

Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points)

Score.....

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = \ln(C)$$

Also, let  $\frac{C_1}{C_0}$  is distributed as log-normal with mean equals  $\mu_c$  and its variance is  $\sigma_c$ .

Please read and answer the following questions carefully and completely.

Score.....

**Question 1.1 ( 10 marks)** Calculate the risk free rate  $R_f$  in terms of the individual's consumption,  $C_0$  and  $C_1$ . Then, explain the relationship between the level of consumption and the risk free rate in this economy.

$$\begin{aligned}
 u'(C_0) &= R_f \delta E[u'(C_1)] \\
 u'(C_0) &= \frac{1}{C_0} \\
 u'(C_1) &= \frac{1}{C_1} \\
 \frac{1}{R_f} &= \delta \left[ \frac{C_0}{C_1} \right] \quad E \left[ \frac{u'(C_1)}{u'(C_0)} \right] \\
 R_f &= \frac{1}{\delta} \left( \frac{C_1}{C_0} \right)
 \end{aligned}$$

implying that when the consumption level is low, the risk free rate is high.

Score.....

**Question 1.2 ( 10 marks)** Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$\begin{aligned} \epsilon &= \frac{R_f}{C_1} \frac{\frac{\partial C_1}{\partial C_0}}{\frac{\partial R_f}{\partial R_f}} = \frac{\frac{\partial \ln(C_1/C_0)}{\partial \ln(R_f)}}{1} \\ &= \frac{1}{\delta} \\ &= \delta \end{aligned}$$

NOTE

$$\frac{\frac{\partial R_f}{\partial C_1}}{\frac{\partial C_1}{\partial C_0}} = \frac{1}{\delta}$$

if interest rate is falling, the individual's consumption level will

1. substitution effect decreases the return from transforming current consumption into future consumption, providing less incentive to save.
2. income effect from lower return of savings makes the individual worse off and would lower consumption in both periods.

Score.....

**Question 1.3 ( 10 marks)** Solve for the pricing kernel  $P_i$  of any risky asset  $i$  in this economy. Then explain the meaning of this pricing kernel.

$$P_i^N = E [ M_{01} X_i^N ]$$

where

$$\begin{aligned} M_{01} &= (\delta / 1_{01}) \frac{u'(c_1)}{u'(c_0)} \\ &= (\delta / 1_{01}) \left( \frac{c_0}{c_1} \right) \end{aligned}$$

SDF for nominal returns equal to the real pricing kernel.

$$u(c) = \ln(c)$$

Score.....

**Question 1.4 (10 marks)** Calculate Hansen-Jaganathan Bound and explain the meaning.

$$M_{01} = \frac{\delta u'(c_1)}{u'(c_0)} = \delta \left( \frac{c_1}{c_0} \right)$$

$$\begin{aligned} S_e &= \frac{E[R_M] - R_f}{\sigma_{R_M}} \\ &= \sigma_{M01} R_f \end{aligned}$$

slope of  
CML