Thammasat University
Faculty of Economics
Bachelor of Economics (International Program)

AC201
Fundamental Accounting

Semester 2/2011

Course Materials

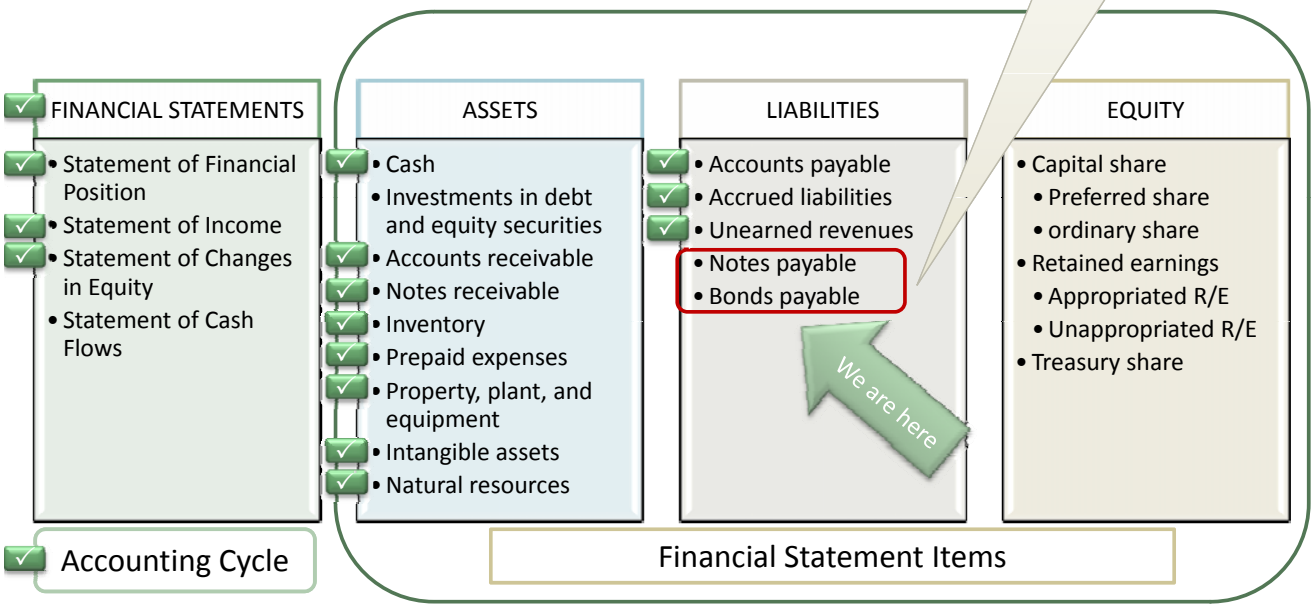
Topic:
Time Value of Money Concept

Session:
Session #10

Instructor:
Assistant Professor Dr. Orapan Yolrabil

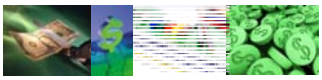
Time Value of Money Concept:
Application of TVM in determining bond issue price

I/S: Interest expense



FINANCIAL STATEMENT ANALYSIS

AC201 Fundamental Accounting



TIME VALUE OF MONEY: PRESENT AND FUTURE VALUES

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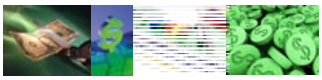
Time Value of Money

- ***Time value of money concept*** **refers to costs or benefits derived from holding or not holding money over time.**
  - ***Interest*** is the cost of using money for a specific period.
    - ***Simple interest*** is the interest cost for one or more periods when the principal sum – the amount on which interest is computed – stays the same from period to period.
    - ***Compound interest*** is the interest cost for two or more periods when after each period, the interest earned in that period is added to the amount on which interest is computed in future periods.

"Time is money"



Simple Interest

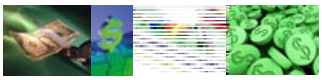
- **Example of simple interest:**
  - The Company accepts an 8 percent, 30,000 Baht note due in 90 days. How much will the Company receive at that time?
    - The interest is calculated as follows:
      - Interest = Principal x Rate x Time
        = 30,000 x 8/100 x 90/365
        = 591.78 Baht
    - Therefore, the total that the Company will receive is 30,591 Baht, calculated as follows:
      - Total = Principal + Interest
        = 30,000 + 591.78
        = 30,591.78 Baht

Compound Interest

- **Example of compound interest:**
  - The Company deposits 5,000 Baht in an account that pays 6 percent interest. It expects to leave the principal and accumulated interest in the account for three years. How much will the account total at the end of three years?
    - Assume that the interest is paid at the end of the year and is added to the principal at that time, and that this total in turn earns interest.
    - The amount at the end of three years is computed as follows:

(1) Year	(2) Principal Amount at Beginning of Year	(3) Annual Amount of Interest (Col. 2 x 6%)	(4) Accumulated Amount at End of Year (Col. 2 + Col. 3)
1	5,000.00	300.00	5,300.00
2	5,300.00	318.00	5,618.00
3	5,618.00	337.08	5,955.08

    - At the end of three years, the Company will have 5,955.08 Baht in its account.

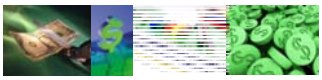


Future Value

- **Future value is the amount an investment will be worth at a future date if invested at compound interest.**
  - An example of question asking about future value is *"what is the future value of a single sum (5,000 Baht) at compound interest (6 percent) for three years?"*

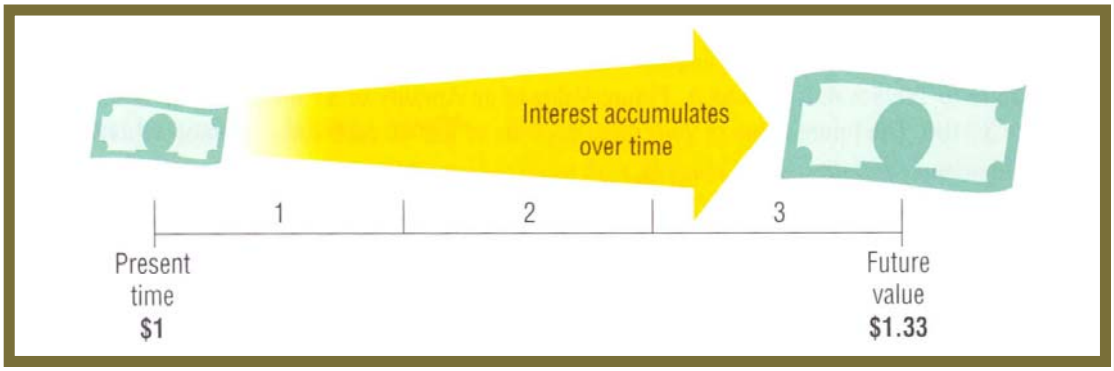
Future Value of a Single Sum (FV)

- **In future value of a single amount problems, you will be asked to calculate how much money you will have in the future as the result of investing a certain amount in the present.**
  - If you were to receive a gift of 10,000 Baht, for instance, you might decide to put it in a savings account and use the money as a down payment on a house after you graduate. The future value computation would tell you how much money will be available when you graduate.
    - To solve a future value problem, you need to know three items:
      - Amount to be invested
      - Interest rate (i) the amount will earn
      - Number of periods (n) in which the amount will earn interest



Future Value of a Single Sum (Cont.)

