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EE325 Section 1 HW 2   Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$y-\bar{y}$	$x-\bar{x}$
1	2.8	63	-0.4125	-14.625
2	3.4	72	0.1875	-5.625
3	3	78	-0.2125	0.375
4	3.5	81	0.2875	3.375
5	3.6	87	0.3875	9.375
6	3.0	75	-0.2125	-2.625
7	2.7	75	-0.5125	-2.625
8	3.7	90	0.4875	12.375

$\bar{y} = 3.2125$ $\bar{x} = 77.625$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$E(\hat{\beta}_1) = \beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{17.4375}{511.875} = 0.034066$$

$$E(\hat{\beta}_0) = \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$= 3.2125 - (0.034066)(77.625)$$

$$\beta_0 = 0.56812675$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \widehat{u}_i = 0$.

$$\hat{Y}_i = \underbrace{\hat{\beta}_0}_{E(\beta_0)} + \hat{\beta}_1 x_i$$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{\beta}_0 = 0.0568132$

$\hat{\beta}_1 = 0.034066$

i	$\hat{y}_i$	$\hat{u}_i$
1	$0.57 + 0.0341(\overset{x_1}{63}) = 2.71$	$Y_1 - \hat{Y}_1 = 0.085714$
2	3.020879	0.379120879
3	3.225275	-0.225274725
4	3.327473	0.172527473
5	3.531868	0.068131868
6	3.123077	-0.123076923
7	3.123077	-0.423076923
8	3.634066	0.065934066

$\rightarrow \Sigma \hat{u}_i = 0$

1.3 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$\sigma^2 = \frac{\Sigma \hat{u}_i^2}{n-2} =$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\Sigma(x_i-\bar{x})^2} =$$

$$var(\hat{\beta}_0) = \frac{\Sigma x_i^2}{n\Sigma(x_i-\bar{x})^2} \cdot \sigma^2 =$$

$$var(\hat{u}_i) = \frac{\Sigma(\hat{u}_i)^2}{n-2} =$$

2. Data is listed in the table

X_i (20)	Y_i (9.1)	$X - \bar{X}$	$Y - \bar{Y}$
10	0	-10	-9.1
12	2	-8	-7.1
14	5	-6	-4.1
16	6	-4	-3.1
18	7	-2	-2.1
22	10	2	0.9
24	10	4	0.9
26	15	6	5.9
28	16	8	6.9
30	20	10	10.9

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$E(\hat{\beta}_1) = \beta_1 = \frac{\Sigma(Y-\bar{Y})(X-\bar{X})}{\Sigma(X-\bar{X})^2} = \frac{394}{440} = 0.8955$$

$$E(\hat{\beta}_0) = \beta_0 = \bar{Y} - \beta_1 \bar{X}$$

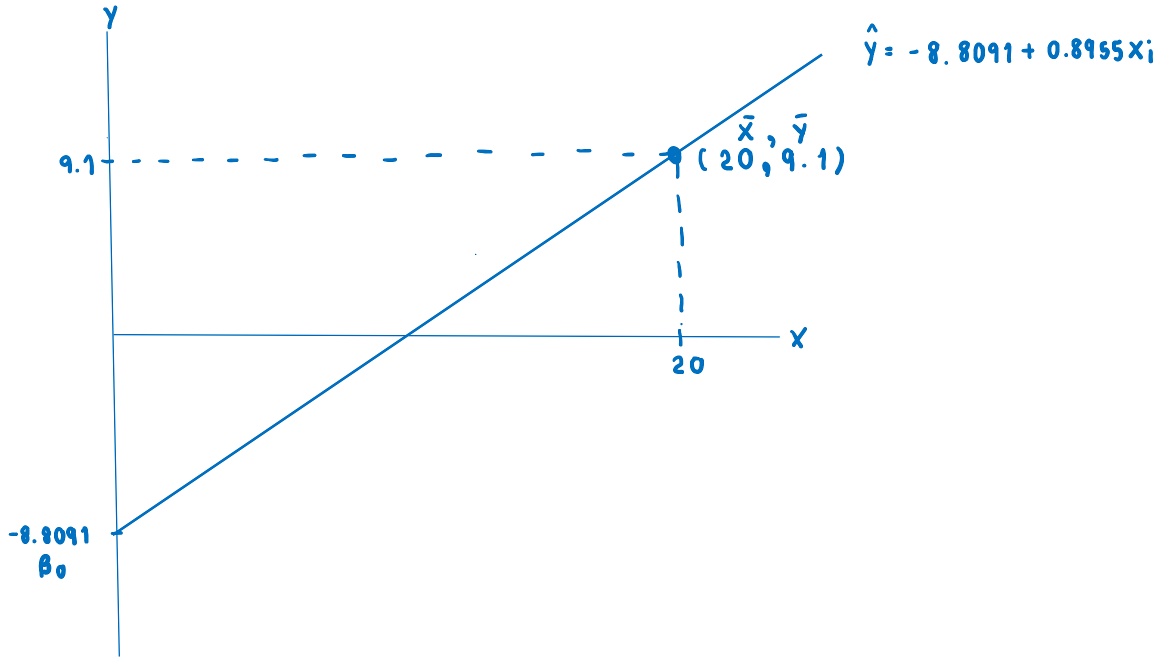
$$= 9.1 - (0.8955)(20)$$

$$= -8.8091$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

i	$\hat{Y}_i$	$\hat{u}_i$
1	0.1455	-0.1455
2	1.9363	0.0636
3	3.7273	1.2727
4	5.5182	0.4818
5	7.3091	-0.3091
6	10.8909	-0.8909
7	12.6818	-2.6818
8	14.4727	0.5273
9	16.2636	-0.2636
10	18.0545	1.9455

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



2.4 If X_i =16, what is the predicted Y?

$\hat{Y}_4 = 5.5182$

2.5 Find $var(\widehat{u}_i), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2}$:

$Var(\hat{\beta}_1) = \frac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2}$:

$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n\sum(x_i - \bar{x})^2} \cdot \sigma^2$:

$var(\hat{u}_i) = \frac{\sum(\hat{u}_i)^2}{n-2}$:

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \text{ ,}$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

proof : $E(\hat{\beta}_1) = \beta_1$