Macroeconomics

Lecture # 13

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The Possibility of Dynamic Inefficiency

- In the Ramsey-Cass-Koopmans Model, the equilibrium in this model also maximizes the welfare of the representative household.
- The equilibrium of Diamond model need not be Pareto-efficient.

The Possibility of Dynamic Inefficiency

- In case that we have logarithmic utility and Cobb-Douglas production and g=0, we can recall from Lecture#11:

$$k_{t+1} = \frac{1}{(1+n)} \frac{1}{2+\rho} (1-\alpha) k_t^{\alpha} \equiv D k_t^{\alpha} \qquad (1)$$

Hence, at k^* *where* $k_{t+1} = k_t$,

$$k^* = D^{\frac{1}{1-\alpha}} = \left[\frac{1}{(1+n)} \frac{1}{2+\rho} (1-\alpha) \right]^{\frac{1}{1-\alpha}} \qquad (2)$$

Thus, the marginal product of capital on the balance growth path, $\alpha k^{*\alpha-1}$, *is*

$$f'(k^*) = \frac{\alpha}{1-\alpha} (1+n)(2+\rho) \qquad (3)$$

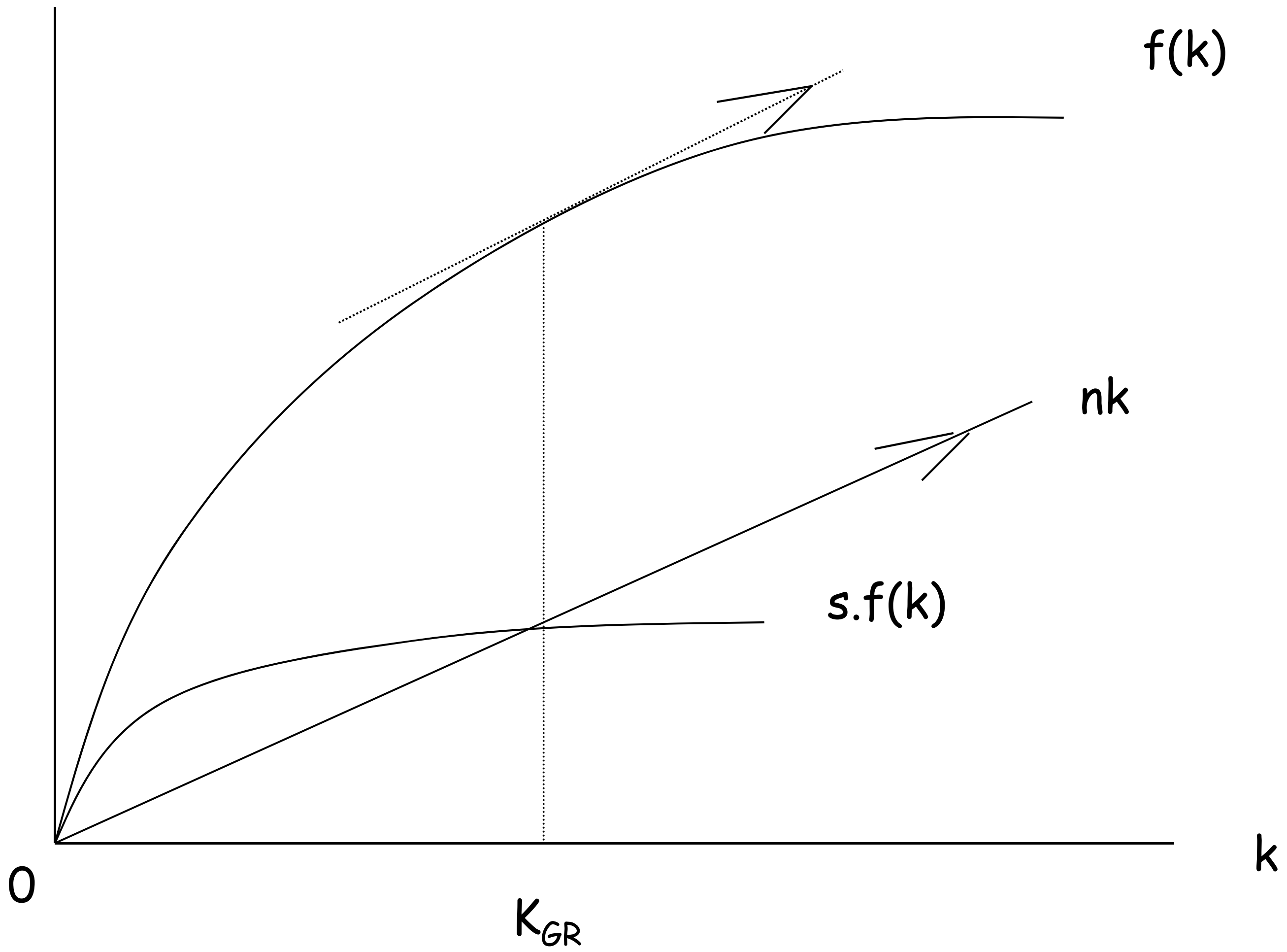
The Possibility of Dynamic Inefficiency

- The golden-rule capital stock, k_{GR}, is defined by

$$f'(k_{GR}) = (n + g + \delta) = n, \quad \because g = \delta = 0 \qquad (3)$$

$$Note\ that\ \ f'(k^*)\ can\ either\ be\ greater\ or\ less\ than\ \ f'(k_{GR})$$

- At k_{GR} , consumption is at its maximum possible level among balanced growth paths.


f(k)
nk
s.f(k)
0
k
K_{GR}

The Possibility of Dynamic Inefficiency

- It is inefficient for k^* to exceed k_{GR}.

$If \quad k^* > k_{GR},\ consumption\ per\ worker\ , c^*, is$

$$c^* = f\left(k^*\right) - nk^*$$

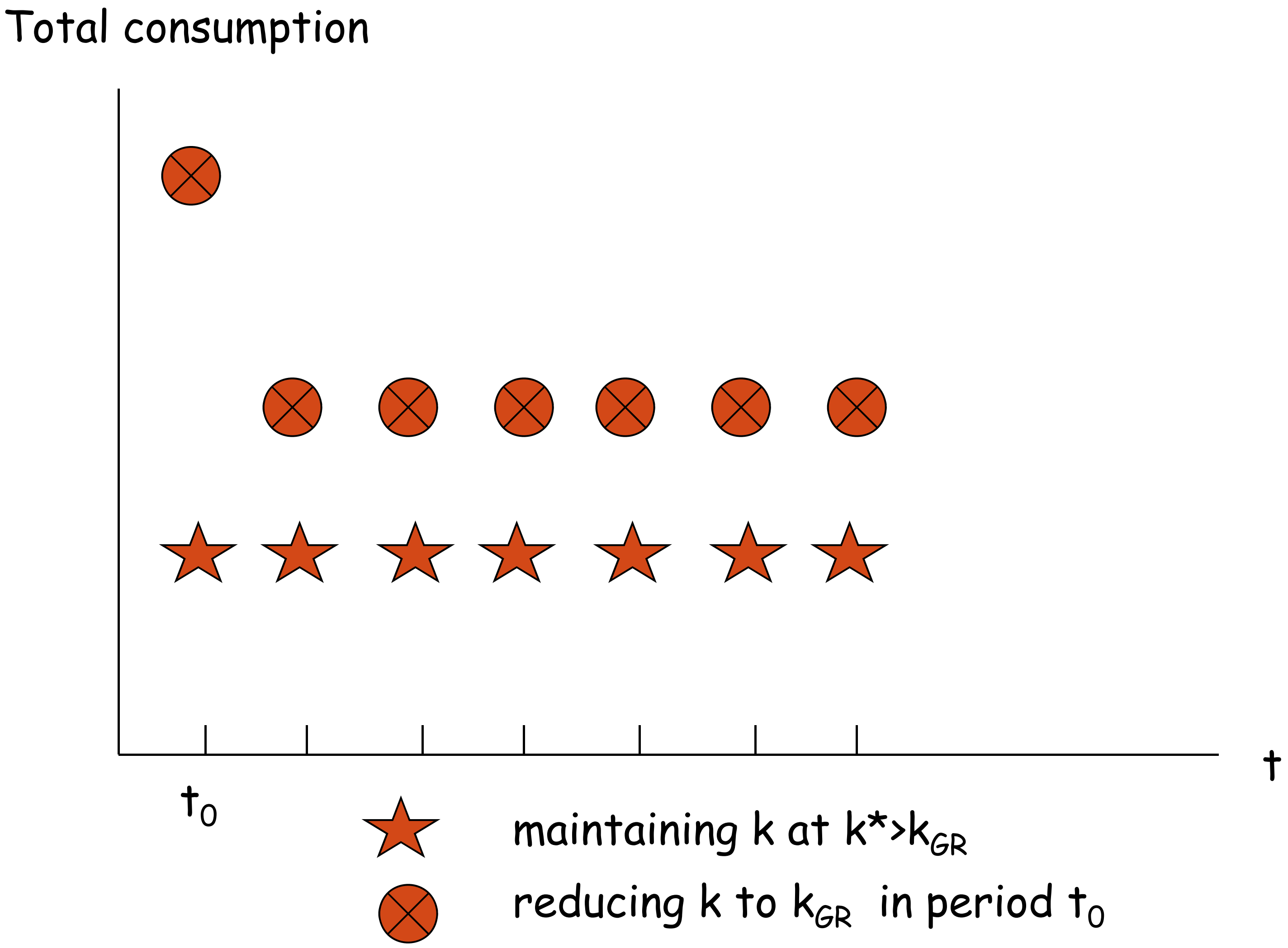
$Suppose\ at\ t_0, the\ planner\ allocates\ more\ resource\ for\ consumption,$
$and\ fewer\ to\ saving\ than\ usual, so\ that\ capital\ per\ worker\ in\ all$
$later\ pepriods\ are\ at\ k_{GR}.$