- **Since the future value concept is based on compound interest, the amount of interest for each period is calculated by multiplying the principal plus any interest not paid out in prior periods.**
  - Graphically, the calculation of the future value of 1 Baht for three periods and an interest rate of 10 percent may be represented as follows:

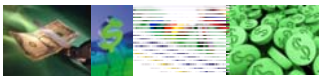


Future Value of a Single Sum (Cont.)

- **Example of Future Value of a Single Sum:**
  - Assume that on January 1, 2009, you deposit 1,000 Baht in a savings account at 10 percent annual interest, compounded annually.
    - At the end of three years, the 1,000 Baht will have increased to 1,331 Baht as follows:

(1) Year	(2) Principal Amount at Beginning of Year	(3) Annual Amount of Interest (Col. 2 x 10%)	(4) Accumulated Amount at End of Year (Col. 2 + Col. 3)
1	1,000.00	100.00	1,100.00
2	1,100.00	110.00	1,210.00
3	1,210.00	121.00	1,331.00

    - FV Table:
      - FV = PV x FV Factor
        1,331 = 1,000 x 1.3310
    - Financial calculator:
      - PV = 1,000, I = 10%, n = 3 → FV = 1,331

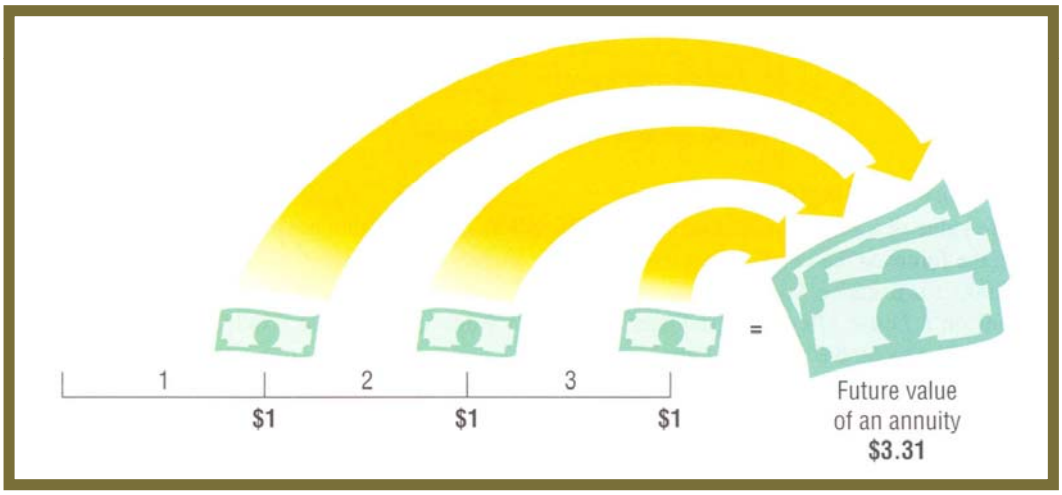


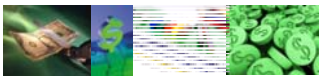
Future Value of An Annuity (FVA)

- **Ordinary annuity is a series of equal payments made at the end of equal intervals of time, with compound interest of these payments.**
  - The future value of an annuity includes compound interest on each payment from the date of payment to the end of the term of the annuity. Each new payment accumulates less interest than prior payments, only because the number of periods remaining in which to accumulate interest decreases.

Future Value of An Annuity (Cont.)

- The future value of annuity of 1 Baht for three periods at 10 percent may be represented graphically as:





Future Value of An Annuity (Cont.)

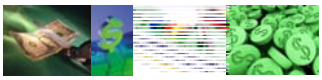
- **Example of Future Value of an Annuity:**
  - Assume that each year for three years, you deposit 1,000 Baht cash in a savings account at 10 percent interest per year. You make the first 1,000 Baht deposit on December 31, 2009, the second one on December 31, 2010, and the third and last one on December 31, 2011.
    - The first 1,000 Baht deposit earns compound interest for two years (for a total principal and interest of 1,210 Baht); the second deposit earns interest for one year (for a total principal and interest of 1,100 Baht). The third deposit earns no interest because it was made on the day that the balance is computed.
    - Thus, the total amount in the savings account at the end of three years is 3,310 Baht (1,210 + 1,100 + 1,000).

Future Value of An Annuity (Cont.)

- Compound interest calculation:

(1) Year	(2) Principal Amount at Beginning of Year	(3) Annual Amount of Interest (Col. 2 x 10%)	(4) Periodic Payment	(5) Total Amount Accumulated at the End of the Year (Col. 2 + Col.3 + Col. 4)
1	-	-	1,000.00	1,000.00
2	1,000.00	100.00	1,000.00	2,100.00
3	2,100.00	210.00	1,000.00	3,310.00

- FV Table:
  - FV = PMT x FVA Factor
    3,310 = 1,000 x 3.3100
- Financial calculator:
  - PMT = 1,000, I = 10%, n = 3 → FV = 3,310

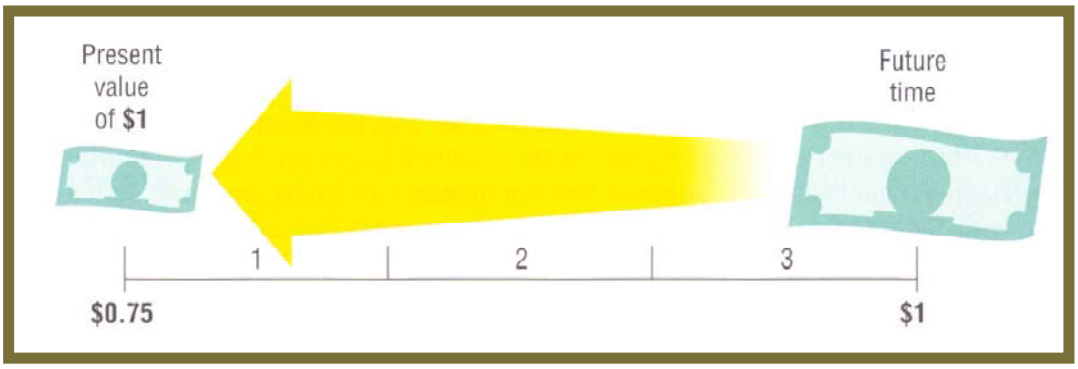


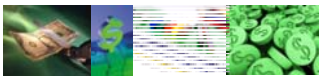
Present Value

- **Present value is the amount that must be invested today at a given rate of interest to produce a given future value.**
  - The amount to be received in the future (future value) is not worth as much as today as an amount received today (present value).
  - Example:
    - The Company needs 1,000 Baht one year from now. How much does she have to invest today to achieve that goal if the interest rate is 5 percent?
      - Recall: $PV \times (1 + i)^n = FV$
      - $PV = FV / (1 + i)^n$
      - Thus, $PV = 1,000 / (1.05)^1$
      - $PV = 952.38$ Baht

Present Value of a Single Sum (PV)

- **Present value of a single sum**
  - The present value of a single amount is the worth to you today of receiving that amount some time in the future.
  - Graphically, the present value of 1 Baht due at the end of the third period with an interest rate of 10 percent can be represented as follows:





Present Value of a Single Sum (Cont.)

