Answer Keys Chp6

1.

a) $\int x^{\frac{2}{3}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

b) $\int (2x-3)^4\,dx = \frac{1}{10}(2x-3)^5 + c$

c) $\int \frac{3x^2+2}{\sqrt{x}}\,dx = \int 3x^{\frac{3}{2}} + 2x^{\frac{-1}{2}}\,dx = \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{6}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + c$

d) $\int \sqrt[3]{x^3+3x^2}\,(x^2+2x)dx = \frac{1}{4}(x^3+3x^2)^{\frac{4}{3}} + c$

e) $\int \frac{x^2-1}{x\sqrt{x}+\sqrt{x}}\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2x^{\frac{3}{2}}}{3} - 2x^{\frac{1}{2}} + c$

2.

a) $\int_0^1 (x^4+2x^3+4x+10)dx = \frac{x^5}{5} + \frac{2x^4}{4} + \frac{4x^2}{2} + 10x \,| =$
$\frac{1}{5} + \frac{2}{4} + \frac{4}{2} + 10 = \frac{127}{10}$

b) $\int_0^8 x^{\frac{2}{3}}\,dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} \,| = \frac{96}{5}$

c) $\int_7^{12} 1\,dx = 12 - 7 = 5$

d) $\int_{-8}^8 (\sqrt[3]{x^2} + 2)\,dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 2x \,| = \frac{(8)^{\frac{5}{3}}}{\frac{5}{3}} + 2(8) - \left[\frac{(-8)^{\frac{5}{3}}}{\frac{5}{3}} + 2(-8)\right] = \frac{352}{5}$

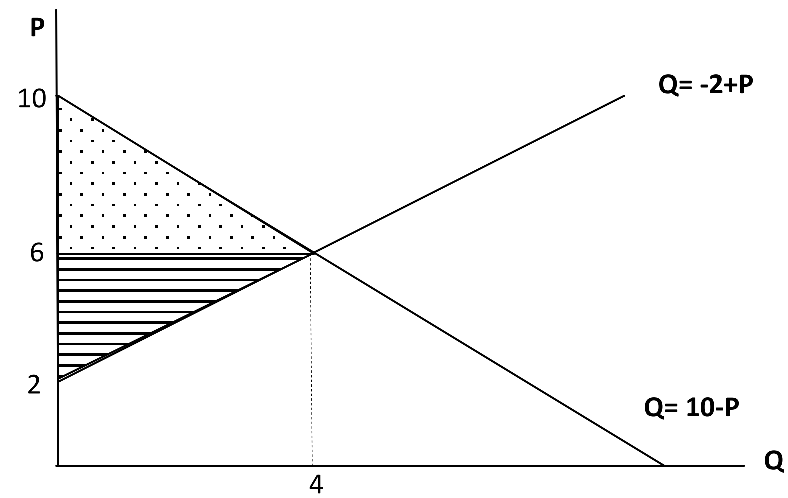
e) $\int_0^{64} (x^{\frac{1}{2}} + 5x^{\frac{-2}{3}})\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{5x^{\frac{1}{3}}}{\frac{1}{3}} \,| = \frac{(64)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{5(64)^{\frac{1}{3}}}{\frac{1}{3}} = 401\frac{1}{3}$

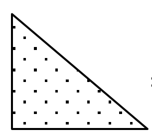
3. Area under the curve = $\int_0^3 x^2+4x\,dx = \left(\frac{x^3}{3} + \frac{4x^2}{2}\right) | = 27$
4. Area under the curve = $\int_1^5 (6x - x^2)\,dx = (3x^2 - \frac{x^3}{3}) | = 75 - \frac{125}{3} - \left[3 - \frac{1}{3}\right] = \frac{92}{3}$
5. Area between two curves = $\int_{-1}^3 \left[\frac{-x^2}{2} + x + \frac{3}{2}\right] - [x^2 - 2x - 3]\,dx$

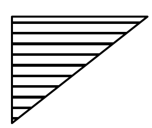
$= \frac{-x^3}{2} + \frac{3}{2}x^2 + \frac{9}{2}x \,|$

$$= \frac{-(3)^3}{2} + \frac{3}{2}(3)^2 + \frac{9}{2}(3) - \left[\frac{-(-1)^3}{2} + \frac{3}{2}(-1)^2 + \frac{9}{2}(-1)\right]$$
$= 16$

6. Q=10 – P and Q= -2 + P, so $Q_d = Q_s$; 10 – P = -2 +P
   P*=6 and Q*=4



Consumer Surplus=  = ½(4)(4)=8

Producer Surplus=  =1/2(4)(4)=8

Total Surplus= 8+8=16

7. $\int (MC)\,dQ = \int (Q^3+2Q^2+2Q+5)\,dQ = \frac{Q^4}{4} + \frac{2Q^3}{3} + \frac{2Q^2}{2} + 5Q + C$ = Total Cost

   Given fixed costs= 150 baht
   Then, total cost = $\frac{Q^4}{4} + \frac{2Q^3}{3} + Q^2 + 5Q + 150$

