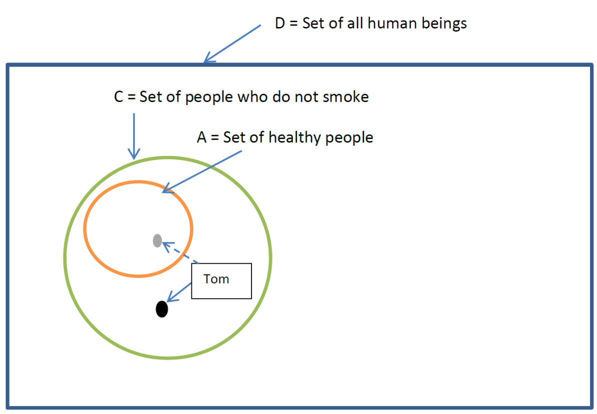
Solution: Quiz 3

1. Let the domain D be the set of all human beings. Suppose Tom $\in D$.
Use the **diagram** to show that the following argument is valid or invalid.
"No healthy people smoke ."
"Tom does not smoke."
$\therefore$ "Tom is healthy."

Solution:

To use the diagram,
let A be the set of healthy people, and
let C be the set of people who do **not** smoke.
Then
- the first premise "No healthy people smoke ." tells us that
"**All healthy people do not smoke**" or "$A \subseteq C$."
- the second premise "Tom does not smoke."
tells us that "Tom is in the set C," or Tom $\in C$,
- the conclusion tells us that "Tom is in the set A."

From the diagram, since we only know that "Tom" is an element in C , it is possible that "Tom" is either an element in A or not an element in A. That is, the conclusion may not happen because it is possible that "Tom" does **not** in A or "Tom" is not healthy. Therefore the argument is **invalid**.



To confirm this by using the universal rules of inferences.
Let $P(x)$ be "x is healthy,"
$Q(x)$ be "x does not smoke."
Then we can transform the given argument in the quantified form of **converse error** as follows.

$$\forall x, P(x) \rightarrow Q(x)$$
$$Q(a) \text{ for a particular } a = \text{ Tom}$$
$$\therefore P(a)$$

That is, this argument is **invalid** by the converse error. ■

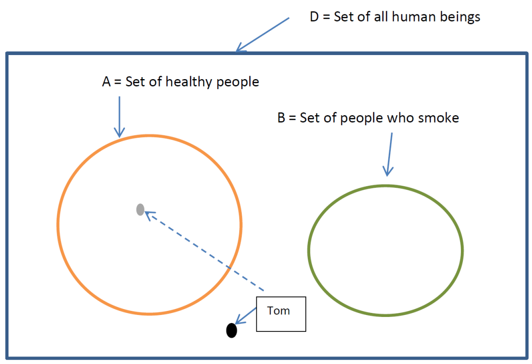
ALTERNATIVE ANSWER USING DIAGRAM:

Suppose we
let A be the set of healthy people, and
let B be the set of people who smoke.

Let B^c be the complement of B, i.e. B^c is the set of elements in D that are not in B. Note that $B \cap B^c = \emptyset$.

- The first premise "No healthy people smoke ." is equivalent to "**All healthy people do not smoke**" or $A \subseteq B^c$, so A cannot have anything in common with B and this implies "$A \cap B = \emptyset$."
- The second premise implies "Tom"$\notin B$ or "Tom"$\in B^c$.
- The conclusion implies "Tom" $\in A$

Hence the following diagram is also **correct** for this argument and it gives the same answer "*invalid*".



From the diagram, since we only know that "Tom" is not an element in B , it is possible that "Tom" is either an element in A or not an element in A. That is, the conclusion may not happen when "Tom" does **not** in A or "Tom" is not healthy. Therefore the argument is **invalid**.