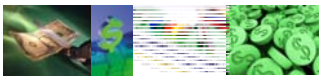
- **Example of Present Value of a Single Sum**
  - Suppose the Company wants to be sure of having 4,000 Baht at the end of three years. How much must it invest today in a 5 percent savings account to achieve this goal?
    - The calculation is presented as follows:

(1) Year	(2) Amount at the End of Year	÷	(3) Divided by (1 + i)	(4) Present Value at the Beginning of Year (Col. 2 ÷ Col. 3)
1	4,000.00	÷	1.05	3,809.52
2	3,809.52	÷	1.05	3,628.12
3	3,628.12	÷	1.05	3,455.35

    - The Company must invest 3,455.35 Baht $(4,000/(1.05)^3)$ today to achieve a value of 4,000 Baht in three years.

Present Value of a Single Sum (Cont.)

- PV Table:
  - PV = FV x PV factor
    3,455.35 = 4,000 x 0.8640
- Financial calculator:
  - FV = 4,000, I = 10%, n = 3 → PV = 3,455.35

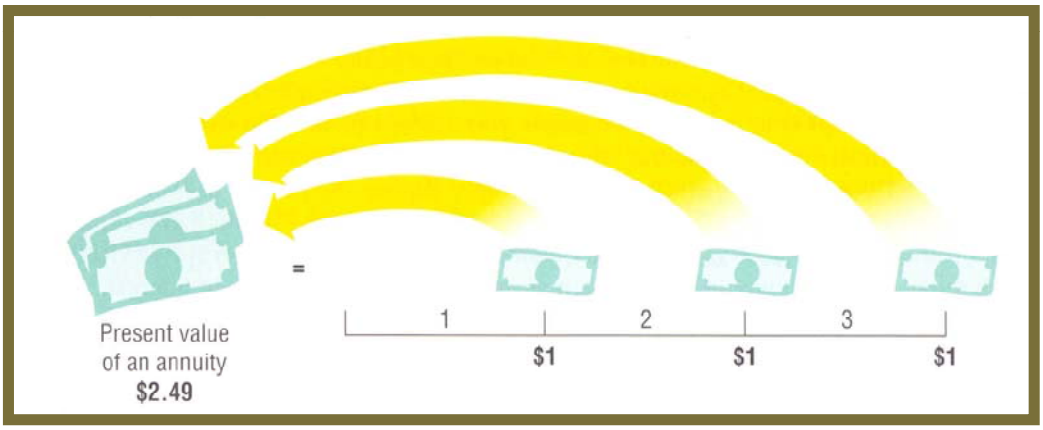


Present Value of an Annuity (PVA)

- **Present value of an ordinary annuity**
  - Recall that an annuity is a series of consecutive payments characterized by
    - An equal dollar amount each interest period.
    - Interest periods of equal length (year, half a year, quarter, or month)
    - An equal interest rate each interest period.
  - The present value of an annuity is the value now of a series of equal amounts to be received (or paid out) for some specified number of periods in the future. It is computed by discounting each of the equal periodic amounts.

Present Value of an Annuity (Cont.)

- The present value of an annuity of 1 Baht for three periods at 10 percent interest may be represented graphically as follows:



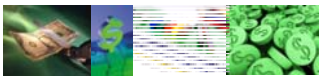
Present Value of an Annuity (Cont.)

- **Example of Present Value of an Annuity:**
  - The Company has sold a piece of property and is to receive 15,000 Baht in three equal annual payments of 5,000 Baht beginning one year from today. What is the present value of this sale if the current interest rate is 5 percent?
    - PV Table:

Future Receipt (Annuity)				Present		Present
Year 1	Year 2	Year 3		Value Factor		Value
5,000.00			x	0.952	=	฿4,760.00
	5,000.00		x	0.907	=	฿4,535.00
		5,000.00	x	0.864	=	฿4,320.00
						฿13,615.00

Present Value of an Annuity (Cont.)

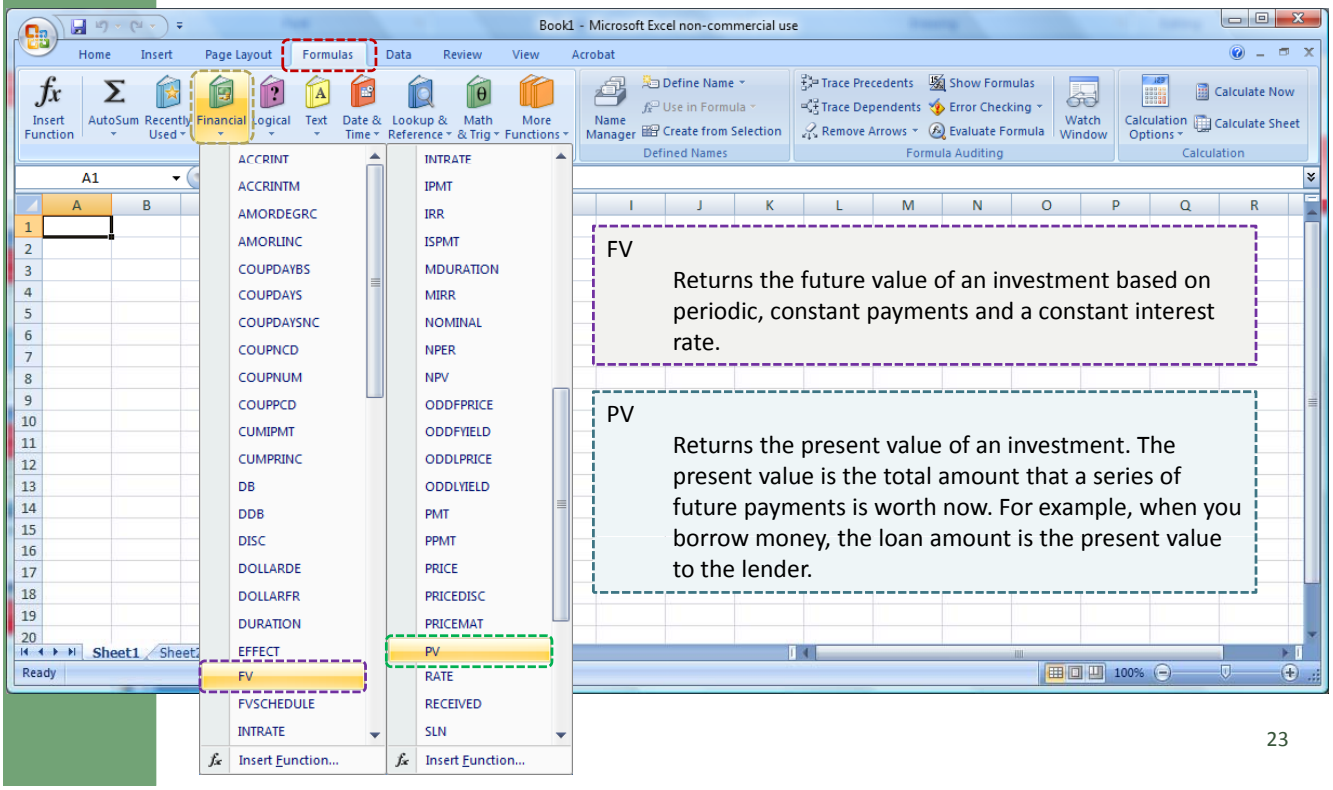
- Or PVA Table:
  - PV = PMT x PVA Factor
  - 13,615 = 5,000 x 2.723
- Financial calculator:
  - PMT = 5,000, I = 5%, n = 3 → PV = 13,615



