Solutions for End-of-Chapter Questions and Problems: Chapter Nine

1. What is the difference between book value accounting and market value accounting? How do interest rate changes affect the value of bank assets and liabilities under the two methods? What is marking to market?

Book value accounting reports assets and liabilities at the original issue values. Market value accounting reports assets and liabilities at their current market values. Current market values may be different from book values because they reflect current market conditions, such as current interest rates. FIs generally report their balance sheets using book value accounting methods. This is a problem if an asset or liability has to be liquidated immediately. If the asset or liability is held until maturity, then the reporting of book values does not pose a problem.

For an FI, a major factor affecting asset and liability values is interest rate changes. If interest rates increase, the value of both loans (assets) and deposits and other debt (liabilities) fall. If assets and liabilities are held until maturity, interest rate changes do not affect the valuation of the FI. However, if deposits or loans have to be refinanced, then market value accounting presents a better picture of the condition of the FI. The process by which changes in the economic value of assets and liabilities are accounted is called marking to market. The changes can be beneficial as well as detrimental to the total economic value of the FI.

2. What are the two different general interpretations of the concept of duration, and what is the technical definition of this term? How does duration differ from maturity?

Duration measures the weighted-average life of an asset or liability in economic terms. As such, duration has economic meaning as the interest rate sensitivity (or interest elasticity) of an asset's value to changes in the interest rate. Duration differs from maturity as a measure of interest rate sensitivity because duration takes into account the time of arrival and the rate of reinvestment of all cash flows during the assets life. Technically, duration is the weighted-average time to maturity using the relative present values of the cash flows as the weights.

3. A one-year, $100,000 loan carries a coupon rate and a market interest rate of 12 percent. The loan requires payment of accrued interest and one-half of the principal at the end of six months. The remaining principal and accrued interest are due at the end of the year.

   a. What will be the cash flows at the end of six months and at the end of the year?

$CF_{1/2}$ = ($100,000 x 0.12 x ½) + $50,000 = $56,000 interest and principal.
CF_1 = ($50,000 x 0.12 x ½) + $50,000 = $53,000 interest and principal.

b. What is the present value of each cash flow discounted at the market rate? What is the total present value?

$PV \text{ of } CF_{1/2} = \$56{,}000/1.06 = \$52{,}830.19$
$PV \text{ of } CF_1 = \$53{,}000/(1.06)^2 = \underline{47{,}169.81}$
$PV \text{ Total } CF = \$100{,}000.00$

c. What proportion of the total present value of cash flows occurs at the end of six months? What proportion occurs at the end of the year?

$X_{1/2} = \$52{,}830.19 \div \$100{,}000 = 0.5283 = 52.83\%$
$X_1 = \$47{,}169.81 \div \$100{,}000 = 0.4717 = 47.17\%$

d. What is the duration of this loan?

Duration = 0.5283(1/2) + 0.4717(1) = 0.7358

OR

t	CF	PV of CF	PV of CF x t
½	$56,000	$52,830.19	$26,415.09
1	53,000	47,169.81	47,169.81
		$100,000.00	$73,584.91

Duration = $73,584.91/$100,000.00 = 0.7358 years

4. Two bonds are available for purchase in the financial markets. The first bond is a two-year, $1,000 bond that pays an annual coupon of 10 percent. The second bond is a two-year, $1,000, zero-coupon bond.

a. What is the duration of the coupon bond if the current yield to maturity (R) is 8 percent? 10 percent? 12 percent? (*Hint*: You may wish to create a spreadsheet program to assist in the calculations.)

<u>Coupon Bond</u>: Par value = $1,000 Coupon rate = 10% Annual payments
R = 8% Maturity = 2 years

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	$100	0.9259	$92.59	$92.59
2	1,100	0.8573	943.07	1,886.15
			$1,035.67	$1,978.74

Duration = $1,978.74/$1,035.67 = 1.9106

R = 10% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$100	0.9091	$90.91	$90.91
2	1,100	0.8264	909.09	1,818.18
			$1,000.00	$1,909.09

Duration = $1,909.09/$1,000.00 = 1.9091

R = 12% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$100	0.8929	$89.29	$89.23
2	1,100	0.7972	876.91	1,753.83
			$966.20	$1,843.11

Duration = $1,843.11/$966.20 = 1.9076

b. How does the change in the yield to maturity affect the duration of this coupon bond?

Increasing the yield to maturity decreases the duration of the bond.

c. Calculate the duration of the zero-coupon bond with a yield to maturity of 8 percent, 10 percent, and 12 percent.

Zero Coupon Bond: Par value = $1,000 Coupon rate = 0%

R = 8% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
2	$1,000	0.8573	$857.34	$1,714.68
			$857.34	$1,714.68

Duration = $1,714.68/$857.34 = 2.0000

R = 10% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
2	$1,000	0.8264	$826.45	$1,652.89
			$826.45	$1,652.89

Duration = $1,652.89/$826.45 = 2.0000

R = 12% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
2	$1,000	0.7972	$797.19	$1,594.39
			$797.19	$1,594.39

Duration = $1,594.39/$797.19 = 2.0000

d. How does the change in the yield to maturity affect the duration of the zero-coupon bond?

Changing the yield to maturity does not affect the duration of the zero coupon bond.

e. Why does the change in the yield to maturity affect the coupon bond differently than it affects the zero-coupon bond?

Increasing the yield to maturity on the coupon bond allows for higher reinvestment income that more quickly recovers the initial investment. The zero-coupon bond, on the other hand, has no cash flow (and therefore no reinvestment of income) until maturity.

5. What is the duration of a five-year, $1,000 Treasury bond with a 10 percent semiannual coupon selling at par? Selling with a yield to maturity of 12 percent? 14 percent? What can you conclude about the relationship between duration and yield to maturity? Plot the relationship. Why does this relationship exist?

Five-year Treasury Bond: Par value = $1,000 Coupon rate = 10% Semiannual payments

R = 10% Maturity = 5 years

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
0.5	50	0.9524	47.620	23.810
1.0	50	0.9070	45.350	45.350
1.5	50	0.8638	43.190	64.785
2.0	50	0.8227	41.135	82.270
2.5	50	0.7835	39.175	97.937
3.0	50	0.7462	37.310	111.930
3.5	50	0.7107	35.535	124.373
4.0	50	0.6768	33.842	135.368
4.5	50	0.6446	32.230	145.035
5.0	1,050	0.6139	644.595	3,222.975
			1,000.00	4,053.833

Duration = $4,053.91/$1,000.00 = 4.0539

R = 12% Maturity = 5 years

t	CF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
0.5	50	47.17	23.58
1.0	50	44.50	44.50
1.5	50	41.98	62.97

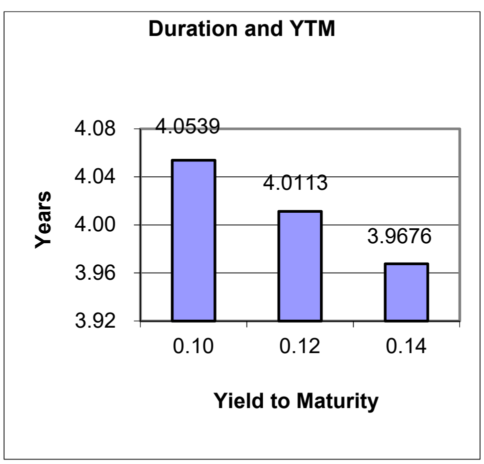
2.0	50	39.60	79.21
2.5	50	37.36	93.41
3.0	50	35.25	105.74
3.5	50	33.25	116.38
4.0	50	31.37	125.48
4.5	50	29.59	133.18
5.0	1,050	586.31	2,931.57
		926.40	3,716.03

Duration = \$3,716.03/\$926.40 = 4.0113

R = 14% Maturity = 5 years

t	CF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	50	46.73	23.36
1.0	50	43.67	43.67
1.5	50	40.81	61.22
2.0	50	38.14	76.29
2.5	50	35.65	89.12
3.0	50	33.32	99.95
3.5	50	31.14	108.98
4.0	50	29.10	116.40
4.5	50	27.20	122.39
5.0	1,050	533.77	2,668.83
		859.53	3,410.22

Duration = \$3, 410.22/\$859.53 = 3.9676



As the yield to maturity increases, the higher yields discount later cash flows more heavily and the relative importance, or weights, of those later cash flows decline when compared with earlier cash flows on the bond.

6. Consider three Treasury bonds each of which has a 10 percent semiannual coupon and trades at par.

a. Calculate the duration for a bond that has a maturity of four years, three years, and two years.

Four-year Treasury Bond: Par value = $1,000 Coupon rate = 10% Semiannual payments

R = 10% Maturity = 4 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	50	0.9524	47.62	23.81
1.0	50	0.9070	45.35	45.35
1.5	50	0.8638	43.19	64.79
2.0	50	0.8227	41.14	82.27
2.5	50	0.7835	39.18	97.94
3.0	50	0.7462	37.31	111.93
3.5	50	0.7107	35.53	124.37
4.0	1,050	0.6768	710.68	2,842.72
			1,000.00	3,393.19

Duration = $3,393.19/$1,000.00 = 3.3932

R = 10% Maturity = 3 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	50	0.9524	47.62	23.81
1.0	50	0.9070	45.35	45.35
1.5	50	0.8638	43.19	64.79
2.0	50	0.8227	41.14	82.27
2.5	50	0.7835	39.18	97.94
3.0	1,050	0.7462	783.53	2,350.58
			1,000.00	2,664.74

Duration = $2,664.74/$1,000.00 = 2.6647

R = 10% Maturity = 2 years

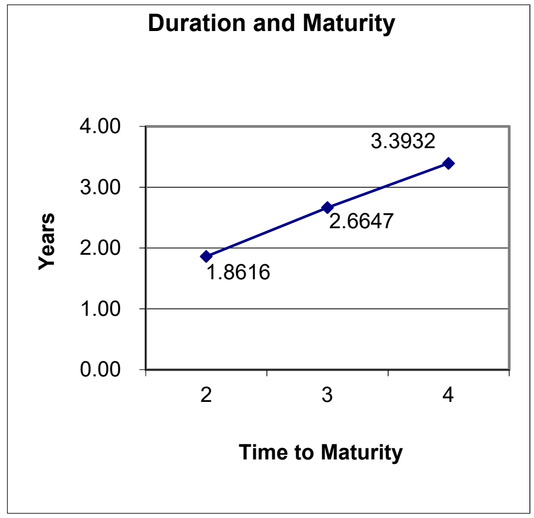
t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	50	0.9524	47.62	23.81
1.0	50	0.9070	45.35	45.35
1.5	50	0.8638	43.19	64.79
2.0	1,050	0.8227	863.84	1,727.68
			1,000.00	1,861.62

Duration = $1,861.62/$1,000.00 = 1.8616

b. What conclusions can you reach about the relationship between duration and the time to maturity? Plot the relationship.

