Student Name Mahabhop Aranchavanich ID 6104640443

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i) = 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

   b. $\sum_{y=0}^{5} f(x+y) = f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

   c. $\sum_{i=1}^{10} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

   $= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 = 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = [2(1)+(2)] + [2(1)+(3)] + [2(2)+(2)] + [2(2)+(3)]$

   $= 4 + 5 + 6 + 7 = 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $\int_{-2}^{4} f(x) = 1 \rightarrow 8b = 1 \rightarrow b = \frac{1}{8} = 0.125$

   b. Find the answer for $P(X \leq 2)$

   $1 - P(X > 2) = 1 - P(X=3) - P(X=4) = 1 - \frac{1}{16} - \frac{1}{32} = \frac{32-2-1}{32} = \frac{29}{32}$

   c. Find the answer for $P(-2 \leq X \leq 3)$

   $1 - P(X=4) = 1 - \frac{1}{32} = \frac{31}{32}$

   d. Find the answer for $P(X \geq 1)$

   $P(X=1) + P(X=2) + P(X=3) + P(X=4)$

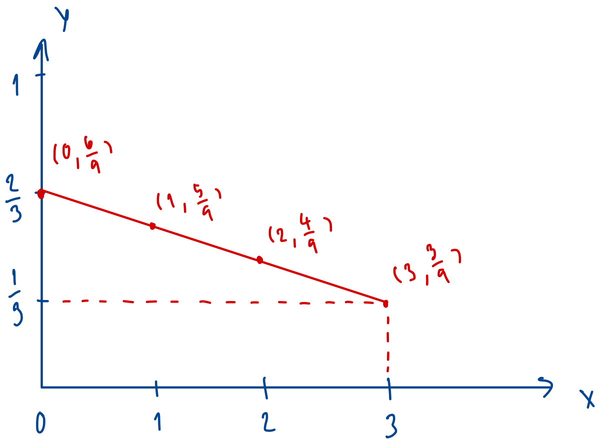
   $\frac{1}{4} + \frac{3}{16} + \frac{1}{16} + \frac{1}{32} = \frac{8+6+2+1}{32} = \frac{17}{32}$

Student Name ______________________________ ID ______________

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$\int_1^3 -\frac{1}{9}x + \frac{6}{9}\,dx$$

$$-\frac{1}{9}\frac{x^2}{2} + \frac{6}{9}x \Big|_1^3 \quad 3 \rightarrow -\frac{1}{2} + 2 = \frac{3}{2}, \quad 1 \rightarrow -\frac{1}{18} + \frac{6}{9} = \frac{11}{18}, \quad \frac{3}{2} - \frac{11}{18} = \frac{16}{18}$$

c. Find the answer for $P(X \ge 2)$

$$\int_2^3 -\frac{1}{9}x + \frac{6}{9}\,dx$$

$$-\frac{1}{9}\frac{x^2}{2} + \frac{6}{9}x \Big|_2^3 \quad 3 \rightarrow -\frac{1}{2} + 2 = \frac{3}{2}, \quad 2 \rightarrow -\frac{2}{9} + \frac{12}{9} = \frac{10}{9}, \quad \frac{3}{2} - \frac{10}{9} = \frac{7}{18}$$

d. Find the expected value of X

$$\int_0^4 x f(x)\,dx$$

$$\int_0^4 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$\int_0^4 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$-\frac{1}{9}\frac{x^3}{3} + \frac{6}{9}\frac{x^2}{2} \Big|_0^3 \quad 3 \rightarrow -1 + 3 = 2, \quad 0 \rightarrow 0, \quad 2 - 0 = 2$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