Using Formulas to Determine FV and PV

- **Future Value of a Single Sum**
  - $FV = PV \times (1 + i)^n$
- **Future Value of an Annuity**
  - $FVA = PMT [((1 + i)^n - 1) / i]$
- **Present Value of a Single Sum**
  - $PV = FV / (1 + i)^n$
- **Present Value of an Annuity**
  - $PVA = PMT [(1 - (1 / (1 + i)^n)) / i]$

Using Excel to determine FV and PV



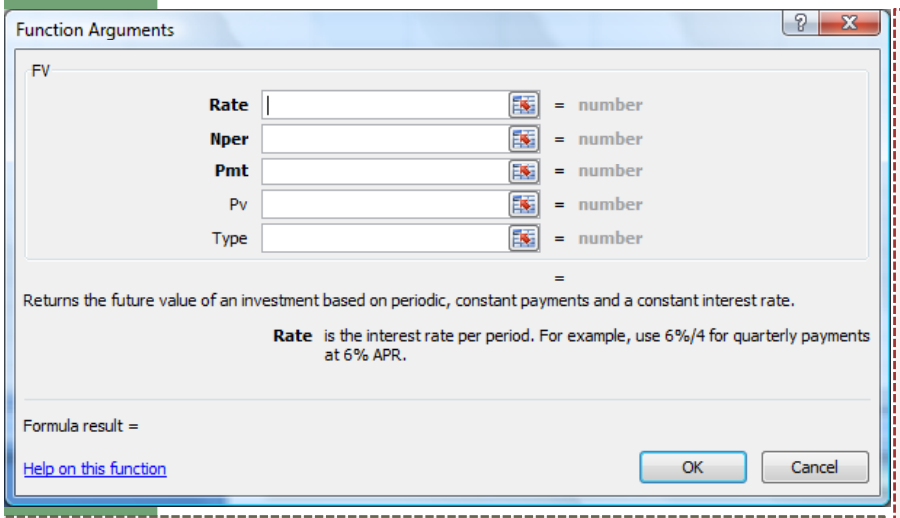
FV

Returns the future value of an investment based on periodic, constant payments and a constant interest rate.

PV

Returns the present value of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender.

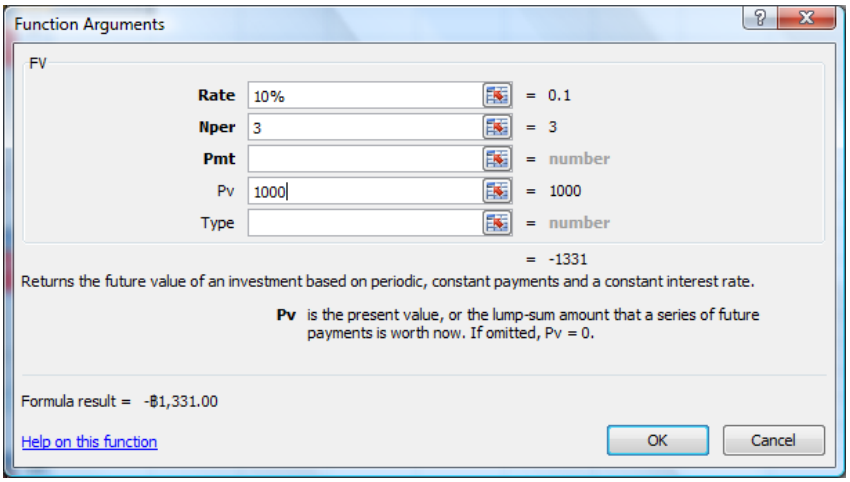
Future Value Calculation with Excel



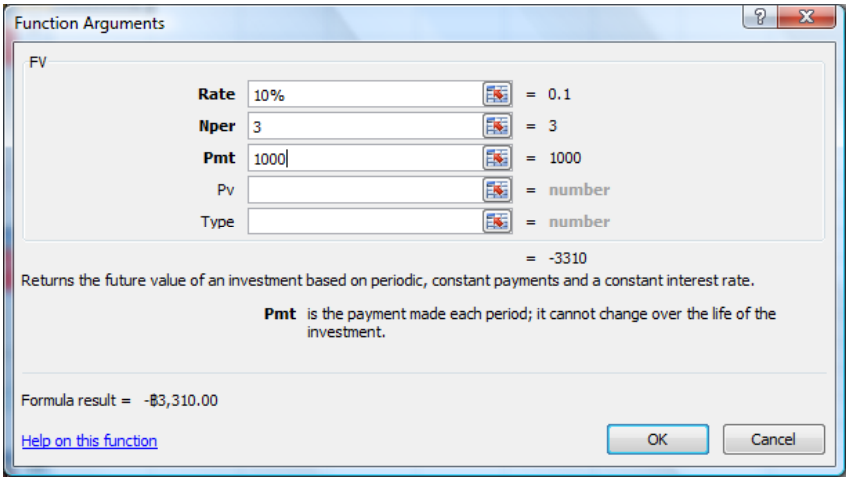
Syntax

FV(rate,nper,pmt,pv,type)

Rate is the interest rate per period.
Nper is the total number of payment periods in an annuity.
Pmt is the payment made each period; it cannot change over the life of the annuity. Typically, pmt contains principal and interest but no other fees or taxes. If pmt is omitted, you must include the pv argument.
Pv is the present value, or the lump-sum amount that a series of future payments is worth right now. If pv is omitted, it is assumed to be 0 (zero), and you must include the pmt argument.
Type is the number 0 or 1 and indicates when payments are due. If type is omitted, it is assumed to be 0.