$Under\ this\ plan, at\ t_0,$

$$c_0 = f\left(k^*\right) + \left(k^* - k_{GR}\right) - nk_{GR}$$

$and\ each\ subsequent\ period,$

$$c_{GR} = f\left(k_{GR}\right) - nk_{GR}$$



Total consumption
t
t_0
maintaining k at $k^*>k_{GR}$
reducing k to k_{GR} in period t_0

The Possibility of Dynamic Inefficiency

- This plan of reducing k to k_{GR} makes more resources available for consumption in every period than the policy of maintaining k at k*.
- Thus the equilibrium of the Diamond model can be Pareto-inefficient. This is because the infinity of generations gives the planner a means to providing for the consumption that is not available to the market.

The Possibility of Dynamic Inefficiency

- The planner can take 1 unit of labor income from each young person and transfer it to the old; since there are 1+n young persons for each old person, this increase the consumption of old person by 1+n units.
- If the marginal product of capital is less than n (that is $k > k_{GR}$), this way of transferring resources between young and old age is more efficient than saving.

The effects of Government Purchases

- Let G_t denote the government's purchases of goods per unit of effective labor in period t. Government finances those purchases by lump-sum taxes on the young

The effects of Government Purchases

$$k_{t+1} = \frac{1}{(1+n)(1+g)}\frac{1}{(2+\rho)}\left[(1-\alpha)k_t^{\alpha} - G_t\right] \qquad (4)$$

- A higher G_t reduces k_{t+1} for a given k_t. (Note that)

$$(1-\alpha)k_t^{\alpha} = wage$$

Tax Versus Bond Finance

- Let b be the stock of bonds per unit of effective labor.

$$k_{t+1} = \frac{1}{(1+n)(1+g)}\frac{1}{(2+\rho)}\left[(1-\alpha)k_t^{\alpha} - T_t\right] - b_{t+1} \qquad (5)$$

- Eq (5) shows that taxes and bonds have different effects on capital formulation.

Tax Versus Bond Finance

- When the government cuts taxes and issues bonds, the taxes to repay those bonds are levied on future generations. Hence, the individuals currently alive are better off, and they therefore increase their consumption. Thus a switch from tax to bond finance reduces the capital stock.

Tax Versus Bond Finance

- The government can use bonds to prevent the economy from accumulating too much capital.
- Without government, for $k^* > k_{GR}$ and the capital stock in some period t equals its golden-rule level, the labor income of the young is $(1-\alpha)k_{GR}^{\alpha}$, and saving fraction of $1/(2+\rho)$ of this. Thus $k_{t+1} + b_{t+1}$ equals

Tax Versus Bond Finance

$$k_{t+1}+b_{t+1}=\frac{1}{(1+n)(1+g)}\frac{1}{(2+\rho)}(1-\alpha)k_{GR}^{\alpha}\equiv a_{GR} \qquad (6)$$

- For the economy to be on a balance growth path with the capital stock at the golden-rule level, b must equal the difference between the total amount the young wish to save when $k=k_{GR}$, and the amount of that saving that must take the form of capital, k_{GR} . So, $b = a_{GR} - k_{GR}$ can cause the balance-growth path value of k to equal its golden- rule value.