As maturity decreases, duration decreases at a decreasing rate. Although the graph below does not illustrate with great precision, the change in duration is less than the change in time to maturity.

Duration	Maturity	Change in Duration
1.8616	2	
2.6647	3	0.8031
3.3932	4	0.7285



7. A six-year, $10,000 CD pays 6 percent interest annually and has a 6 percent yield to maturity. What is the duration of the CD? What would be the duration if interest were paid semiannually? What is the relationship of duration to the relative frequency of interest payments?

Six-year CD: Par value = $10,000 Coupon rate = 6%

R = 6%		Maturity = 6 years		Annual payments
t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	600	0.9434	566.04	566.04
2	600	0.8900	534.00	1,068.00
3	600	0.8396	503.77	1,511.31
4	600	0.7921	475.26	1,901.02
5	600	0.7423	448.35	2,241.77
6	10,600	0.7050	7,472.58	44,835.49
			10,000.00	52,123.64

Duration = $52,123.64/$1,000.00 = 5.2124

R = 6% Maturity = 6 years Semiannual payments

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
0.5	300	0.9709	291.26	145.63
1	300	0.9425	282.78	282.78
1.5	300	0.9151	274.54	411.81
2	300	0.8885	266.55	533.09
2.5	300	0.8626	258.78	646.96
3	300	0.8375	251.25	753.74
3.5	300	0.8131	243.93	853.75
4	300	0.7894	236.82	947.29
4.5	300	0.7664	229.93	1,034.66
5	300	0.7441	223.23	1,116.14
5.5	300	0.7224	216.73	1,192.00
6	10,300	0.7014	7,224.21	43,345.28
			10,000.00	51,263.12

Duration = $51,263.12/$10,000.00 = 5.1263

Duration decreases as the frequency of payments increases. This relationship occurs because (a) cash is being received more quickly and (b) reinvestment income will occur more quickly from the earlier cash flows.

8. What is a consol bond? What is the duration of a consol bond that sells at a yield to maturity of 8 percent? 10 percent? 12 percent? Would a consol bond trading at a yield to maturity of 10 percent have a greater duration than a 20-year zero-coupon bond trading at the same yield to maturity? Why?

A consol bond is a bond that pays a fixed coupon each year forever. A consol bond trading at a yield to maturity of 10 percent has a duration of 11 years, while a 20-year zero-coupon bond trading at a ytm of 10 percent, or any other ytm, has a duration of 20 years because no cash flows occur before the twentieth year.

	Consol Bond
R	D = 1 + 1/R
0.08	13.50 years
0.10	11.00 years
0.12	9.33 years

9. Maximum Pension Fund is attempting to manage one of the bond portfolios under its management. The fund has identified three bonds that have five year maturities and trade at a yield to maturity of 9 percent. The bonds differ only in that the coupons are 7 percent, 9 percent, and 11 percent.

   a. What is the duration for each bond?

Five-year Bond: Par value = $1,000 Maturity = 5 years Annual payments

R = 9% Coupon rate = 7%

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	70	0.9174	64.22	64.22
2	70	0.8417	58.92	117.84
3	70	0.7722	54.05	162.16
4	70	0.7084	49.59	198.36
5	1,070	0.6499	695.43	3,477.13
			922.21	4,019.71

Duration = $4,019.71/$922.21 = 4.3588

R = 9% Coupon rate = 9%

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$90	0.9174	82.57	82.57
2	$90	0.8417	75.75	151.50
3	$90	0.7722	69.50	208.49
4	$90	0.7084	63.76	255.03
5	$1,090	0.6499	708.43	3,542.13
			1,000.00	4,239.72

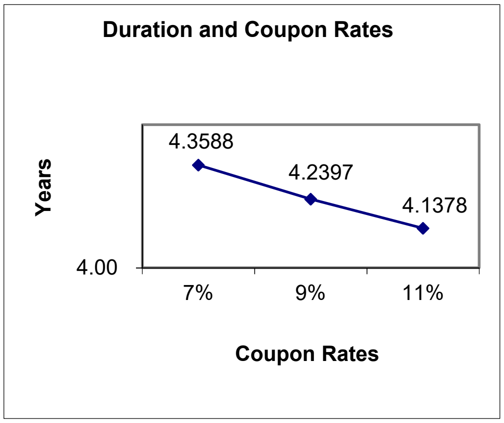
Duration = $4,239.72/$1,000.00 = 4.2397

R = 9% Coupon rate = 11%

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$110	0.9174	100.92	100.92
2	$110	0.8417	92.58	185.17
3	$110	0.7722	84.94	254.82
4	$110	0.7084	77.93	311.71
5	$1,110	0.6499	721.42	3,607.12
			1,077.79	4,459.73

Duration = $4,459.73/$1,077.79 = 4.1378

b. What is the relationship between duration and the amount of coupon interest that is paid? Plot the relationship.



Duration decreases as the amount of coupon interest increases.

Duration	Coupon	Change in Duration
4.3588	7%	
4.2397	9%	-0.1191
4.1378	11%	-0.1019

10. An insurance company is analyzing three bonds and is using duration as the measure of interest rate risk. All three bonds trade at a yield to maturity of 10 percent, have $10,000 par values, and have five years to maturity. The bonds differ only in the amount of annual coupon interest that they pay: 8, 10, and 12 percent.

    a. What is the duration for each five-year bond?

Five-year Bond: Par value = $10,000   R = 10%   Maturity = 5 years   Annual payments

Coupon rate = 8%

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	800	0.9091	727.27	727.27
2	800	0.8264	661.16	1,322.31
3	800	0.7513	601.06	1,803.16
4	800	0.6830	546.41	2,185.64
5	10,800	0.6209	6,705.95	33,529.75
			9,241.84	39,568.14

Duration = $39,568.14/9,241.84 = 4.2814

Coupon rate = 10%

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$1,000	0.9091	909.09	909.09
2	$1,000	0.8264	826.45	1,652.89
3	$1,000	0.7513	751.31	2,253.94
4	$1,000	0.6830	683.01	2,732.05
5	$11,000	0.6209	6,830.13	34,150.67
			10,000.00	41,698.65

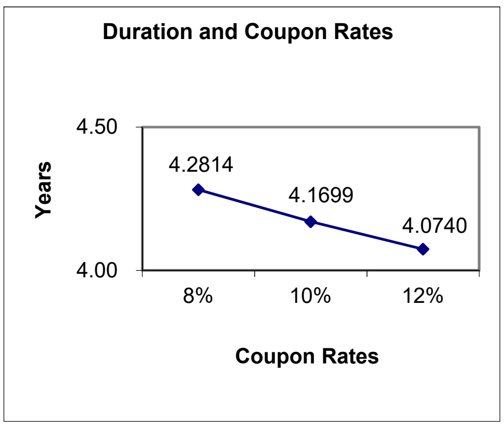
Duration = $41.698.65/10,000.00 = 4.1699

Coupon rate = 12%

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	$1,200	0.9091	1,090.91	1,090.91
2	$1,200	0.8264	991.74	1,983.47
3	$1,200	0.7513	901.58	2,704.73
4	$1,200	0.6830	819.62	3,278.46
5	$11,200	0.6209	6,954.32	34,771.59
			10,758.16	43,829.17

Duration = $43,829.17/10,758.16 = 4.0740

b. What is the relationship between duration and the amount of coupon interest that is paid?



Duration decreases as the amount of coupon interest increases.

Duration	Coupon	Change in Duration
4.2814	8%	
4.1699	10%	-0.1115
4.0740	12%	-0.0959

11. You can obtain a loan of $100,000 at a rate of 10 percent for two years. You have a choice of (i) paying the interest (10 percent) each year and the total principal at the end of the second year or (ii) amortizing the loan, that is, paying interest (10 percent) and principal in equal payments each year. The loan is priced at par.

a. What is the duration of the loan under both methods of payment?

Two-year loan: Interest at end of year one; Principal and interest at end of year two

Par value = $100,000 Coupon rate = 10% Annual payments

R = 10% Maturity = 2 years

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	$10,000	0.9091	9,090.91	9,090.91
2	$110,000	0.8264	90,909.09	181,818.18
			100,000.00	190,909.09

Duration = $190,909.09/$100,000 = 1.9091

Two-year loan: Amortized over two years

Par value = \$100,000 Coupon rate = 10% Annual amortized payments

R = 10% Maturity = 2 years = \$57,619.05

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	\$57,619.05	0.9091	52,380.95	52,380.95
2	\$57,619.05	0.8264	47,619.05	95,238.10
			100,000.00	147,619.05

Duration = \$147,619.05/\$100,000 = 1.4762

b. Explain the difference in the two results

Duration decreases dramatically when a portion of the principal is repaid at the end of year one. Duration is the weighted-average maturity of an asset. If more weight is given to early payments, the effective maturity of the asset is reduced.

12. How is duration related to the interest elasticity of a fixed-income security? What is the relationship between duration and the price of the fixed-income security?

Taking the first derivative of a bond's (or any fixed-income security) price (P) with respect to the yield to maturity (R) provides the following:

$$\frac{dP/P}{dR/(1+R)} = -D$$

The economic interpretation is that D is a measure of the percentage change in the price of a bond for a given percentage change in yield to maturity (interest elasticity). This equation can be rewritten to provide a practical application:

$$dP = -D\left[\frac{dR}{1+R}\right]P$$

In other words, if duration is known, then the change in the price of a bond due to small changes in interest rates, R, can be estimated using the above formula.

13. You have discovered that the price of a bond rose from \$975 to \$995 when the yield to maturity fell from 9.75 percent to 9.25 percent. What is the duration of the bond?

We know $-D = \dfrac{\Delta P/P}{\Delta R/(1+R)} = \dfrac{20/975}{-0.005/1.0975} = -4.5 \text{ years} \Rightarrow D = 4.5 \text{ years}$

14. A 10-year, 10 percent annual coupon, $1,000 bond trades at a yield to maturity of 8 percent. The bond has a duration of 6.994 years. What is the modified duration of this bond? What is the practical value of calculating modified duration? Does modified duration change the result of using the duration relationship to estimate price sensitivity?

