

Solution: Quiz 2

1. Let the set of positive integers $\mathbb{Z}^+$ be the domain of x . Determine if the following statement

$$x^2 > 4 \Rightarrow x > 2.$$

is true or false and explain your answer.

Answer: Let T_h be the truth set of the predicate: “ $x^2 > 4$ ”

and T_c be the truth set of the predicate: “ $x > 2$ ”

For $x \in \mathbb{Z}^+$, “ $x^2 > 4$ ” is true when $x \in \{3, 4, 5, \dots\}$ and “ $x > 2$ ” is true when $x \in \{3, 4, 5, \dots\}$.

That is, $T_h = \{3, 4, 5, \dots\} = \mathbb{Z}^+ / \{1, 2\}$ and $T_c = \{3, 4, 5, \dots\} = \mathbb{Z}^+ / \{1, 2\}$.

Since $T_h = T_c$, every element in the truth set of “ $x^2 > 4$ ” is an element in the truth set of “ $x > 2$ ” for $x \in \mathbb{Z}^+$. This implies that the statement is true.

2. Let $D = \{-1, 0, 1\}$ and $E = \{-1, 0, 2\}$. Consider the statement

$$\exists x \in D, \forall y \in E \text{ such that } x \cdot y = y.$$

- (a) Write the negation for the above statement (without using negation symbol “ $\sim$ ” in the final answer).
 (b) Determine the truth value of the above statement. Explain your answer.

Answer:

(a)

$$\begin{aligned} \sim (\exists x \in D, \forall y \in E \text{ such that } x \cdot y = y) &\equiv \forall x \in D \sim (\forall y \in E \text{ such that } x \cdot y = y) \\ &\equiv \forall x \in D, \exists y \in E \text{ such that } \sim (x \cdot y = y) \\ &\equiv \forall x \in D, \exists y \in E \text{ such that } x \cdot y \neq y \end{aligned}$$

- (b) This statement is true because if we let $x = 1 \in D$, then $1 \cdot y = y$ for all $y = -1, 0, 2 \in E$.
 I.e.

$$\begin{aligned} y = -1, \quad 1 \cdot (-1) &= -1 \\ y = 0, \quad 1 \cdot (0) &= 0 \\ y = 2, \quad 1 \cdot (2) &= 2 \end{aligned}$$