**Chp4Optimization without constraint**

1. Find the stationary point of the following functions and using second order condition to identify the type of the stationary point;
   a) $Y=-x^2+4x+91$
   b) $Y=2x^3+3x^2-12x+4$
   c) $Y=-3x^2+4x-2$
   d) $Y=(2x-7)^3$
   e) $Y=\frac{1}{9}x^3-\frac{1}{6}x^2-\frac{2}{3}x+1$
2. Given a utility function as follows
   $U=\frac{5}{2}Q^2-\frac{1}{3}Q^3$
   Find a) Marginal utility
   b)The maximum utility of this consumer

3. Given a production function as follows
   $Q=6L^2-0.2L^3$ where Q=output and L=Labor
   Find a) The level of labors that maximizes the output
   b)Show that when average product is maximized, average product is equal to marginal product
   c)The range of level of labors that represents as efficient production

4. Given a short-run production cost function as follows
   $TC=\frac{Q^3}{20}-\frac{3Q^2}{10}+2Q+4$ where Q=output and TC=production cost
   Find,
   a) Total variable cost, total fixed cost, average variable cost, average fixed cost, average cost and marginal cost
   b) The output levels that minimize short run average variable cost
   c) Show that at the minimum average variable cost, average variable cost is equal to marginal cost

5. Given a production cost of a competitive firm in the short run as
   $TC=Q^3-12Q^2+60Q+10$

Find,

a) If the market price is 39, how much the firm will produce and profits the firm can earn
b) The shut-down price of the firm

6. A monopolist faces a market demand curve, $P=50 + 0.5Q$ and a short-run total cost of this monopolist is $TC(Q)= Q^3- 8Q^2+60Q+2$ In the short run, what is the equilibrium price, quantity, monopolist's profit, and a shut down price?

7. A firm has the following total cost and demand functions

$$C= \frac{1}{3} Q^3 – 7Q^2 +11Q +50$$

$$Q=100 – P$$

a) Write down the total revenue function in terms of Q
b) Formulate the total profit maximization in term of Q

8. Given marginal cost function of a competitive firm is $MC=3Q^2 – 12Q+19$
   a) Find the level of output that firm can minimize its average variable cost. Show that the second order condition is satisfied.
   b) If the market price of this product =34 baht/unit and given fixed cost is 50 baht, determine the profit maximizing level of output and the profits of the firm. Show that the second order condition is satisfied.

9. Find the extrema of the following functions
   a) $F(x,y)=-2x^2-2xy-2y^2+36x+42y-158$
   b) $F(x,y)=8x^3+2xy – 3x^2+y^2+1$
   c) $F (x, y, z)= xz+x^2- y+yz+y^2+3z^2$
   d) $F(x,y, z) = 2x^2+xy+4y^2+xz+z^2+3$

10. Let $F=4xy+12x+16y-2x^2-3y^2$ be the soybean yield function where f is a kilogram of soybean; x is a kilogram of fertilizer x and y is a kilogram of fertilizer y. Find the quantity of x and y that yields maximum f. Proof your

result by using second order condition.

11. Consider a competitive firm produces 2 goods where prices of each good are $P_1$ =70 and $P_2$= 50. Firm's production cost is given as $TC=Q_1^2+Q_1Q_2+Q_2^2$ Find $Q_1$ and $Q_2$ that maximizes firm's profits and calculate the maximum profits of this firm.

12. Consider a noncompetitive firm produces 2 goods. Given a market demand of each good as

    $Q_1= 40 – 2P_1+P_2$ and $Q_2=15+P_1-P_2$

    Firm's production cost is given as $TC= Q_1^2+Q_1Q_2^+Q_2^2$, find $Q_1$ and $Q_2$ that maximizes firm's profits and calculate the maximum profits of this firm

13. A producer lets two factories producing identical car and the total cost of production for two factories are as follows
    $TC_1 = 500Q_1 + 2Q_1^2$ and $TC_2 = 100Q_2 + Q_2^2$
    a)If the selling price of car is 1000 baht, what is the quantity of car produced in each factories so that the firm will maximize their profit
    b)If the producer owns the monopoly power and the supply of the car is $Q=1575 – P$(if $Q=Q_1+Q_2$), what is the quantity of car produced to maximizes the profit ?

14. A firm sells a product into two markets. A market demand curve for each market is given as

    $P_1 = 40-2Q_1$ and $P_2=90-4Q_2$

    Production cost of this product is $TC=10+18Q$ where $Q=Q_1+Q_2$, find price and quantity in each market that the firm can maximize the profit and find total profit they receive.

15. Given the production function as $Q=5K^{1/2}L^{1/4}$ (K=capital and L=labor) in the competitive market and the price of good is 4 baht/unit, the price of capital

is 5baht/unit and the wage is 10 baht/unit. Find the quantity of capital and labor that maximizes the profit of the firm.

16. A firm produces goods for selling in two markets which have demand function as $Q_1 = 24 – 0.2P_1$ and $Q_2 = 10 – 0.05P_2$ and their total cost in production is $TC = 35+40Q$ where $Q = (Q_1+Q_2)$.

    a) If there is a price discrimination between two markets, find the quantity of the goods in each market to maximize the firm's profit.
    b) If there is not a price discrimination between two markets, find the quantity of the goods in each market to maximize the firm's profit.
    c) Which way of those two approaches give the firm higher profit?