Modified duration = Duration/(1+ R) = 6.994/1.08 = 6.4759. Some practitioners find this value easier to use because the percentage change in value can be estimated simply by multiplying the existing value times the basis point change in interest rates rather than by the relative change in interest rates. Using modified duration will not change the estimated price sensitivity of the asset.

15. What is dollar duration? How is dollar duration different from duration?

Dollar duration is the *dollar value change* in the price of a security to a one percent change in the return on the security. Duration is a measure of the *percentage change* in the price of a security for a one percent change in the return on the security. The dollar duration is intuitively appealing in that we multiply the dollar duration by the change in the interest rate to get the actual dollar change in the value of a security to a change in interest rates.

16. Calculate the duration of a two-year, $1,000 bond that pays an annual coupon of 10 percent and trades at a yield of 14 percent. What is the expected change in the price of the bond if interest rates fall by 0.50 percent (50 basis points)?

<u>Two-year Bond:</u> Par value = $1,000 Coupon rate = 10% Annual payments
R = 14% Maturity = 2 years

t	CF_t	DF_t	$CF_t \times DF_t$	$CF_t \times DF_t \times t$
1	100	0.8772	87.72	87.72
2	1,100	0.7695	846.41	1,692.83
			934.13	1,780.55

Duration = $1,780.55/$934.13 = 1.9061

The expected change in price = - dollar duration x □R = $-D\dfrac{\Delta R}{1+R}P = $ - MD x □R =
- (1.9061/1.14) x (-0.0050) x $934.13 = $7.81. This implies a new price of $941.94 ($934.13 + $7.81). The actual price using conventional bond price discounting would be $941.99. The difference of $0.05 is due to convexity, which is not considered in the duration elasticity measure.

17. The duration of an 11-year, $1,000 Treasury bond paying a 10 percent semiannual coupon and selling at par has been estimated at 6.763 years.

a. What is the modified duration of the bond? What is the dollar duration of the bond?

Modified duration = D/(1 + R/2) = 6.763/(1 + 0.10/2) = 6.441 years
Dollar duration = MD x P = 6.441 x $1,000 = 6441

b. What will be the estimated price change on the bond if interest rates increase 0.10 percent (10 basis points)? If rates decrease 0.20 percent (20 basis points)?

For an interest rate increase of 0.10 percent:

Estimated change in price = - dollar duration x ΔR = -6441 x 0.001 = -$6.441
=> new price = $1,000 - $6.441 = $993.559

For an interest rate decrease of 0.20 percent:

Estimated change in price = -6441 x -0.002 = $12.882
=> new price = $1,000 + $12.882 = $1,012.882

c. What would the actual price of the bond be under each rate change situation in part (b) using the traditional present value bond pricing techniques? What is the amount of error in each case?

Rate Change	Price Estimated	Actual Price	Error
+0.001	$993.559	$993.535	$0.024
-0.002	$1,012.882	$1,013.111	-$0.229

18. Suppose you purchase a six-year, 8 percent coupon bond (paid annually) that is priced to yield 9 percent. The face value of the bond is $1,000.

a. Show that the duration of this bond is equal to five years.

Six-year Bond: Par value = $1,000 Coupon rate = 8% Annual payments
R = 9% Maturity = 6 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	80	0.9174	73.39	73.39
2	80	0.8417	67.33	134.67
3	80	0.7722	61.77	185.32
4	80	0.7084	56.67	226.70
5	80	0.6499	51.99	259.97
6	1,080	0.5963	643.97	3,863.81
			955.14	4,743.87

Duration = $4,743.87/955.14 = 4.97 $\approx$ 5 years

b. Show that if interest rates rise to 10 percent within the next year and your investment horizon is five years from today, you will still earn a 9 percent yield on your investment.

Value of bond at end of year five: PV = (\$80 + \$1,000)/1.10 = \$981.82
Future value of interest payments at end of year five: \$80$FV_{n=4,\ i=10\%}$ = \$488.41
Future value of all cash flows at n = 5:

Coupon interest payments over five years	\$400.00
Interest on interest at 10 percent	88.41
Value of bond at end of year five	\$981.82
Total future value of investment	\$1,470.23

Yield on purchase of asset at \$955.14 = \$1,470.23x$PV_{n=5,\ i=?\%}$ $\Rightarrow$ i = 9.00924%

c. Show that a 9 percent yield also will be earned if interest rates fall next year to 8 percent.

Value of bond at end of year five: PV = (\$80 + \$1,000)/1.08 = \$1,000
Future value of interest payments at end of year five: \$80x$FV_{n=5,\ i=8\%}$ = \$469.33
Future value of all cash flows at n = 5:

Coupon interest payments over five years	\$400.00
Interest on interest at 8 percent	69.33
Value of bond at end of year five	\$1,000.00
Total future value of investment	\$1,469.33

Yield on purchase of asset at \$955.14 = \$1,469.33x$PV_{n=5,\ i=?\%}$ $\Rightarrow$ i = 8.99596 percent.

19. Suppose you purchase a five-year, 15 percent coupon bond (paid annually) that is priced to yield 9 percent. The face value of the bond is \$1,000.

a. Show that the duration of this bond is equal to four years.

Five-year Bond: Par value = \$1,000 Coupon rate = 15% Annual payments
R = 9% Maturity = 5 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	\$150	0.9174	137.62	137.62
2	\$150	0.8417	126.25	252.50
3	\$150	0.7722	115.83	347.48
4	\$150	0.7084	106.26	425.06

5	$1,150	0.6499	747.42	3,737.10
			1,233.38	4,899.76

Duration = $4899.76/1,233.38 = 3.97 ≈ 4 years

b. Show that if interest rates rise to 10 percent within the next year and your investment horizon is four years from today, you will still earn a 9 percent yield on your investment.

Value of bond at end of year four: PV = ($150 + $1,000)/1.10 = $1,045.45

Future value of interest payments at end of year four: $150x$FV_{n=4,\ i=10\%}$ = $696.15

Future value of all cash flows at n = 4:

Coupon interest payments over four years	$600.00
Interest on interest at 10 percent	96.15
Value of bond at end of year four	$1,045.45
Total future value of investment	$1,741.60

Yield on purchase of asset at $1,233.38 = $1,741.60x$PV_{n=4,\ i=?\%}$ ⇒ i = 9.00%

c. Show that a 9 percent yield also will be earned if interest rates fall next year to 8 percent.

Value of bond at end of year four: PV = ($150 + $1,000)/1.08 = $1,064.81

Future value of interest payments at end of year four: $150x$FV_{n=4,\ i=8\%}$ = $675.92

Future value of all cash flows at n = 4:

Coupon interest payments over four years	$600.00
Interest on interest at 8 percent	75.92
Value of bond at end of year four	$1,064.81
Total future value of investment	$1,740.73

Yield on purchase of asset at $1,233.38 = $1,740.73x$PV_{n=4,\ i=?\%}$ ⇒ i = 9.00 percent.

20. Consider the case in which an investor holds a bond for a period of time longer than the duration of the bond, that is, longer than the original investment horizon.

a. If interest rates rise, will the return that is earned exceed or fall short of the original required rate of return? Explain.

In this case, the actual return earned would exceed the yield expected at the time of purchase. The benefits from a higher reinvestment rate would exceed the price reduction effect if the investor holds the bond for a sufficient length of time.

b. What will happen to the realized return if interest rates decrease? Explain.

If interest rates decrease, the realized yield on the bond will be less than the expected yield because the decrease in reinvestment earnings will be greater than the gain in bond value.

c. Recalculate parts (b) and (c) of problem 18 above, assuming that the bond is held for all five years, to verify your answers to parts (a) and (b) of this problem.

The case where interest rates rise to 10 percent, n = five years:

Future value of interest payments at end of year five: $\$150 \text{x} FV_{n=5,\, i=10\%} = \$915.76$

Future value of all cash flows at n = 5:

Coupon interest payments over five years	$750.00
Interest on interest at 10 percent	165.76
Value of bond at end of year five	$1,000.00
Total future value of investment	$1,915.76

Yield on purchase of asset at $\$1{,}233.38 = \$1{,}915.76 \text{x} PV_{n=5,\, i=?\%} \Rightarrow i = 9.2066\%$

The case where interest rates fall to 8 percent, n = five years:

Future value of interest payments at end of year five: $\$150 \text{x} FV_{n=5,\, i=8\%} = \$879.99$

Future value of all cash flows at n = 5:

Coupon interest payments over five years	$750.00
Interest on interest at 8 percent	129.99
Value of bond at end of year five	$1,000.00
Total future value of investment	$1,879.99

Yield on purchase of asset at $\$1{,}233.38 = \$1{,}879.99 \text{x} PV_{n=5,\, i=?\%} \Rightarrow i = 8.7957\%$

d. If either calculation in part (c) is greater than the original required rate of return, why would an investor ever try to match the duration of an asset with his or her investment horizon?

The answer has to do with the ability to forecast interest rates. Forecasting interest rates is a very difficult task, one that most financial institution money managers are unwilling to do. For most managers, betting that rates would rise to 10 percent to provide a realized yield of 9.20 percent over five years is not a sufficient return to offset the possibility that rates could fall to 8 percent and thus give a yield of only 8.8 percent over five years.

21. Two banks are being examined by regulators to determine the interest rate sensitivity of their balance sheets. Bank A has assets composed solely of a 10-year $1 million loan with a coupon rate and yield of 12 percent. The loan is financed with a 10-year $1 million CD with a coupon rate and yield of 10 percent. Bank B has assets composed solely of a 7-year, 12 percent zero-coupon bond with a current (market) value of $894,006.20 and a maturity (principal) value of $1,976,362.88. The bond is financed with a 10-year, 8.275 percent coupon $1,000,000 face value CD with a yield to maturity of 10 percent. The loan and the CDs pay interest annually, with principal due at maturity.

    a. If market interest rates increase 1 percent (100 basis points), how do the market values of the assets and liabilities of each bank change? That is, what will be the net effect on the market value of the equity for each bank?