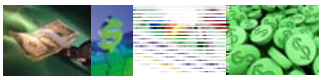


Future Value of a Single Sum

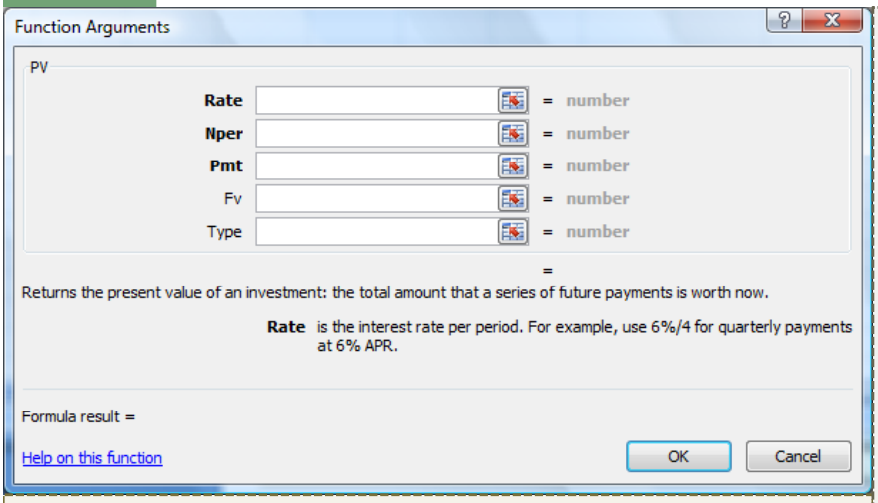


Future Value of an Annuity

AC201 Fundamental Accounting



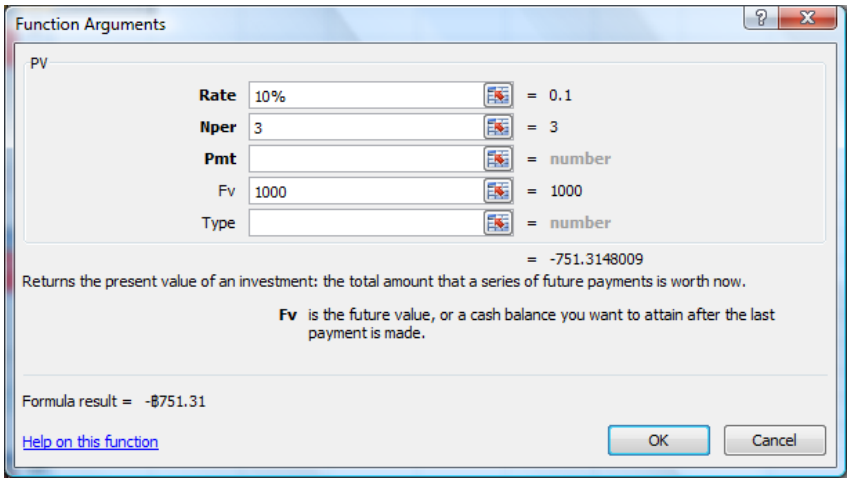
Present Value Calculation with Excel



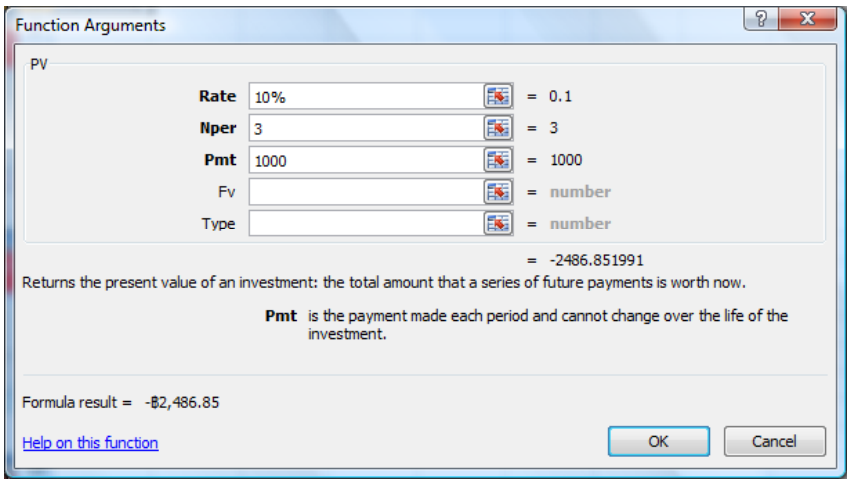
Syntax

PV(rate,nper,pmt,fv,type)

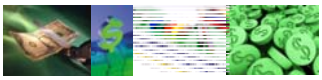
Rate is the interest rate per period.
Nper is the total number of payment periods in an annuity.
Pmt is the payment made each period and cannot change over the life of the annuity. Typically, pmt includes principal and interest but no other fees or taxes. If pmt is omitted, you must include the fv argument.
Fv is the future value, or a cash balance you want to attain after the last payment is made. If fv is omitted, it is assumed to be 0 (the future value of a loan, for example, is 0). You could then make a conservative guess at an interest rate and determine how much you must save each month. If fv is omitted, you must include the pmt argument.
Type is the number 0 or 1 and indicates when payments are due.



Present Value of a Single Sum



Present Value of an Annuity



BA II Plus™ Calculator

BA II PLUS™ Calculator

TVM and Amortization Worksheet Variables

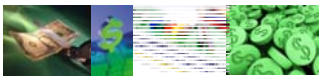
Variable	Key	Display	Type of Variable
Number of periods	[N]	**N**	Enter-or-compute
Interest rate per year	[I/Y]	**I/Y**	Enter-or-compute
Present value	[PV]	**PV**	Enter-or-compute
Payment	[PMT]	**PMT**	Enter-or-compute
Future value	[FV]	**FV**	Enter-or-compute
Number of payments per year	[2nd] [P/Y]	**P/Y**	Enter-only
Number of compounding periods per year	[↓]	**C/Y**	Enter-only
End-of-period payments	[2nd] [BGN]	**END**	Setting
Beginning-of-period payments	[2nd] [SET]	**BGN**	Setting
Starting payment	[2nd] [AMORT]	**P1**	Enter-only
Ending payment	[↓]	**P2**	Enter-only
Balance	[↓]	**BAL**	Auto-compute
Principal paid	[↓]	**PRN**	Auto-compute
Interest paid	[↓]	**INT**	Auto-compute

Resetting the TVM and Amortization Worksheet Variables

- To reset all calculator variables and formats to default values (including TVM and amortization variables), press [2nd] [RESET] [ENTER]:

Variable	Default	Variable	Default
N	0	**END/BGN**	**END**
I/Y	0	**P1**	1
PV	0	**P2**	1
PMT	0	**BAL**	0
FV	0	**PRN**	0
P/Y	1	**INT**	0
C/Y	1		

- To reset only the TVM variables (**N**, **I/Y**, **PV**, **PMT**, **FV**) to default values, press [2nd] [CLR TVM].
- To reset **P/Y** and **C/Y** to default values, press [2nd] [P/Y] [2nd] [CLR WORK].
- To reset the Amortization worksheet variables (**P1**, **P2**, **BAL**, **PRN**, **INT**) to default values, press [2nd] [CLR WORK] while in the Amortization worksheet.
- To reset **END/BGN** to the default value, press [2nd] [BGN] [2nd] [CLR WORK].



BA II Plus™ Calculator (Cont.)

Examples: Computing Value in Savings

These examples show you how to compute the future and present values of a savings account paying 0.5% compounded at the end of each year with a 20-year time frame.

Computing Future Value

Example: If you open the account with $5,000, how much will you have after 20 years?

To	Press	Display	
Set all variables to defaults.	[2nd] [RESET] [ENTER]	**RST**	**0.00**
Enter number of payments.	**20** [N]	**N=**	**20.00**◁
Enter interest rate.	**.5** [I/Y]	**I/Y=**	**0.50**◁
Enter beginning balance.	**5000** [+/−] [PV]	**PV=**	**-5,000.00**◁
Compute future value.	[CPT] [FV]	**FV=**	**5,524.48**∗

Answer: The account will be worth $5,524.48 after 20 years.

