln k$$

$$\frac{\overset{\circ}{y}}{y} = (1-\alpha)\frac{\overset{\circ}{A}}{A} + \alpha\frac{\overset{\circ}{k}}{k}$$

$$g_y = (1-\alpha)g_A + \alpha g_k$$

$$g_y = (1-\alpha)g_A + \alpha g_y$$

$$(1-\alpha)g_y = (1-\alpha)g_A$$

$$g_y = g_k = g_A$$

If there is no technological progress, no growth. So, what is the rate of technological progress?

$$\overset{\circ}{A} = \delta L_A^{\lambda} A^{\phi} \Rightarrow \frac{\overset{\circ}{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}}.$$

- Along balanced growth path, $\frac{\overset{\circ}{A}}{A} = g_A = \text{constant}$

$$g_A = \delta \frac{L_A^{\lambda}}{A^{1-\phi}}$$

$$\ln g_A = \ln \delta + \ln L_A^{\lambda} - \ln A^{1-\phi}$$

$$0 = 0 + \lambda \frac{\overset{\circ}{L}_A}{L_A} - (1-\phi) \frac{\overset{\circ}{A}}{A}$$

$$(1-\phi) g_A = \lambda n$$

$$g_A = \frac{\lambda n}{1-\phi}$$

- Thus the long-run growth rate of this economy is determined by λ, n, and ϕ

  (1) parameters of the production function for ideas (λ and ϕ)

$$\overset{\circ}{A} = \delta L_A^{\lambda} A^{\phi}$$

  (2) rate of growth of researchers, which is given by population growth (n)

- To understand the intuition, let look at the simple case where $\lambda=1$ $\phi=0$

$$\overset{\circ}{A} = \delta L_A^{\lambda} A^{\phi} \Rightarrow \overset{\circ}{A} = \delta L_A$$

- Productivity of researchers (Adot divided by L_A) is constant.
- New ideas keep coming. There is no duplication problem in research and the productivity of a researcher today is independent of the stock of ideas that have been discovered in the past.
- For sustained growth, the number of new ideas must be expanding over time. This occurs if the number of researchers is increasing because of world population growth. In this case, the growth of ideas is clearly related to the growth in population.
- Problem with this model

- Romer (1990) when $\lambda=1$, $\phi=1$, $\bar{\delta}=\delta A$

$$\mathring{A} = \delta L_A^{\lambda} A^{\phi} \Rightarrow \mathring{A} = \delta L_A A \Rightarrow \frac{\mathring{A}}{A} = \delta L_A$$

- Growth of ideas is proportional to the exiting stock of ideas . With this assumption, growth of ideas grows over time, even if L_A is constant.
- Rejected by the evidence
- Notice that the LR growth rate is not affected by policies such as subsidies to r&D or changes in the investment rates as in the Solow model
- Many works in new growth theory has sought to develop models in which policy changes can have effects on long-run growth.

Increase in the R&D share

- Say government subsidy for R&D that increases the % of labor doing research
- λ=1 and ϕ=0