For Bank A, an increase of 100 basis points in interest rate will cause the market values of assets and liabilities to decrease as follows:

Loan: $\$120{,}000 \text{x} PVA_{n=10,i=13\%} + \$1{,}000{,}000 \text{x} PV_{n=10,i=13\%} = \$945{,}737.57$

CD: $\$100{,}000 \text{x} PVA_{n=10,i=11\%} + \$1{,}000{,}000 \text{x} PV_{n=10,i=11\%} = \$941{,}107.68$

The loan value decreases $54,262.43 and the CD value falls $58,892.32. Therefore, the decrease in value of the asset is $4,629.89 less than the liability, which is, in turn, the increase in the market value of equity for Bank A.

For Bank B:

Bond: $\$1{,}976{,}362.88 \text{x} PV_{n=7,i=13\%} = \$840{,}074.08$

CD: $\$82{,}750 \text{x} PVA_{n=10,i=11\%} + \$1{,}000{,}000 \text{x} PV_{n=10,i=11\%} = \$839{,}518.43$

The bond value decreases $53,932.12 and the CD value falls $54,487.79. Therefore, the decrease in value of the asset is $555.67 less than the liability, which is, in turn, the increase in the market value of equity for Bank B.

    b. What accounts for the differences in the changes in the market value of equity between the two banks?

The assets and liabilities of Bank A change in value by different amounts because the durations of the assets and liabilities are not the same, even though the face values and maturities are the same. For Bank B, the maturities of the assets and liabilities are different, but the current market values and durations are the same. Thus, the change in interest rates causes a smaller change in value for both liabilities and assets.

c. Verify your results above by calculating the duration for the assets and liabilities of each bank, and estimate the changes in value for the expected change in interest rates. Summarize your results.

Ten-year CD Bank B (values in thousands of $s)

Par value = $1,000 Coupon rate = 8.275% Annual payments

R = 10% Maturity = 10 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	82.75	0.9091	75.23	75.23
2	82.75	0.8264	68.39	136.78
3	82.75	0.7513	62.17	186.51
4	82.75	0.6830	56.52	226.08
5	82.75	0.6209	51.38	256.91
6	82.75	0.5645	46.71	280.26
7	82.75	0.5132	42.46	297.25
8	82.75	0.4665	38.60	308.83
9	82.75	0.4241	35.09	315.85
10	1,082.75	0.3855	417.45	4,174.47
			894.01	6,258.15

Duration = $6,258.15/894.01 = 7.00

The duration on the CD of Bank B is calculated above to be 7.00 years. Since the bond is a zero-coupon, the duration is equal to the maturity of 7 years.

Using the duration formula to estimate the change in value:

Bond: $\Delta\text{Value} = -D\frac{\Delta R}{1+R}P = -7.00\frac{0.01}{1.12}\$894{,}006.20 = -\$55{,}875.39$

CD: $\Delta\text{Value} = -D\frac{\Delta R}{1+R}P = -7.00\frac{0.01}{1.10}\$894{,}006.20 = -\$56{,}891.30$

The difference in the change in value of the assets and liabilities for Bank B is $1,015.91 using the duration estimation model. The difference in this estimate and the estimate found in part (a) above is due to the convexity of the two financial assets.

The duration estimates for the loan and CD for Bank A are presented below:

Ten-year Loan Bank A (values in thousands of $s)

Par value = $1,000 Coupon rate = 12% Annual payments

R = 12% Maturity = 10 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	120	0.8929	107.14	107.14
2	120	0.7972	95.66	191.33
3	120	0.7118	85.41	256.24
4	120	0.6355	76.26	305.05
5	120	0.5674	68.09	340.46
6	120	0.5066	60.80	364.77
7	120	0.4523	54.28	379.97
8	120	0.4039	48.47	387.73
9	120	0.3606	43.27	389.46
10	1,120	0.3220	360.61	3,606.10
			1,000.00	6,328.25

Duration = \$6,328.25/\$1,000 = 6.3282

Ten-year CD Bank A (values in thousands of \$s)

Par value = \$1,000 Coupon rate = 10% Annual payments

R = 10% Maturity = 10 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	100	0.9091	90.91	90.91
2	100	0.8264	82.64	165.29
3	100	0.7513	75.13	225.39
4	100	0.6830	68.30	273.21
5	100	0.6209	62.09	310.46
6	100	0.5645	56.45	338.68
7	100	0.5132	51.32	359.21
8	100	0.4665	46.65	373.21
9	100	0.4241	42.41	381.69
10	1,100	03855	424.10	4,240.98
			1,000.00	6,759.02

Duration = \$6,759.02/\$1,000 = 6.7590

Using the duration formula to estimate the change in value:

Loan: $$\Delta \text{Value} = -D\frac{\Delta R}{1+R}P = -6.3282\frac{0.01}{1.12}\$1{,}000{,}000 = -\$56{,}501.79$$

CD: $$\Delta \text{Value} = -D\frac{\Delta R}{1+R}P = -6.7590\frac{0.01}{1.10}\$1{,}000{,}000 = -\$61{,}445.45$$

The difference in the change in value of the assets and liabilities for Bank A is $4,943.66 using the duration estimation model. The difference in this estimate and the estimate found in part (a) above is due to the convexity of the two financial assets. The reason the change in asset values for Bank A is considerably larger than for Bank B is because of the difference in the durations of the loan and CD for Bank A.

22. If an FI uses only duration to immunize its portfolio, what three factors affect changes in the net worth of the FI when interest rates change?

The change in net worth for a given change in interest rates is given by the following equation:

$$\Delta E = -\left[D_A - D_L k\right] * A * \frac{\Delta R}{1+R} \qquad where\ k = \frac{L}{A}$$

Thus, three factors are important in determining ΔE.

1) $[D_A - D_L k]$ or the leveraged adjusted duration gap. The larger this gap, the more exposed is the FI to changes in interest rates.
2) *A*, or the size of the FI. The larger is *A*, the larger is the exposure to interest rate changes.
3) $\Delta R/(1 + R)$, or the interest rate shock. The larger is the shock, the larger is the interest rate risk exposure.

23. Financial Institution XY has assets of $1 million invested in a 30-year, 10 percent semiannual coupon Treasury bond selling at par. The duration of this bond has been estimated at 9.94 years. The assets are financed with equity and a $900,000, two-year, 7.25 percent semiannual coupon capital note selling at par.

a. What is the leverage adjusted duration gap of Financial Institution XY?

The duration of the capital note is 1.8975 years.

Two-year Capital Note (values in thousands of $s)

Par value = $900 Coupon rate = 7.25% Semiannual payments
R = 7.25% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	32.625	0.9650	31.48	15.74
1	32.625	0.9313	30.38	30.38
1.5	32.625	0.8987	29.32	43.98
2	932.625	0.8672	808.81	1,617.63

900.00	1,707.73

Duration = $1,707.73/$900.00 = 1.8975

The leverage-adjusted duration gap can be found as follows:

$$\text{Leverage} - \text{adjusted duration gap} = [D_A - D_L k] = 9.94 - 1.8975 \frac{\$900{,}000}{\$1{,}000{,}000} = 8.23225 \text{years}$$

b. What is the impact on equity value if the relative change in all market interest rates is a decrease of 20 basis points? *Note*: The relative change in interest rates is ΔR/(1+R/2) = -0.0020.

The change in net worth using leverage adjusted duration gap is given by:

$$\Delta E = -[D_A - D_L k] * A * \frac{\Delta R}{1 + \frac{R}{2}} = -\left[9.94 - (1.8975)\left\langle {}^{9}\!/_{10} \right\rangle\right](1{,}000{,}000)(-0.0020) = \$16{,}464$$

c. Using the information calculated in parts (a) and (b), what can be said about the desired duration gap for the financial institution if interest rates are expected to increase or decrease.

If the FI wishes to be immune from the effects of interest rate risk (either positive or negative changes in interest rates), a desirable leverage-adjusted duration gap (DGAP) is zero. If the FI is confident that interest rates will fall, a positive DGAP will provide the greatest benefit. If the FI is confident that rates will increase, then negative DGAP would be beneficial.

d. Verify your answer to part (c) by calculating the change in the market value of equity assuming that the relative change in all market interest rates is an increase of 30 basis points.

$$\Delta E = -[D_A - D_L k] * A * \frac{\Delta R}{1 + \frac{R}{2}} = -[8.23225](1{,}000{,}000)(0.003) = -\$24{,}697$$

e. What would the duration of the assets need to be to immunize the equity from changes in market interest rates?

Immunizing the equity from changes in interest rates requires that the DGAP be 0. Thus, $(D_A - D_L k) = 0 \Rightarrow D_A = D_L k$, or $D_A = 1.8975 \times 0.9 = 1.70775$ years.

24. The balance sheet for Gotbucks Bank, Inc. (GBI), is presented below ($ millions):

<u>Assets</u> <u>Liabilities and Equity</u>

Cash	$30	Core deposits	$20
Federal funds	20	Federal funds	50
Loans (floating)	105	Euro CDs	130
Loans (fixed)	65	Equity	20
Total assets	$220	Total liabilities and equity	$220

Notes to the balance sheet: The fed funds rate is 8.5 percent, the floating loan rate is LIBOR + 4 percent, and currently LIBOR is 11 percent. Fixed rate loans have five-year maturities, are priced at par, and pay 12 percent annual interest. The principal is repaid at maturity. Core deposits are fixed rate for two years at 8 percent paid annually. The principal is repaid at maturity. Euro CDs currently yield 9 percent.

a. What is the duration of the fixed-rate loan portfolio of Gotbucks Bank?

Five-year Loan (values in millions of $s)

Par value = $65 Coupon rate = 12% Annual payments

R = 12% Maturity = 5 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	7.8	0.8929	6.964	6.964
2	7.8	0.7972	6.218	12.436
3	7.8	0.7118	5.552	16.656
4	7.8	0.6355	4.957	19.828
5	72.8	05674	41.309	206.543
			65.000	262.427

Duration = $262.427/$65.000 = 4.0373

b. If the duration of the floating-rate loans and fed funds is 0.36 year, what is the duration of GBI's assets?

D_A = [$30(0) + $20(0.36) + $105(0.36) + $65(4.0373)]/$220 = 1.3974 years

c. What is the duration of the core deposits if they are priced at par?

Two-year Core Deposits (values in millions of $s)

Par value = $20 Coupon rate = 8% Annual payments

R = 8% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	1.6	0.9259	1.481	1.481
2	21.6	0.8573	18.519	37.037

20.000	38.519

Duration = \$38.519/\$20.000 = 1.9259

d. If the duration of the Euro CDs and fed funds liabilities is 0.401 year, what is the duration of GBI's liabilities?