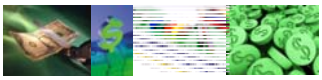
BA II Plus™ Calculator (Cont.)

Computing Present Value

Example: How much money must you deposit to have $10,000 in 20 years?

To	Press		Display
Enter final balance.	**10000** [FV]	**FV=**	**10,000.00**◁
Compute present value.	[CPT] [PV]	**PV=**	**-9,050.63**∗

Answer: You must deposit $9,050.63.

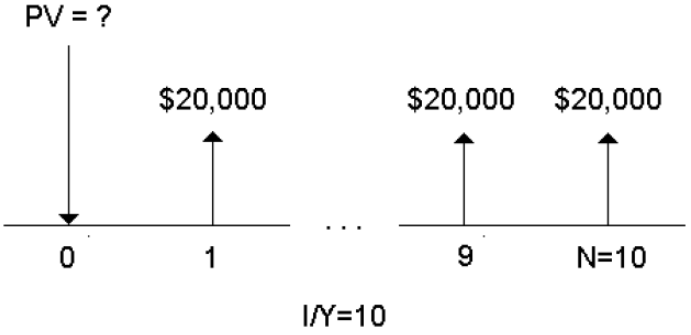


BA II Plus™ Calculator (Cont.)

Example: Computing Present Value in Annuities

The Furros Company purchased equipment providing an annual savings of $20,000 over 10 years. Assuming an annual discount rate of 10%, what is the present value of the savings using an ordinary annuity and an annuity due?

Cost Savings for a Present-Value Ordinary Annuity



BA II Plus™ Calculator (Cont.)

To	Press		Display
Set all variables to defaults.	2nd [RESET] ENTER	RST	0.00
Enter number of payments.	10 N	N=	10.00◁
Enter interest rate per payment period.	10 I/Y	I/Y=	10.00◁
Enter payment.	20000 +/− PMT	PMT=	-20,000.00◁
Compute present value (ordinary annuity).	CPT PV	PV=	122,891.34*

TABLE 1 Future Value of $1
$FV = \$1\ (1 + i)^n$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.01000	1.01500	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.05500	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000	1.20000
2	1.02010	1.03022	1.04040	1.05063	1.06090	1.07123	1.08160	1.09203	1.10250	1.11303	1.12360	1.14490	1.16640	1.18810	1.21000	1.23210	1.25440	1.44000
3	1.03030	1.04568	1.06121	1.07689	1.09273	1.10872	1.12486	1.14117	1.15763	1.17424	1.19102	1.22504	1.25971	1.29503	1.33100	1.36763	1.40493	1.72800
4	1.04060	1.06136	1.08243	1.10381	1.12551	1.14752	1.16986	1.19252	1.21551	1.23882	1.26248	1.31080	1.36049	1.41158	1.46410	1.51807	1.57352	2.07360
5	1.05101	1.07728	1.10408	1.13141	1.15927	1.18769	1.21665	1.24618	1.27628	1.30696	1.33823	1.40255	1.46933	1.53862	1.61051	1.68506	1.76234	2.48832
6	1.06152	1.09344	1.12616	1.15969	1.19405	1.22926	1.26532	1.30226	1.34010	1.37884	1.41852	1.50073	1.58687	1.67710	1.77156	1.87041	1.97382	2.98598
7	1.07214	1.10984	1.14869	1.18869	1.22987	1.27228	1.31593	1.36086	1.40710	1.45468	1.50363	1.60578	1.71382	1.82804	1.94872	2.07616	2.21068	3.58318
8	1.08286	1.12649	1.17166	1.21840	1.26677	1.31681	1.36857	1.42210	1.47746	1.53469	1.59385	1.71819	1.85093	1.99256	2.14359	2.30454	2.47596	4.29982
9	1.09369	1.14339	1.19509	1.24886	1.30477	1.36290	1.42331	1.48610	1.55133	1.61909	1.68948	1.83846	1.99900	2.17189	2.35795	2.55804	2.77308	5.15978
10	1.10462	1.16054	1.21899	1.28008	1.34392	1.41060	1.48024	1.55297	1.62889	1.70814	1.79085	1.96715	2.15892	2.36736	2.59374	2.83942	3.10585	6.19174
11	1.11567	1.17795	1.24337	1.31209	1.38423	1.45997	1.53945	1.62285	1.71034	1.80209	1.89830	2.10485	2.33164	2.58043	2.85312	3.15176	3.47855	7.43008
12	1.12683	1.19562	1.26824	1.34489	1.42576	1.51107	1.60103	1.69588	1.79586	1.90121	2.01220	2.25219	2.51817	2.81266	3.13843	3.49845	3.89598	8.91610
13	1.13809	1.21355	1.29361	1.37851	1.46853	1.56396	1.66507	1.77220	1.88565	2.00577	2.13293	2.40985	2.71962	3.06580	3.45227	3.88328	4.36349	10.69932
14	1.14947	1.23176	1.31948	1.41297	1.51259	1.61869	1.73168	1.85194	1.97993	2.11609	2.26090	2.57853	2.93719	3.34173	3.79750	4.31044	4.88711	12.83918
15	1.16097	1.25023	1.34587	1.44830	1.55797	1.67535	1.80094	1.93528	2.07893	2.23248	2.39656	2.75903	3.17217	3.64248	4.17725	4.78459	5.47357	15.40702
16	1.17258	1.26899	1.37279	1.48451	1.60471	1.73399	1.87298	2.02237	2.18287	2.35526	2.54035	2.95216	3.42594	3.97031	4.59497	5.31089	6.13039	18.48843
17	1.18430	1.28802	1.40024	1.52162	1.65285	1.79468	1.94790	2.11338	2.29202	2.48480	2.69277	3.15882	3.70002	4.32763	5.05447	5.89509	6.86604	22.18611
18	1.19615	1.30734	1.42825	1.55966	1.70243	1.85749	2.02582	2.20848	2.40662	2.62147	2.85434	3.37993	3.99602	4.71712	5.55992	6.54355	7.68997	26.62333
19	1.20811	1.32695	1.45681	1.59865	1.75351	1.92250	2.10685	2.30786	2.52695	2.76565	3.02560	3.61653	4.31570	5.14166	6.11591	7.26334	8.61276	31.94800
20	1.22019	1.34686	1.48595	1.63862	1.80611	1.98979	2.19112	2.41171	2.65330	2.91776	3.20714	3.86968	4.66096	5.60441	6.72750	8.06231	9.64629	38.33760
21	1.23239	1.36706	1.51567	1.67958	1.86029	2.05943	2.27877	2.52024	2.78596	3.07823	3.39956	4.14056	5.03383	6.10881	7.40025	8.94917	10.80385	46.00512
25	1.28243	1.45095	1.64061	1.85394	2.09378	2.36324	2.66584	3.00543	3.38635	3.81339	4.29187	5.42743	6.84848	8.62308	10.83471	13.58546	17.00006	95.39622
30	1.34785	1.56308	1.81136	2.09757	2.42726	2.80679	3.24340	3.74532	4.32194	4.98395	5.74349	7.61226	10.06266	13.26768	17.44940	22.89230	29.95992	237.37631
40	1.48886	1.81402	2.20804	2.68506	3.26204	3.95926	4.80102	5.81636	7.03999	8.51331	10.28572	14.97446	21.72452	31.40942	45.25926	65.00087	93.05097	1469.77160

