

Approximation of change of y as a result of change of x

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

$$\begin{aligned} \Delta y &\approx f'(x_1) \cdot \Delta x \\ &= f'(2) \cdot 0.1 \\ &= 2(2) \cdot 0.1 = 0.4 \end{aligned}$$

- What is the real Δy ?

$$\begin{aligned} y_2 &= f(2.1) = 10 + (2.1)^2 = 14.41 \\ \Delta y &= y_2 - y_1 = \end{aligned}$$

- We underestimate the real change of y .
- What if $\Delta x = -0.2$? Approximate the change of y .

HW Given $y = 10 + \sqrt{x}$,

- Find the derivative $f'(x)$.
- Fill in the table

| Point | X | Y | $f'(x)$ |
|-------|---|--------|---------|
| | 0 | 10 | |
| A | 1 | 11 | 0.5 |
| B | 2 | 11.414 | 0.3536 |
| C | 3 | 11.732 | 0.2887 |

a). $y = 10 + \sqrt{x}$
 $y' = \frac{x}{2}$
 $y' = \frac{1}{2} \left(\frac{1}{\sqrt{x}} \right)$

not exact
 Formula: $\Delta y \approx f'(x_1) \times \Delta x$

- Does the slope increase as x increases? No
- Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate?

Find ΔY known $\Delta x = 0.2$ $x = 3$
 $= f'(3) \times 0.2$
 $= 0.2887 \times 0.2 = 0.05774$

Note: If the function $f(x)$ is linear, the approximation is exact.

ΔY
 $y_2 = f(3.2) = 10 + \sqrt{3.2} = 11.7889$
 $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11.7889 - 11.732}{3.2 - 3} = 0.0569$