D_L = [\$20(1.9259) + \$50(0.401) + \$130(0.401)]/\$200 = 0.5535 years

e. What is GBI's duration gap? What is its interest rate risk exposure?

GBI's leveraged adjusted duration gap is: 1.3974 - 200/220 x (0.5535) = 0.8942 years

Since GBI's duration gap is positive, an increase in interest rates will lead to a decrease in the market value of equity.

f. What is the impact on the market value of equity if the relative change in all interest rates is an increase of 1 percent (100 basis points)? Note that the relative change in interest rates is $\Delta R/(1+R) = 0.01$.

For a 1 percent increase, the change in equity value is:

ΔE = -0.8942 x \$220,000,000 x (0.01) = -\$1,967,280 (new net worth will be \$18,032,720).

g. What is the impact on the market value of equity if the relative change in all interest rates is a decrease of 0.5 percent (-50 basis points)?

For a 0.5 percent decrease, the change in equity value is:

ΔE = -0.8942 x \$220,000,000 x (-0.005) = \$983,647 (new net worth will be \$20,983,647).

h. What variables are available to GBI to immunize the bank? How much would each variable need to change to get DGAP equal to zero?

Immunization requires the bank to have a leverage adjusted duration gap of 0. Therefore, GBI could reduce the duration of its assets to 0.5032 (0.5535 x 200/220) years by using more fed funds and floating rate loans. Or GBI could use a combination of reducing asset duration and increasing liability duration in such a manner that DGAP is 0.

25. Hands Insurance Company issued a \$90 million, one-year note at 8 percent add-on annual interest (paying one coupon at the end of the year) or with an 8 percent yield. The proceeds,

plus \$10 million in equity, were used to fund a \$100 million, two-year commercial loan with a 10 percent coupon rate and a 10 percent yield. Immediately after these transactions were simultaneously closed, all market interest rates increased 1.5 percent (150 basis points).

a. What is the true market value of the loan investment and the liability after the change in interest rates?

The market value of the loan decreases by \$2,551,831 to \$97,448,169.

$MV_A = \$10,000,000 \text{ x } PVA_{n=2,\ i=11.5\%} + \$100,000,000 \text{ x } PV_{n=2,\ i=11.5\%} = \$97,448,169$

The market value of the note decrease \$1,232,877 to \$88,767,123

$MV_L = \$97,200,000 \text{ x } PV_{n=1,\ i=9.5\%} = \$88,767,123$

b. What impact did these changes in market value have on the market value of the FI's equity?

$\Delta E = \Delta A - \Delta L = -\$2,551,831 - (-\$1,232,877) = -\$1,318,954$

The increase in interest rates caused the asset to decrease in value more than the liability which caused the market value of equity to decrease by \$1,318,954.

c. What was the duration of the loan investment and the liability at the time of issuance?

Two-year Loan (values in millions of \$s)

Par value = \$100 Coupon rate = 10% Annual payments

R = 10% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	\$10	0.9091	9.091	9.091
2	\$110	0.8264	90.909	181.818
			100.000	190.909

Duration = \$190.909/\$100.00 = 1.9091

The duration of the loan investment is 1.9091 years. The duration of the liability is one year since it is a one year note that pays interest and principal at the end of the year.

d. Use these duration values to calculate the expected change in the value of the loan and the liability for the predicted increase of 1.5 percent in interest rates.

The approximate change in the market value of the loan for a 1.5 percent change is:

$\Delta A = -1.9091 * \frac{.015}{1.10} * \$100{,}000{,}000 = -\$2{,}603{,}300$. The expected market value of the loan using the above formula is $97,396,700.

The approximate change in the market value of the note for a 1.5 percent change is:

$\Delta L = -1.0 \text{ x } \frac{0.015}{1.08} \text{ x } \$90{,}000{,}000 = -\$1{,}250{,}000$. The expected market value of the note using the above formula is $88,750,000.

e. What is the duration gap of Hands Insurance Company after the issuance of the asset and note?

The leverage adjusted duration gap is [1.9091 – (0.9)1.0] = 1.0091 years.

f. What is the change in equity value forecasted by this duration gap for the predicted increase in interest rates of 1.5 percent?

ΔE = -1.0091 x $100,000,000 x [0.015/(1.10)] = -$1,376,045. Note that this calculation assumes that the change in interest rates is relative to the rate on the loan. Further, this estimated change in equity value compares with the estimates above in part (d) as follows:

ΔE = ΔA - ΔL = -$2,603,300 - (-$1,250,000) = -$1,353,300.

g. If the interest rate prediction had been available during the time period in which the loan and the liability were being negotiated, what suggestions would you have offered to reduce the possible effect on the equity of the company? What are the difficulties in implementing your ideas?

Obviously, the duration of the loan could be shortened relative to the liability, or the liability duration could be lengthened relative to the loan, or some combination of both. Shortening the loan duration would mean the possible use of variable rates, or some earlier payment of principal. The duration of the liability cannot be lengthened without extending the maturity life of the note. In either case, the loan officer may have been up against market or competitive constraints in that the borrower or investor may have had other options. Other methods to reduce the interest rate risk under conditions of this nature include using derivatives such as options, futures, and swaps.

26. The following balance sheet information is available (amounts in thousands of dollars and duration in years) for a financial institution:

	Amount	Duration
T-bills	$90	0.50
T-notes	55	0.90

T-bonds	176	*x*
Loans	2,724	7.00
Deposits	2,092	1.00
Federal funds	238	0.01
Equity	715	

Treasury bonds are five-year maturities paying 6 percent semiannually and selling at par.

a. What is the duration of the T-bond portfolio?

Five-year Treasury Bond

Par value = \$176 Coupon rate = 6% Semiannual payments

R = 6% Maturity = 5 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
0.5	5.28	0.9709	5.13	2.56
1	5.28	0.9426	4.98	4.98
1.5	5.28	0.9151	4.83	7.25
2	5.28	0.8885	4.69	9.38
2.5	5.28	0.8626	4.55	11.39
3	5.28	0.8375	4.42	13.27
3.5	5.28	0.8131	4.29	15.03
4	5.28	0.7894	4.17	16.67
4.5	5.28	0.7664	4.05	18.21
5	181.28	0.7441	134.89	674.45
			176.00	773.18

Duration = \$773.18/\$176.00 = 4.3931

b. What is the average duration of all the assets?

[(0.5)(\$90) + (0.9)(\$55) + (4.3931)(\$176) + (7)(\$2,724)]/\$3,045 = 6.5470 years

c. What is the average duration of all the liabilities?

[(1)(\$2,092) + (0.01)(\$238)]/\$2,330 = 0.8989 years

d. What is the leverage adjusted duration gap? What is the interest rate risk exposure?

DGAP = $D_A - kD_L$ = 6.5470 - (\$2,330/\$3,045)(0.8989) = 5.8592 years

The duration gap is positive, indicating that an increase in interest rates will lead to a decrease in the market value of equity.

e. What is the forecasted impact on the market value of equity caused by a relative upward shift in the entire yield curve of 0.5 percent [i.e., $\Delta R/(1+R) = 0.0050$]?

The market value of the equity will change by:

$\Delta MVE = -DGAP \times A \times \Delta R/(1+R) = -5.8592(\$3{,}045)(0.0050) = -\$89.207$. The loss in equity of $89,207 will reduce the market value of equity to $625,793.

f. If the yield curve shifts downward by 0.25 percent [i.e., $\Delta R/(1+R) = -0.0025$], what is the forecasted impact on the market value of equity?

The change in the value of equity is $\Delta MVE = -5.8592(\$3{,}045)(-0.0025) = \$44{,}603$. Thus, the market value of equity will increase by $44,603, to $759,603.

g. What variables are available to the financial institution to immunize the balance sheet? How much would each variable need to change to get DGAP equal to 0?

Immunization requires the bank to have a leverage adjusted duration gap of 0. Therefore, the FI could reduce the duration of its assets to 0.6878 years by using more T-bills and floating rate loans. Or, the FI could try to increase the duration of its deposits possibly by using fixed-rate CDs with a maturity of 3 or 4 years. Finally, the FI could use a combination of reducing asset duration and increasing liability duration in such a manner that DGAP is 0. This duration gap of 5.8592 years is quite large and it is not likely that the FI will be able to reduce it to zero by using only balance sheet adjustments. For example, even if the FI moved all of its loans into T-bills, the duration of the assets still would exceed the duration of the liabilities after adjusting for leverage. This adjustment in asset mix would imply foregoing a large yield advantage from the loan portfolio relative to the T-bill yields in most economic environments.

27. Refer again to the financial institution in problem 26.

a. What is the change in the value of the firm's assets for relative upward shift in the entire yield curve of 0.5 percent?

$\Delta A = -6.5470 * 0.005 * \$3{,}045 = -\$99.678428$. The expected change in the market value of the assets using the above formula is $99,678,428.

b. What is the change in the value of the firm's liabilities for relative upward shift in the entire yield curve of 0.4 percent?

$\Delta L = -0.8989 \text{ x } 0.004 \text{ x } \$2{,}330 = -\$8.37752$. The expected change in the market value of the liabilities using the above formula is $8,377,520.

c. What is the resulting change in the value of equity for the firm?

$\Delta MVE = \Delta MVA - \Delta MVL = \$99{,}678{,}428 - (\$8{,}377{,}520) = \$91{,}300{,}908$

28. Assume that a goal of the regulatory agencies of financial institutions is to immunize the ratio of equity to total assets, that is, $\Delta(E/A) = 0$. Explain how this goal changes the desired duration gap for the institution. Why does this differ from the duration gap necessary to immunize the total equity? How would your answers to part (h) in problem 24 and part (g) in problem 26 change if immunizing equity to total assets was the goal?

In this case, the duration of the assets and liabilities should be equal. Thus, if $\Delta E = \Delta A$, then by definition the leveraged adjusted duration gap is positive, since ΔE would exceed $k\Delta A$ by the amount of $(1 - k)$ and the FI would face the risk of increases in interest rates. In reference to problems 24 and 26, the adjustments on the asset side of the balance sheet would not need to be as strong, although the difference likely would not be large if the FI in question is a depository institution such as a bank or savings institution.

29. Identify and discuss three criticisms of using the duration gap model to immunize the portfolio of a financial institution.