TABLE 2 Present Value of $1

$$PV = \frac{\$1}{(1 + i)^n}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	0.98030	0.97066	0.96117	0.95181	0.94260	0.93351	0.92456	0.91573	0.90703	0.89845	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719	0.69444
3	0.97059	0.95632	0.94232	0.92860	0.91514	0.90194	0.88900	0.87630	0.86384	0.85161	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178	0.57870
4	0.96098	0.94218	0.92385	0.90595	0.88849	0.87144	0.85480	0.83856	0.82270	0.80722	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552	0.48225
5	0.95147	0.92826	0.90573	0.88385	0.86261	0.84197	0.82193	0.80245	0.78353	0.76513	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743	0.40188
6	0.94205	0.91454	0.88797	0.86230	0.83748	0.81350	0.79031	0.76790	0.74622	0.72525	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663	0.33490
7	0.93272	0.90103	0.87056	0.84127	0.81309	0.78599	0.75992	0.73483	0.71068	0.68744	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235	0.27908
8	0.92348	0.88771	0.85349	0.82075	0.78941	0.75941	0.73069	0.70319	0.67684	0.65160	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388	0.23257
9	0.91434	0.87459	0.83676	0.80073	0.76642	0.73373	0.70259	0.67290	0.64461	0.61763	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061	0.19381
10	0.90529	0.86167	0.82035	0.78120	0.74409	0.70892	0.67556	0.64393	0.61391	0.58543	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197	0.16151
11	0.89632	0.84893	0.80426	0.76214	0.72242	0.68495	0.64958	0.61620	0.58468	0.55491	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748	0.13459
12	0.88745	0.83639	0.78849	0.74356	0.70138	0.66178	0.62460	0.58966	0.55684	0.52598	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668	0.11216
13	0.87866	0.82403	0.77303	0.72542	0.68095	0.63940	0.60057	0.56427	0.53032	0.49856	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917	0.09346
14	0.86996	0.81185	0.75788	0.70773	0.66112	0.61778	0.57748	0.53997	0.50507	0.47257	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462	0.07789
15	0.86135	0.79985	0.74301	0.69047	0.64186	0.59689	0.55526	0.51672	0.48102	0.44793	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270	0.06491
16	0.85282	0.78803	0.72845	0.67362	0.62317	0.57671	0.53391	0.49447	0.45811	0.42458	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312	0.05409
17	0.84438	0.77639	0.71416	0.65720	0.60502	0.55720	0.51337	0.47318	0.43630	0.40245	0.37136	0.31657	0.27027	0.23107	0.19784	0.16963	0.14564	0.04507
18	0.83602	0.76491	0.70016	0.64117	0.58739	0.53836	0.49363	0.45280	0.41552	0.38147	0.35034	0.29586	0.25025	0.21199	0.17986	0.15282	0.13004	0.03756
19	0.82774	0.75361	0.68643	0.62553	0.57029	0.52016	0.47464	0.43330	0.39573	0.36158	0.33051	0.27651	0.23171	0.19449	0.16351	0.13768	0.11611	0.03130
20	0.81954	0.74247	0.67297	0.61027	0.55368	0.50257	0.45639	0.41464	0.37689	0.34273	0.31180	0.25842	0.21455	0.17843	0.14864	0.12403	0.10367	0.02608
21	0.81143	0.73150	0.65978	0.59539	0.53755	0.48557	0.43883	0.39679	0.35894	0.32486	0.29416	0.24151	0.19866	0.16370	0.13513	0.11174	0.09256	0.02174
24	0.78757	0.69954	0.62172	0.55288	0.49193	0.43796	0.39012	0.34770	0.31007	0.27666	0.24698	0.19715	0.15770	0.12640	0.10153	0.08170	0.06588	0.01258
25	0.77977	0.68921	0.60953	0.53939	0.47761	0.42315	0.37512	0.33273	0.29530	0.26223	0.23300	0.18425	0.14602	0.11597	0.09230	0.07361	0.05882	0.01048
28	0.75684	0.65910	0.57437	0.50088	0.43708	0.38165	0.33348	0.29157	0.25509	0.22332	0.19563	0.15040	0.11591	0.08955	0.06934	0.05382	0.04187	0.00607
29	0.74934	0.64936	0.56311	0.48866	0.42435	0.36875	0.32065	0.27902	0.24295	0.21168	0.18456	0.14056	0.10733	0.08215	0.06304	0.04849	0.03738	0.00506
30	0.74192	0.63976	0.55207	0.47674	0.41199	0.35628	0.30832	0.26700	0.23138	0.20064	0.17411	0.13137	0.09938	0.07537	0.05731	0.04368	0.03338	0.00421
31	0.73458	0.63031	0.54125	0.46511	0.39999	0.34423	0.29646	0.25550	0.22036	0.19018	0.16425	0.12277	0.09202	0.06915	0.05210	0.03935	0.02980	0.00351
40	0.67165	0.55126	0.45289	0.37243	0.30656	0.25257	0.20829	0.17193	0.14205	0.11746	0.09722	0.06678	0.04603	0.03184	0.02209	0.01538	0.01075	0.00068

