$*\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ $\beta_0 = \bar{Y} - \beta_1\bar{X}$ $U = Y - \hat{Y}$

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student. $\bar{x} = 77.625$ $\bar{y} = 3.2125$

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

from 1.1 from table!

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

① $x-\bar{x}$	② $y-\bar{y}$	① × ②	①^2	$\hat{Y}$	$\hat{U}$
-14.625	-0.4125	6.0328125	213.891	2.71525	0.08475
-5.625	0.1875	-1.0546875	31.641	3.02125	0.37875
0.375	-0.2125	-0.0796875	0.141	3.22525	-0.22525
3.375	0.2875	0.9703125	11.391	3.32725	0.17275
9.375	0.3875	3.6328125	87.891	3.53125	0.06875
-2.625	-0.2125	0.5578125	6.891	3.12325	-0.12325
-2.625	-0.5125	1.3453125	6.891	3.12325	-0.42325
12.375	0.4875	6.0328125	153.141	3.63325	0.06675
		17.4375	511.878		0

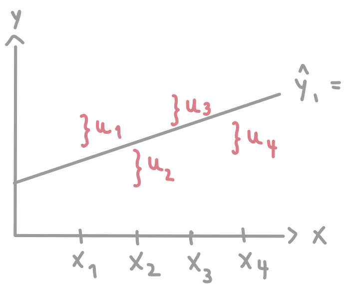
1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

OLS $\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ $\beta_0 = \bar{Y} - \beta_1\bar{X}$

$\beta_1 = \frac{17.4375}{511.878} = 0.034$

$\beta_0 = 3.2125 - (0.034)(77.625)$
$= 0.57325$

ใส่หมวก β
ตอนทำจาก sample



$\hat{Y}_i = \beta_0 + \beta_1 x_i$
↓
$\therefore Y_i = \beta_0 + \beta_1 x_i + u_i$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y} = \beta_0 + \beta_1 x_i$ $\hat{U} = Y_i - \hat{Y}$

$\hat{Y}$	$\hat{U}$
2.71525	0.08475
3.02125	0.37875
3.22525	-0.22525
3.32725	0.17275
3.53125	0.06875
3.12325	-0.12325
3.12325	-0.42325
3.63325	0.06675
	0

$\sum_{i=0}^{n} \hat{u}_i = 0.08475 + 0.37875 \ldots\ldots + 0.06675$

$= 0$

$* \ \sigma^2 = var(\hat{u}_i)$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$\hat{\sigma}^2_{u_i} = \frac{\sum_{i=1}^{n} \hat{u}^2}{n-2} = \frac{(0.08475)^2 + (0.37875)^2 + (-0.22525)^2 + \dots + (0.06675)^2}{6}$$

$$\therefore \sigma^2 = 0.0724546$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = 0.00014154661$$

$$var(\hat{\beta}_0) = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2 =$$

2. Data is listed in the table

X_i	Y_i	① $x_i - \bar{x}$	② $y_i - \bar{y}$	① × ②	$(x_i - \bar{x})^2$	$\hat{y}_i$
10	0	-10	-9.1	91	100	0.145
12	2	-8	-7.1	56.8	64	1.936
14	5	-6	-4.1	24.6	36	3.727
16	6	-4	-3.1	12.4	16	5.518
18	7	-2	-2.1	4.2	4	7.309
22	10	2	0.9	1.8	4	10.891
24	10	4	5.9	3.6	16	12.682
26	15	6	6.9	35.4	36	14.473
28	16	8		55.2	64	16.264
30	20	10	10.9	109	100	18.055
					440	

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 \approx \frac{394}{440} \approx 0.8955$$

$$\hat{\beta}_0 \approx 9.1 - (0.8955)(20)$$
$$\approx -8.81$$

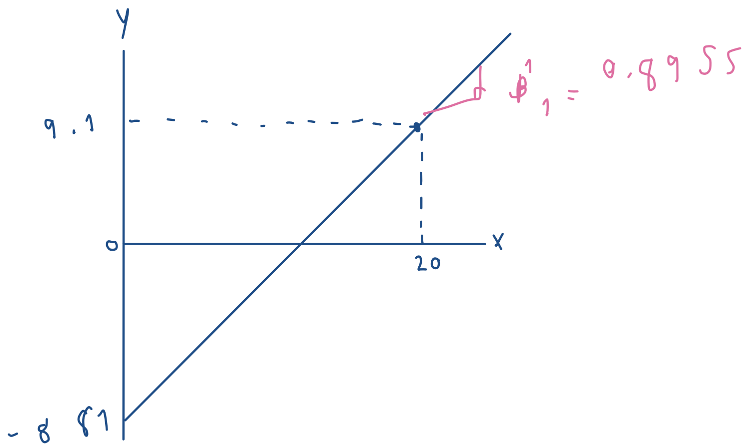
2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

x_i	y_i	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-.0145
12	2	1.936	0.064
14	5	3.727	1.275
16	6	5.518	0.482
18	7	7.309	0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	4.473	0.527
28	15	15.254	-0.164
30	20	19.055	1.945
			0

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$(20, 9.1)$ $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1$

$\hat{y} = -8.81 + 0.8955 X_i$



2.4 If X_i =16, what is the predicted Y?

$X_1 = 16$ $\hat{Y} = 5.518$

2.5 Find $var(\widehat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \ ,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.