The three criticisms are:

a Immunization is a dynamic problem because duration changes over time. Thus, it is necessary to rebalance the portfolio as the duration of the assets and liabilities change over time.

b Duration matching can be costly because it is not easy to restructure the balance sheet periodically, especially for large FIs.

c Duration is not an appropriate tool for immunizing portfolios when the expected interest rate changes are large because of the existence of convexity. Convexity exists because the relationship between security price changes and interest rate changes is not linear, which is assumed in the estimation of duration. Using convexity to immunize a portfolio will reduce the problem.

30. In general, what changes have occurred in the financial markets that would allow financial institutions to restructure their balance sheets more rapidly and efficiently to meet desired

goals? Why is it critical for an FI manager who has a portfolio immunized to match a desired investment horizon to rebalance the portfolio periodically? What is convexity? Why is convexity a desirable feature to capture in a portfolio of assets?

The growth of purchased funds markets, asset securitization, and loan sales markets have considerably increased the speed of major balance sheet restructurings. Further, as these markets have developed, the cost of the necessary transactions has also decreased. Finally, the growth and development of the derivative securities markets provides significant alternatives to managing the risk of interest rate movements only with on-balance-sheet adjustments.

Assets approach maturity at different rates of speed than the duration of the same assets approaches zero. Thus, after a period of time, a portfolio of assets that was immunized against interest rate risk will no longer be immunized. In fact, portfolio duration will exceed the remaining time in the investment or target horizon, and changes in interest rates could prove costly to the institution.

Convexity is a property of fixed-rate assets that reflects nonlinearity in the reflection of price-yield relationships. This characteristic is similar to buying insurance to cover part of the interest rate risk faced by the FI. The more convexity in the price-yield relationship for a given asset, the more insurance against interest rate changes is purchased.

31. A financial institution has an investment horizon of two years 9.33 months (or 2.777 years). The institution has converted all assets into a portfolio of 8 percent, $1,000, three-year bonds that are trading at a yield to maturity of 10 percent. The bonds pay interest annually. The portfolio manager believes that the assets are immunized against interest rate changes.

a. Is the portfolio immunized at the time of bond purchase? What is the duration of the bonds?

Three-year Bonds

Par value = $1,000 Coupon rate = 8% Annual payments

R = 10% Maturity = 3 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	80	0.9091	72.73	72.73
2	80	0.8264	66.12	132.23
3	1,080	0.7513	811.42	2,434.26
			950.26	2,639.22

Duration = $2,639.22/$950.26 = 2.777

The bonds have a duration of 2.777 years, which is 33.33 months. For practical purposes, the bond investment horizon is immunized at the time of purchase.

b. Will the portfolio be immunized one year later?

After one year, the investment horizon will be 1 year, 9.33 months (or 1.777 years). At this time, the bonds will have a duration of 1.9247 years, or 1 year, 11+ months. Thus, the bonds will no longer be immunized.

Two-year Bonds

Par value = $1,000 Coupon rate = 8% Annual payments

R = 10% Maturity = 2 years

t	CF_t	DF_t	CF_t x DF_t	CF_t x DF_t x t
1	$80	0.9091	72.73	72.73
2	$1,080	0.8264	892.56	1,785.12
			965.29	1,857.85

Duration = $1,857.85/$965.29 = 1.9247

c. Assume that one-year, 8 percent zero-coupon bonds are available in one year. What proportion of the original portfolio should be placed in these bonds to rebalance the portfolio?

The investment horizon is 1 year, 9.33 months, or 21.33 months. Thus, the proportion of bonds that should be replaced with the zero-coupon bonds can be determined by the following analysis:

21.33 months = w_{zero} x 12 months + (1 – w_{zero})x1.9247x12 months $\Rightarrow$ w_{zero} = 15.92 percent

Thus, 15.92 percent of the bond portfolio should be replaced with the zero-coupon bonds after one year.

The following questions and problems are based on material in Appendix 9A, at the book's website (www.mhhe.com/saunders9e).

32. Consider a 12-year, 12 percent annual coupon bond with a required return of 10 percent. The bond has a face value of $1,000.

a. What is the price of the bond?

PV = $120 x $PVA_{i=10\%,n=12}$ + $1,000 x $PV_{i=10\%,n=12}$ = $1,136.27

b. If interest rates rise to 11 percent, what is the price of the bond?

PV = \$120 x $PVA_{i=11\%,n=12}$ + \$1,000 x $PV_{i=11\%,n=12}$ = \$1,064.92

c. What has been the percentage change in price?

ΔP = (\$1,064.92 - \$1,136.27)/\$1,136.27 = -0.0628 or –6.28 percent.

d. Repeat parts (a), (b), and (c) for a 16-year bond.

PV = \$120 x $PVA_{i=10\%,n=16}$ + \$1,000 x $PV_{i=10\%,n=16}$ = \$1,156.47
PV = \$120 x $PVA_{i=11\%,n=16}$ + \$1,000 x $PV_{i=11\%,n=16}$ = \$1,073.79
ΔP = (\$1,073.79 - \$1,156.47)/\$1,156.47 = -0.0715 or –7.15 percent.

e. What do the respective changes in bond prices indicate?

For the same change in interest rates, longer-term fixed-rate assets experience a greater change in price.

33. Consider a five-year, 15 percent annual coupon bond with a face value of \$1,000. The bond is trading at a yield to maturity of 12 percent.

a. What is the price of the bond?

PV = \$150 x $PVA_{i=12\%,n=5}$ + \$1,000 x $PV_{i=12\%,n=5}$ = \$1,108.14

b. If the yield to maturity increases 1 percent, what will be the bond's new price?

PV = \$150 x $PVA_{i=13\%,n=5}$ + \$1,000 x $PV_{i=13\%,n=5}$ = \$1,070.34

c. Using your answers to parts (a) and (b), what is the percentage change in the bond's price as a result of the 1 percent increase in interest rates?

ΔP = (\$1,070.34 - \$1,108.14)/\$1,108.14 = -0.0341 or –3.41 percent.

d. Repeat parts (b) and (c) assuming a 1 percent decrease in interest rates.

PV = \$150 x $PVA_{i=11\%,n=5}$ + \$1,000 x $PV_{i=11\%,n=5}$ = \$1,147.84
ΔP = (\$1,147.84 - \$1,108.14)/\$1,108.14 = 0.0358 or 3.58 percent

e. What do the differences in your answers indicate about the interest rate-price relationships of fixed-rate assets?

For a given percentage change in interest rates, the absolute value of the increase in price caused by a decrease in rates is greater than the absolute value of the decrease in price caused by an increase in rates.

34. Consider a $1,000 bond with a fixed-rate 10 percent annual coupon rate and a maturity (N) of 10 years. The bond currently is trading at a yield to maturity (YTM) of 10 percent.

a. Complete the following table:

N	Change Coupon Rate	YTM	Price	$ Change in Price from Par	% Change in Price from Par
8	10%	9%	$1,055.35	$55.35	5.535%
9	10	9	1,059.95	59.95	5.995
10	10	9	1,064.18	64.18	6.418
10	10	10	1,000.00	0.00	0.00
10	10	11	941.11	-58.89	-5.889
11	10	11	937.93	-62.07	-6.207
12	10	11	935.07	-64.93	-6.493

b. Use this information to verify the principles of interest rate-price relationships for fixed-rate financial assets.

Rule 1. Interest rates and prices of fixed-rate financial assets move inversely.

See the change in price from $1,000 to $941.11 for the change in interest rates from 10 percent to 11 percent, or from $1,000 to $1,064.18 when rates change from 10 percent to 9 percent.

Rule 2. The longer is the maturity of a fixed-income financial asset, the greater is the change in price for a given change in interest rates.

A change in rates from 10 percent to 11 percent caused the 10-year bond to decrease in value $58.89, but the 11-year bond decreased in value $62.07, and the 12-year bond decreased $64.93.

Rule 3. The change in value of longer-term fixed-rate financial assets increases at a decreasing rate.

For the increase in rates from 10 percent to 11 percent, the difference in the change in price between the 10-year and 11-year assets is \$3.18 (\$62.07 - \$58.89), while the difference in the change in price between the 11-year and 12-year assets is \$2.86 (\$64.93 - \$62.07).

Rule 4. Although not mentioned in Appendix 9A, for a given percentage (±) change in interest rates, the increase in price for a decrease in rates is greater than the decrease in value for an increase in rates.

For rates decreasing from 10 percent to 9 percent, the 10-year bond increases \$64.18. But for rates increasing from 10 percent to 11 percent, the 10-year bond decreases \$58.89.

The following questions and problems are based on material in Appendix 9B to the chapter.

35. MLK Bank has an asset portfolio that consists of \$100 million of 30-year, 8 percent coupon, \$1,000 bonds that sell at par.

a. What will be the bonds' new prices if market yields change immediately by ± 0.10 percent? What will be the new prices if market yields change immediately by ± 2.00 percent?