TABLE 3 Future Value of an Ordinary Annuity of $1

$$FVA = \frac{(1 + i)^n - 1}{i}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0150	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0550	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200	2.2000
3	3.0301	3.0452	3.0604	3.0756	3.0909	3.1062	3.1216	3.1370	3.1525	3.1680	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744	3.6400
4	4.0604	4.0909	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3423	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793	5.3680
5	5.1010	5.1523	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.5811	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528	7.4416
6	6.1520	6.2296	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.8881	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152	9.9299
7	7.2135	7.3230	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.2669	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.0890	12.9159
8	8.2857	8.4328	8.5830	8.7361	8.8923	9.0517	9.2142	9.3800	9.5491	9.7216	9.8975	10.2598	10.6366	11.0285	11.4359	11.8594	12.2997	16.4991
9	9.3685	9.5593	9.7546	9.9545	10.1591	10.3685	10.5828	10.8021	11.0266	11.2563	11.4913	11.9780	12.4876	13.0210	13.5795	14.1640	14.7757	20.7989
10	10.4622	10.7027	10.9497	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	12.8754	13.1808	13.8164	14.4866	15.1929	15.9374	16.7220	17.5487	25.9587
11	11.5668	11.8633	12.1687	12.4835	12.8078	13.1420	13.4864	13.8412	14.2068	14.5835	14.9716	15.7836	16.6455	17.5603	18.5312	19.5614	20.6546	32.1504
12	12.6825	13.0412	13.4121	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.3856	16.8699	17.8885	18.9771	20.1407	21.3843	22.7132	24.1331	39.5805
13	13.8093	14.2368	14.6803	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.2868	18.8821	20.1406	21.4953	22.9534	24.5227	26.2116	28.0291	48.4966
14	14.9474	15.4504	15.9739	16.5190	17.0863	17.6770	18.2919	18.9321	19.5986	20.2926	21.0151	22.5505	24.2149	26.0192	27.9750	30.0949	32.3926	59.1959
15	16.0969	16.6821	17.2934	17.9319	18.5989	19.2957	20.0236	20.7841	21.5786	22.4087	23.2760	25.1290	27.1521	29.3609	31.7725	34.4054	37.2797	72.0351
16	17.2579	17.9324	18.6393	19.3802	20.1569	20.9710	21.8245	22.7193	23.6575	24.6411	25.6725	27.8881	30.3243	33.0034	35.9497	39.1899	42.7533	87.4421
17	18.4304	19.2014	20.0121	20.8647	21.7616	22.7050	23.6975	24.7417	25.8404	26.9964	28.2129	30.8402	33.7502	36.9737	40.5447	44.5008	48.8837	105.9306
18	19.6147	20.4894	21.4123	22.3863	23.4144	24.4997	25.6454	26.8551	28.1324	29.4812	30.9057	33.9990	37.4502	41.3013	45.5992	50.3959	55.7497	128.1167
19	20.8109	21.7967	22.8406	23.9460	25.1169	26.3572	27.6712	29.0636	30.5390	32.1027	33.7600	37.3790	41.4463	46.0185	51.1591	56.9395	63.4397	154.7400
20	22.0190	23.1237	24.2974	25.5447	26.8704	28.2797	29.7781	31.3714	33.0660	34.8683	36.7856	40.9955	45.7620	51.1601	57.2750	64.2028	72.0524	186.6880
21	23.2392	24.4705	25.7833	27.1833	28.6765	30.2695	31.9692	33.7831	35.7193	37.7861	39.9927	44.8652	50.4229	56.7645	64.0025	72.2651	81.6987	225.0256
30	34.7849	37.5387	40.5681	43.9027	47.5754	51.6227	56.0849	61.0071	66.4388	72.4355	79.0582	94.4608	113.2832	136.3075	164.4940	199.0209	241.3327	1181.8816
40	48.8864	54.2679	60.4020	67.4026	75.4013	84.5503	95.0255	107.0303	120.7998	136.6056	154.7620	199.6351	259.0565	337.8824	442.5926	581.8261	767.0914	7343.8578

TABLE 4 Present Value of an Ordinary Annuity of $1

$$PVA = \frac{1 - \frac{1}{(1 + i)^n}}{i}$$

n/i	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	4.5%	5.0%	5.5%	6.0%	7.0%	8.0%	9.0%	10.0%	11.0%	12.0%	20.0%
1	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	0.96154	0.95694	0.95238	0.94787	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286	0.83333
2	1.97040	1.95588	1.94156	1.92742	1.91347	1.89969	1.88609	1.87267	1.85941	1.84632	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005	1.52778
3	2.94099	2.91220	2.88388	2.85602	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183	2.10648
4	3.90197	3.85438	3.80773	3.76197	3.71710	3.67308	3.62990	3.58753	3.54595	3.50515	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735	2.58873
5	4.85343	4.78264	4.71346	4.64583	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478	2.99061
6	5.79548	5.69719	5.60143	5.50813	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732	4.76654	4.62288	4.48592	4.35526	4.23054	4.11141	3.32551
7	6.72819	6.59821	6.47199	6.34939	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238	5.38929	5.20637	5.03295	4.86842	4.71220	4.56376	3.60459
8	7.65168	7.48593	7.32548	7.17014	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979	5.97130	5.74664	5.53482	5.33493	5.14612	4.96764	3.83716
9	8.56602	8.36052	8.16224	7.97087	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169	6.51523	6.24689	5.99525	5.75902	5.53705	5.32825	4.03097
10	9.47130	9.22218	8.98259	8.75206	8.53020	8.31661	8.11090	7.91272	7.72173	7.53763	7.36009	7.02358	6.71008	6.41766	6.14457	5.88923	5.65022	4.19247
11	10.36763	10.07112	9.78685	9.51421	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687	7.49867	7.13896	6.80519	6.49506	6.20652	5.93770	4.32706
12	11.25508	10.90751	10.57534	10.25776	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384	7.94269	7.53608	7.16073	6.81369	6.49236	6.19437	4.43922
13	12.13374	11.73153	11.34837	10.98319	10.63496	10.30274	9.98565	9.68285	9.39357	9.11708	8.85268	8.35765	7.90378	7.48690	7.10336	6.74987	6.42355	4.53268
14	13.00370	12.54338	12.10625	11.69091	11.29607	10.92052	10.56312	10.22283	9.89864	9.58965	9.29498	8.74547	8.24424	7.78615	7.36669	6.98187	6.62817	4.61057
15	13.86505	13.34323	12.84926	12.38138	11.93794	11.51741	11.11839	10.73955	10.37966	10.03758	9.71225	9.10791	8.55948	8.06069	7.60608	7.19087	6.81086	4.67547
16	14.71787	14.13126	13.57771	13.05500	12.56110	12.09412	11.65230	11.23402	10.83777	10.46216	10.10590	9.44665	8.85137	8.31256	7.82371	7.37916	6.97399	4.72956
17	15.56225	14.90765	14.29187	13.71220	13.16612	12.65132	12.16567	11.70719	11.27407	10.86461	10.47726	9.76322	9.12164	8.54363	8.02155	7.54879	7.11963	4.77463
18	16.39827	15.67256	14.99203	14.35336	13.75351	13.18968	12.65930	12.15999	11.68959	11.24607	10.82760	10.05909	9.37189	8.75563	8.20141	7.70162	7.24967	4.81219
19	17.22601	16.42617	15.67846	14.97889	14.32380	13.70984	13.13394	12.59329	12.08532	11.60765	11.15812	10.33560	9.60360	8.95011	8.36492	7.83929	7.36578	4.84350
20	18.04555	17.16864	16.35143	15.58916	14.87747	14.21240	13.59033	13.00794	12.46221	11.95038	11.46992	10.59401	9.81815	9.12855	8.51356	7.96333	7.46944	4.86958
21	18.85698	17.90014	17.01121	16.18455	15.41502	14.69797	14.02916	13.40472	12.82115	12.27524	11.76408	10.83553	10.01680	9.29224	8.64869	8.07507	7.56200	4.89132
25	22.02316	20.71961	19.52346	18.42438	17.41315	16.48151	15.62208	14.82821	14.09394	13.41393	12.78336	11.65358	10.67478	9.82258	9.07704	8.42174	7.84314	4.94759
30	25.80771	24.01584	22.39646	20.93029	19.60044	18.39205	17.29203	16.28889	15.37245	14.53375	13.76483	12.40904	11.25778	10.27365	9.42691	8.69379	8.05518	4.97894
40	32.83469	29.91585	27.35548	25.10278	23.11477	21.35507	19.79277	18.40158	17.15909	16.04612	15.04630	13.33171	11.92461	10.75736	9.77905	8.95105	8.24378	4.99660