Y \ X	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

   b. Find the marginal probability distribution function (PDF) of X

$$P(X = 1,2,3,4,5) = \frac{1}{6}$$

   c. Find the marginal probability distribution function (PDF) of Y

$$P(Y = 0) = \frac{1}{2}$$
$$P(Y = 1) = \frac{1}{2}$$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X \mid Y = 1) = \frac{P(X = x_i, Y = 1)}{P(Y = 1)} = \frac{\frac{1}{12}}{\frac{1}{2}} = \frac{1}{6}$$

   e. Find the expected value of X given Y is equal to 1

$$E(X = x_i \mid Y = 1) = \sum_{i=1}^{6} x_i P(X = x_i \mid Y = 1) = \sum_{i=1}^{6} x_i \frac{P(X = x_i, Y = 1)}{P(Y = 1)} = \frac{1(\frac{1}{12}) + 2(\frac{1}{12}) + \dots + 6(\frac{1}{12})}{\frac{1}{2}} = \frac{\frac{1}{12} + \frac{2}{12} + \frac{3}{12} + \frac{4}{12} + \frac{5}{12} + \frac{6}{12}}{\frac{1}{2}}$$

$$= \frac{\frac{21}{12}}{\frac{1}{2}} = \frac{21}{6} = 3.5$$

   f. Find the variance of X given Y is equal to 1

$$Var(X) = \sum_{i=1}^{6} (x - E(x - \mu)^2) \cdot f(x \mid y = 1)$$

$$= (1-3.5)^2 \frac{1}{6} + (2-3.5)^2 \frac{1}{6} + (3-3.5)^2 \frac{1}{6} + (4-3.5)^2 \frac{1}{6} + (5-3.5)^2 \frac{1}{6} + (6-3.5)^2 \frac{1}{6}$$

$$= 1.04167 + 0.375 + 0.04167 + 0.04167 + 0.375 + 1.04167$$

$$= 2.91666$$

Student Name ______________________________________ ID ___________________

5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(\bar{x}) = E\left(\frac{x_1 + x_2 + x_3}{3}\right)$

$= \frac{1}{3} E(x_1 + x_2 + x_3)$

$= \frac{1}{3}(3\mu_x)$

$= \mu_x$

$Var(x) = Var\left(\frac{1}{3}(x_1 + x_2 + x_3)\right)$

$= \frac{1}{9}\Big(Var(x_1) + Var(x_2) + Var(x_3)$

$+ 2Cov(x_1, x_2) + 2Cov(x_2, x_3)$

$+ 2Cov(x_1, x_3)\Big)$

$= \frac{1}{9}\left(\sigma^2 + \sigma^2 + \sigma^2 + 2\left(\frac{1}{4}\sigma^2\right)(3)\right)$

$= \frac{1}{9}\left(3\sigma^2 + \frac{3}{2}\sigma^2\right)$

$= \frac{1}{9}\left(\frac{9}{2}\sigma^2\right)$

$= \frac{1}{2}\sigma^2$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

   a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$E(\bar{x}) = E\left(\frac{1}{4}(x_1 + x_2 + x_3 + x_4)\right) = \frac{1}{4}E(x_1 + x_2 + x_3 + x_4) = \frac{4\mu}{4} = \mu$

$Var(\bar{x}) = Var\left(\frac{1}{4}(x_1 + x_2 + x_3 + x_4)\right) = \frac{1}{16}Var(x_1 + x_2 + x_3 + x_4) = \frac{4\sigma^2}{16} = \frac{\sigma^2}{4}$

Student Name ____ ID ____________________

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{x}) = E\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right)$$

$$= \frac{1}{8}E(x_1) + \frac{1}{4}E(x_2) + \frac{1}{8}E(x_3) + \frac{1}{2}E(x_4)$$

$$= \mu\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right)$$

$$= \mu\left(\frac{1}{8} + \frac{2}{8} + \frac{1}{8} + \frac{4}{8}\right)$$

$$= \mu$$

so, $\tilde{x}$ is an unbiased estimator of u

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$Var(\bar{x}) = \frac{\sigma^2}{4}$$

$$Var(\tilde{x}) = \frac{1}{64}Var(x_1) + \frac{1}{16}Var(x_2) + \frac{1}{64}Var(x_3) + \frac{1}{4}Var(x_4)$$

$$= \sigma^2\left(\frac{1}{64} + \frac{1}{16} + \frac{1}{64} + \frac{1}{4}\right) = \sigma^2\left(\frac{1}{64} + \frac{4}{64} + \frac{1}{64} + \frac{16}{64}\right) = \frac{22}{64}\sigma^2$$

The estimator with lower Varience is considered to be the better estimator

therefore, $\bar{x}$ is the better estimator