At +0.10%: Price = $\$80 \times PVA_{n=30,\, i=8.1\%} + \$1{,}000 \times PV_{n=30,\, i=8.1\%} = \$988.85$

At –0.10%: Price = $\$80 \times PVA_{n=30,\, i=7.9\%} + \$1{,}000 \times PV_{n=30,\, i=7.9\%} = \$1{,}011.36$

At +2.0%: Price = $\$80 \times PVA_{n=30,\, i=10\%} + \$1{,}000 \times PV_{n=30,\, i=10\%} = \$811.46$

At –2.0%: Price = $\$80 \times PVA_{n=30,\, i=6.0\%} + \$1{,}000 \times PV_{n=30,\, i=6.0\%} = \$1{,}275.30$

b. The duration of these bonds is 12.1608 years. What are the predicted bond prices in each of the four cases using the duration rule? What is the amount of error between the duration prediction and the actual market values?

$$\Delta P = -D \times [\Delta R/(1+R)] \times P$$

At +0.10%: $\Delta P = -12.1608 \times 0.001/1.08 \times \$1{,}000 = -\$11.26 \Rightarrow P' = \$988.74$

At -0.10%: $\Delta P = -12.1608 \times (-0.001/1.08) \times \$1{,}000 = \$11.26 \Rightarrow P' = \$1{,}011.26$

At +2.0%: $\Delta P = -12.1608 \times 0.02/1.08) \times \$1{,}000 = -\$225.20 \Rightarrow P' = \$774.80$

At -2.0%: $\Delta P = -12.1608 \times (-0.02/1.08) \times \$1{,}000 = \$225.20 \Rightarrow P' = \$1{,}225.20$

	Price - market determined	Price - duration estimation	Amount of error
At +0.10%:	$988.85	$988.74	$0.11
At -0.10%:	$1,011.36	$1,011.26	$0.10
At +2.0%:	$811.46	$774.80	$36.66
At -2.0%:	$1,275.30	$1,225.20	$50.10

c. Given that convexity is 212.4, what are the bond price predictions in each of the four cases using the duration plus convexity relationship? What is the amount of error in these predictions?

$$\Delta P = \{-D \times [\Delta R/(1+R)] + ½ \times CX \times (\Delta R)^2\} \times P$$

At +0.10%: $\Delta P = \{-12.1608 \times 0.001/1.08 + 0.5 \times 212.4 \times (0.001)^2\} \times \$1,000 = -\$11.15$

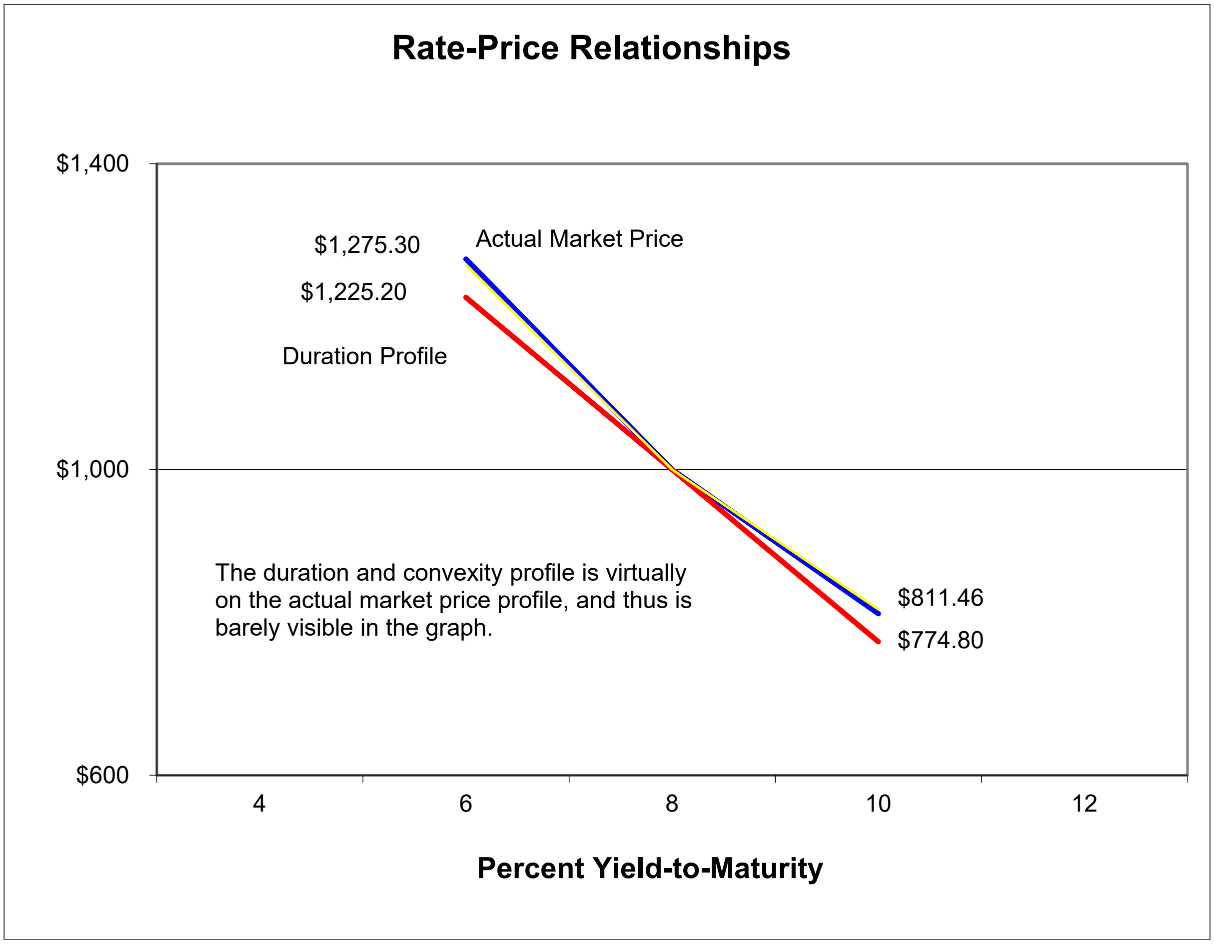
At -0.10%: $\Delta P = \{-12.1608 \times (-0.001/1.08) + 0.5 \times 212.4 \times (-0.001)^2\} \times \$1,000 = \$11.366$

At +2.0%: $\Delta P = \{-12.1608 \times 0.02/1.08 + 0.5 \times 212.4 \times (0.02)^2\} \times \$1,000 = -\$182.72$

At -2.0%: $\Delta P = \{-12.1608 \times (-0.02/1.08) + 0.5 \times 212.4 \times (-0.02)^2\} \times \$1,000 = \$267.68$

	Price market determined	ΔPrice duration & convexity estimation	Price duration & convexity estimation	Amount of error
At +0.10%:	$988.85	-$11.15	$988.85	$0.00
At -0.10%:	$1,011.36	$11.37	$1,011.37	$0.01
At +2.0%:	$811.46	-$182.72	$817.28	$5.82
At -2.0%:	$1,275.30	$267.68	$1,267.68	$7.62

d. Diagram and label clearly the results in parts (a), (b) and (c).



The profiles for the estimates based on only ± 0.10 percent changes in rates are very close together and do not show clearly in a graph. However, the profile relationship would be similar to that shown above for the ± 2.0 percent changes in market rates.

36. Estimate the convexity for each of the following three bonds, all of which trade at yield to maturity of 8 percent and have face values of $1,000.

A 7-year, zero-coupon bond.
A 7-year, 10 percent annual coupon bond.
A 10-year, 10 percent annual coupon bond that has a duration value of 6.994 years (i.e., approximately 7 years).

	ΔMarket Value at 8.01 percent	ΔMarket Value at 7.99 percent	Capital Loss + Capital Gain Divided by Original Price
7-year zero	-0.37804819	0.37832833	0.00000048

7-year coupon	-0.55606169	0.55643682	0.00000034
10-year coupon	-0.73121585	0.73186329	0.00000057

Convexity = 10^8 x (Capital Loss + Capital Gain) ÷ Original Price at 8.00 percent

7-year zero	CX = 100,000,000 x 0.00000048 = 48
7-year coupon	CX = 100,000,000 x 0.00000034 = 34
10-year coupon	CX = 100,000,000 x 0.00000057 = 57

An alternative method of calculating convexity for these three bonds using the following equation is illustrated at the end of this problem.

$$\text{Convexity} = \frac{1}{P \text{ x } (1+R)^2} \text{ x} \sum_{t=1}^{n} \left[\frac{CF_t}{(1+R)^t} \times t \times (1+t) \right]$$

Rank the bonds in terms of convexity, and express the convexity relationship between zeros and coupon bonds in terms of maturity and duration equivalencies.

Ranking, from least to most convexity: 7-year coupon bond, 7-year zero, 10-year coupon

Convexity relationships:

Given the same yield-to-maturity, a zero-coupon bond with the same maturity as a coupon bond will have more convexity.

Given the same yield-to-maturity, a zero-coupon bond with the same duration as a coupon bond will have less convexity.

Zero-coupon Bond

Par value = $1,000 Coupon = 0%

R = 8% Maturity = 7 years

t	CF	PV of CF	PV of CF x t	x(1+t)	x(1+R)^2
1	0.00	0.00	0.00	0.00	
2	0.00	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	
4	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	0.00	
6	0.00	0.00	0.00	0.00	
7	1,000.00	583.49	4,084.43	32,675.46	
		583.49	4,084.43	32,675.46	680.58

Duration = 7.0000

Convexity = 48.011

7-year Coupon Bond

Par value = $1,000 Coupon = 10%

R = 8% Maturity = 7 years

t	CF	PVof CF	PV of CF x t	x(1+t)	x(1+R)^2
1	100.00	92.59	92.59	185.19	
2	100.00	85.73	171.47	514.40	
3	100.00	79.38	238.15	952.60	
4	100.00	73.50	294.01	1,470.06	
5	100.00	68.06	340.29	2,041.75	
6	100.00	63.02	378.10	2,646.71	
7	1,100.00	641.84	4,492.88	35,943.01	
		1,104.13	6,007.49	43,753.72	1287.9

Duration = 5.4409

Convexity = 33.974

10-year Coupon Bond

Par value = $1,000 Coupon = 10%

R = 8% Maturity = 10 years

t	CF	PV of CF	PV of CF x t	x(1+t)	x(1+R)^2
1	100.00	92.59	92.59	185.19	
2	100.00	85.73	171.47	514.40	
3	100.00	79.38	238.15	952.60	
4	100.00	73.50	294.01	1,470.06	
5	100.00	68.06	340.29	2,041.75	
6	100.00	63.02	378.10	2,646.71	
7	100.00	58.35	408.44	3,267.55	
8	100.00	54.03	432.22	3,889.94	
9	100.00	50.02	450.22	4,502.24	
10	1,100.0	509.51	5,095.13	56,046.41	
		1,134.20	7,900.63	75,516.84	1322.9

Duration = 6.9658

Convexity = 57.083

Integrated Mini Case: Calculating and Using Duration GAP

State Bank's balance sheet is listed below. Market yields and durations (in years) are in parenthesis, and amounts are in millions.

Assets Liabilities and Equity

Cash	$20	Demand deposits	$250
Fed funds (1.05%, 0.02)	150	MMDAs (2.5%, 0.50)	
T-bills (5.25%, 0.22)	300	(no minimum balance requirement)	360
T-bonds (7.50%, 7.55)	200	CDs (4.3%, 0.48)	715
Consumer loans (6%, 2.50)	900	CDs (6%, 4.45)	1,105
C&I loans (5.8%, 6.58)	475	Fed funds (1%, 0.02)	515
Fixed-rate mortgages (7.85%, 19.50)	1,200	Commercial paper (3%, 0.45)	400
Variable-rate mortgages,		Subordinated debt:	
repriced @ quarter (6.3%, 0.25)	580	Fixed-rate (7.25%, 6.65)	200
Premises and equipment	120	Total liabilities	$3,545
		Equity	400
Total assets	$3,945	Total liabilities and equity	$3,945

a. What is State Bank's duration gap?