Average Cost = $\frac{Q^3}{4} + \frac{2Q^2}{3} + Q + 5 + \frac{150}{Q}$

Average Fixed Cost = $\frac{150}{Q}$

Average Variable cost = $\frac{Q^3}{4} + \frac{2Q^2}{3} + Q + 5$

8. $C=0.25Y^d+100$
9. P=45 – 0.5Q , then Q=90 – 2P

   From p=32.5 to (Q=0, P=45)

   Consumer surplus = $\int_{32.5}^{45}(90 - 2P)\ dP$ = 90P $-P^2$ |

   = $90(45) - (45)^2 - [90(32.5)-(32.5)^2]$

   =156.25
10. $P=(Q+3)^2$, then Q=$\sqrt{P}$ - 3

   From P=81 to (Q=0, P=9)

   Producer Surplus=$\int_{9}^{81}(\sqrt{P} - 3)$ dP = $\frac{2P^{\frac{3}{2}}}{3} - 3P$ |

   = $\frac{2(81)^{\frac{3}{2}}}{3} - 3(81) - [\frac{2(9)^{\frac{3}{2}}}{3} - 3(9)\ ]$

   = 252

11. $Q=5-P^{1/3}$, $P= (5 – Q)^3$

   So, a) Consumer surplus at p=1(at p=1, Q=5)

   is =$\int_0^5 (5-Q)^3\ dQ$= $\int_0^5 (125 - 75Q + 15Q^2 - Q^3\ )dQ$

   = 125(5) - $\frac{75(5)^2}{2} + \frac{15(5)^3}{3} - \frac{(5)^4}{4}$

   =156.25

   b)Consumer surplus at p=27(at p=27, Q= 2)

   is =$\int_0^2 (5-Q)^3\ dQ$= $\int_0^2 (125 - 75Q + 15Q^2 - Q^3\ )dQ$

   = 125(2) - $\frac{75(2)^2}{2} + \frac{15(2)^3}{3} - \frac{(2)^4}{4}$

   =136

c) Change in Consumer surplus

=156.25 – 136 =20.25

12. $Q_d=5 - P/3$

$P=(Q_s+1)^2$; Then, $Q_d=Q_s$; $5 - P/3=\sqrt{P} - 1$

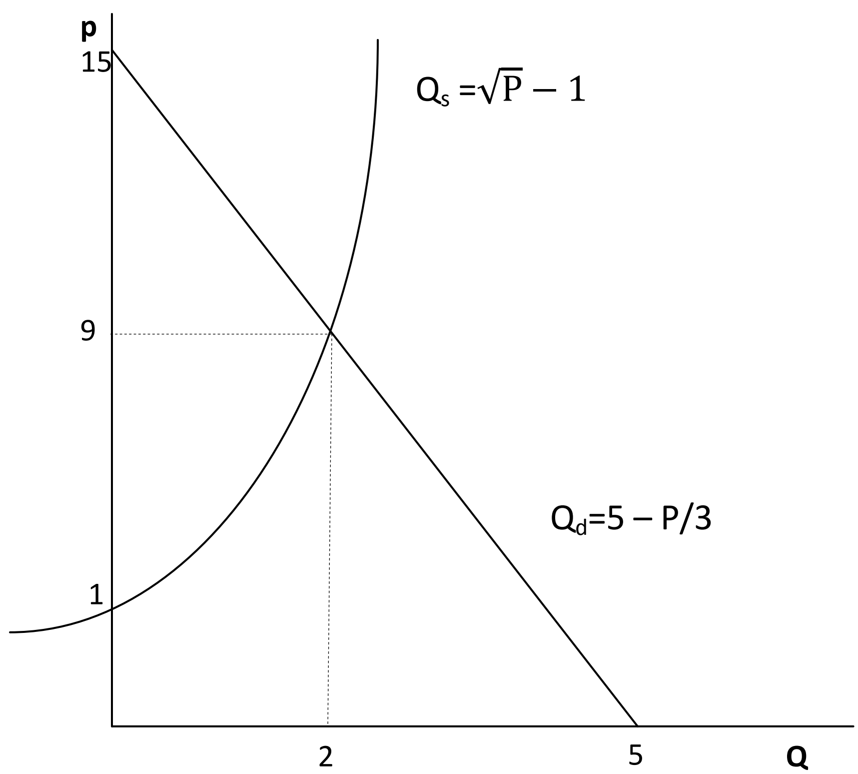
$P^2 - 45P+324=0$

$P_1=36, P_2=9$

So, $Q_1=5$, $Q_2=2$

But, P_1 and Q_1 is not corresponding to Q_d

Then, $P^*=9$ and $Q^*=2$



Consumer Surplus= ½(6)(2)=6

Producer Surplus= $\int_1^9 \sqrt{P} - 1\, dP = \frac{P^{\frac{3}{2}}}{\frac{3}{2}} - P \,\Big|$

$= \frac{(9)^{\frac{3}{2}}}{\frac{3}{2}} - (9) - [\frac{(1)^{\frac{3}{2}}}{\frac{3}{2}} - (1)] = \frac{28}{3}$