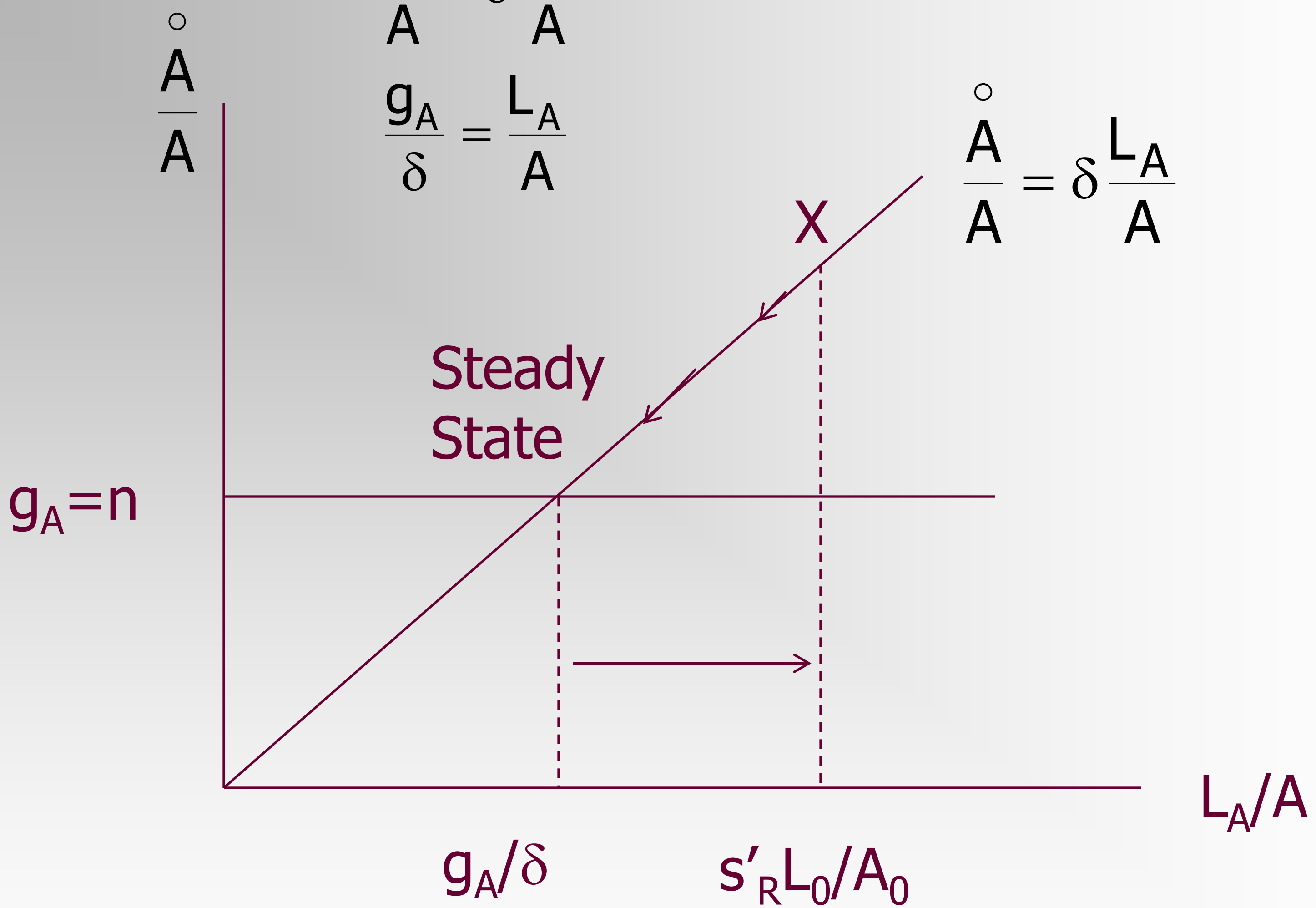
$$\frac{\overset{\circ}{A}}{A} = \delta \frac{L_A^{\lambda}}{A^{1-\phi}}$$

$$\frac{\overset{\circ}{A}}{A} = \delta \frac{s_R L}{A}$$

- Where $L_A = s_R L$

$$\frac{\overset{\circ}{A}}{A} = \delta \frac{L_A}{A}$$

$$\frac{g_A}{\delta} = \frac{L_A}{A}$$



- in steady state, $\frac{L_A}{A} = \frac{g_A}{\delta}$
- as s_R rises to s'_R, $\frac{L_A}{A} = \frac{s'_R L}{A}$
- as $g_A > n$, (L_A/A) declines over time to steady state again
- Thus, a permanent increase in the share of the population devoted to research raises the rate of technological progress temporarily, but not in the long run.
- As in the Solow model, we can find the ratio y/A be constant along the balanced growth path as

$$\left(\frac{y}{A}\right)^{*} = \left(\frac{s_K}{n + g_A + d}\right)^{\frac{\alpha}{1-\alpha}} (1 - s_R)$$

- From balanced growth path,

$$\frac{g_A}{\delta} = \frac{L_A}{A}$$

$$A = \frac{\delta s_R L}{g_A}$$

- Substitute A we get

$$\left(\frac{y}{A}\right)^{*} = \left(\frac{s_K}{n + g_A + d}\right)^{\frac{\alpha}{1-\alpha}} (1 - s_R)$$

$$y^{*}(t) = \left(\frac{s_K}{n + g_A + d}\right)^{\frac{\alpha}{1-\alpha}} (1 - s_R) \frac{\delta s_R L(t)}{g_A}$$

- Per capita output is proportional to the population of the world economy along a balanced growth path.
- The model shows a scale effect: a larger world economy will be a richer world economy.
- From the last equation, the first term , as in the Solow, shows that economies that invest more in capital will be richer.
- Two terms involve the share of labor in research, S_R. The first one implies negative output as fewer workers producing outputs, while the second shows more ideas from more researchers, thus more output.