D_A = [20(0) + 150(0.02) + 300(0.22) + 200(7.55) + 900(2.50) + 475(6.85) + 1,200(19.50) + 580(0.25) + 120(0)]/3,945 = 7.76369 year

D_L = [250(0) + 360(0.50) + 715(0.48) + 1,105(4.45) + 515(0.02) + 400(0.45) + 200(6.65))]/3,545 = 1.96354 years

DGAP = D_A - kD_L = 7.76369 - ($3,545/$3,945)(1.96354) = 5.99924 years

b. Use these duration values to calculate the expected change in the value of the assets and liabilities of State Bank for a predicted increase of 1.5 percent in interest rates.

$\Delta MV_{fedfunds}$ = -0.02 x 0.015/1.0105 x 150m = -$44,532
$\Delta MV_{T.bills}$ = -0.22 x 0.015/1.0525 x 300m = -$940,618
$\Delta MV_{T.bonds}$ = -7.55 x 0.015/1.0750 x 200m = -$21,069,767
$\Delta MV_{consumerloans}$ = -2.50 x 0.015/1.0600 x 900m = -$31,839,623
$\Delta MV_{C\&Iloans}$ = -6.58 x 0.015/1.0580 x 475m = -$44,312,382
$\Delta MV_{fixed.ratemortgages}$ = -19.50 x 0.015/1.0785 x 1,200m = -$325,452,017
$\Delta MV_{variable.ratemortgages}$ = -0.25 x 0.015/1.0630 x 580m = -$2,046,096

=>ΔMVA = -$425,705,035

ΔMV_{MMDAs} = -0.50 x 0.015/1.025 x 360m = -$2,634,146
ΔMV_{CDs} = -0.48 x 0.015/1.0430 x 715m = -$4,935,762
ΔMV_{CDs} = -4.45 x 0.015/1.0600 x 1,105m = -$69,583,726

$\Delta MV_{fedfunds} = -0.02 \times 0.015/1.0100 \times 515m = -\$152,970$
$\Delta MV_{commericalpaper} = -0.45 \times 0.015/1.0300 \times 400m = -\$2,621,359$
$\Delta MV_{fixed.ratesubordinatedebt} = -6.65 \times 0.015/1.0725 \times 200m = \underline{-\$18,601,399}$

$=> \Delta MVL = -\$98,529,362$

c. What is the change in equity value forecasted from the duration values for a predicted increase in interest rates of 1.5 percent?

$\Delta MVE = \Delta MVA - \Delta MVL = -\$425,705,035 - (-\$98,529,362) = -\$327,175,673$

Integrated Mini Case Chapters 8 and 9: Calculating and Using Repricing and Duration GAP

State Bank's balance sheet is listed below. Market yields and durations (in years) are in parenthesis, and amounts are in millions.

Assets		Liabilities and Equity	
Cash	$31	Demand deposits	$253
Fed funds (2.05%, 0.02)	150	Savings accounts (0.5%, 1.25)	50
3-month T-bills (3.25%, 0.22)	200	MMDAs (3.5%, 0.50)	
8-year T-bonds (6.50%, 7.55)	250	(no minimum balance requirement)	460
5-year munis (7.20%, 4.25)	50	3-month CDs (3.2%, 0.20)	175
6-month consumer loans (5%, 0.42)	250	1-year CDs (3.5%, 0.95)	375
5-year car loans (6%, 3.78)	350	5-year CDs (5%, 4.85)	350
7-month C&I loans (4.8%, 0.55)	200	Fed funds (2%, 0.02)	225
2-year C&I loans (4.15%, 1.65)	275	Repos (2%, 0.05)	290
Fixed-rate mortgages (5.10%, 0.48)		6-month commercial paper	
(maturing in 5 months)	450	(4.05%, 0.55)	300
Fixed-rate mortgages (6.85%, 0.85)		Subordinate notes:	
(maturing in 1 year)	300	1-year fixed rate (5.55%, 0.92)	200
Fixed-rate mortgages (5.30%, 4.45)		Subordinated debt:	
(maturing in 5 years)	275	7-year fixed rate (6.25%, 6.65)	100
Fixed-rate mortgages (5.40%, 18.25)		Total liabilities	$2,778
(maturing in 20 years)	355		
Premises and equipment	20	Equity	378
Total assets	$3,156	Total liabilities and equity	$3,156

a. What is the repricing gap if the planning period is six months? One year?

Assets Repricing period

Cash	$31	Not rate sensitive
Fed funds (2.05%)	150	6-months
3-month T-bills (3.25%)	200	6-months
8-year T-bonds (6.50%)	250	Not rate sensitive
5-year munis (7.20%)	50	Not rate sensitive
6-month consumer loans (5%)	250	6-months
5-year car loans (6%)	350	Not rate sensitive
7-month C&I loans (4.8%)	200	1-year
2-year C&I loans (4.15%)	275	Not rate sensitive
Fixed-rate mortgages (5.10%) (maturing in 5 months)	450	6-months
Fixed-rate mortgages (6.85%) (maturing in 1 year)	300	1-year
Fixed-rate mortgages (5.30%) (maturing in 5 years)	275	Not rate sensitive
Fixed-rate mortgages (5.40%) (maturing in 20 years)	355	Not rate sensitive
Premises and equipment	20	Not rate sensitive

Liabilities and Equity		Repricing Period
Demand deposits	$253	Not rate sensitive
Savings accounts (0.5%)	50	6-months
MMDAs (3.5%) (no minimum balance requirement)	460	6-months
3-month CDs (3.2%)	175	6-months
1-year CDs (3.5%)	375	1-year
5-year CDs (5%)	350	Not rate sensitive
Fed funds (2%)	225	6-months
Repos (2%)	290	6-months
6-month commercial paper (4.05%)	300	6-months
Subordinate notes 1-year fixed rate (5.55%)	200	1-year
Subordinated debt 7-year fixed rate (6.25%)	100	Not rate sensitive
Equity	400	Not rate sensitive

6-month repricing gap: RSAs = $150m. + $200m. + $250m. + $450m. = $1050m.

RSLs = $50m. + $460m. + $175m. + $225m. + $290m. + $300m. = $1500m.

CGAP = $1050m. - $1500m. = -$450m.

1-year repricing gap: RSAs = \$1050m. + \$200m. + \$300m. = \$1550m.
RSLs = \$1500m. + \$375m. + \$200m. = \$2075m.
CGAP = \$1550m. - \$2075m. = -\$525m.

b. What is State Bank's duration gap?

D_A = [31(0) + 150(0.02) + 200(0.22) + 250(7.55) + 50(4.25) + 250(0.42) + 350(3.78) + 200(0.55) + 275(1.65) + 450(0.48) + 300(0.85) + 275(4.45) + 355(18.25) + 20(0)]/3,156 = 3.90122 years

D_L = [253(0) + 50(1.25) + 460(0.50) + 175(0.20) + 375(0.95) + 350(4.85) + 225(0.02) + 290(0.05) + 300(0.55) + 200(0.92) + 100(6.65))]/2,778 = 1.22903 years

DGAP = D_A - kD_L = 3.90122 - (\$2,778/\$3,156)(1.22903) = 2.81939 years

c. What is the impact over the next six months on net interest income if interest rates on RSAs increase 50 basis points and on RSLs increase 35 basis points? Explain the results.

ΔNII (6-months) = ΔII (6-months) – ΔIE (6-months)
= \$1050m.(0.0050) - \$1500m.(0.0035) = \$0m.

GAP is negative and interest rates are expected to go up. Therefore, the GAP effect pushes NII down. However, at the same time the spread increases. So, the spread affect pushes NII up. Thus, despite the fact that the GAP works against the bank, reducing its NII, the spread affect exactly offsets this, increasing NII by the same amount. The net effect is no change in NII.

d. What is the impact over the next year on net interest income if interest rates on RSAs decrease (increase) 35 basis points and on RSLs decrease (increase) 50 basis points? Explain the results.

ΔNII (1-year) = ΔII (1-year) – ΔIE (1-year)
= \$1505m.(-0.0035) - \$2075m.(-0.0050) = \$4.95m.

GAP is negative and interest rates are expected to go down. Therefore, the GAP effect pushes NII up. At the same time the spread decreases. So, the spread affect pushes NII up. Thus, both the GAP effect and the spread affect work to increase NII.

ΔNII (1-year) = ΔII (1-year) – ΔIE (1-year)
= \$1550m.(0.0035) - \$2075m.(0.0050) = \$4.95m.

GAP is negative and interest rates are expected to go up. Therefore, the GAP effect pushes NII down. At the same time the spread decreases. So, the spread affect pushes NII down. Thus, both the GAP effect and the spread affect work to decrease NII.

e. Use these duration values to calculate the expected change in the value of the assets and liabilities of State Bank for a predicted decrease of 0.35 percent in interest rates on assets and 0.50 percent on liabilities.

$\Delta MV_{fedfunds} = -0.02 \times -0.0035/1.0205 \times 150m = \$10,289$

$\Delta MV_{T.bills} = -0.22 \times -0.0035/1.0325 \times 200m = \$149,153$

$\Delta MV_{T.bonds} = -7.55 \times -0.0035/1.0650 \times 250m = \$6,203,052$

$\Delta MV_{munis} = -4.55 \times -0.0035/1.0720 \times 50m = \$742,771$

$\Delta MV_{consumerloans} = -0.42 \times -0.0035/1.0500 \times 250m = \$350,000$

$\Delta MV_{carloans} = -3.78 \times -0.0035/1.0600 \times 350m = \$4,368,396$

$\Delta MV_{C\&Iloans} = -0.55 \times -0.0035/1.0480 \times 200m = \$367,366$

$\Delta MV_{C\&Iloans} = -1.65 \times -0.0035/1.0415 \times 275m = \$1,524,844$

$\Delta MV_{fixed.ratemortgages} = -0.48 \times -0.0035/1.0510 \times 450m = \$719,315$

$\Delta MV_{fixed.ratemortgages} = -0.85 \times -0.0035/1.0685 \times 300m = \$835,283$

$\Delta MV_{fixed.ratemortgages} = -4.45 \times -0.0035/1.0530 \times 275m = \$4,067,544$

$\Delta MV_{fixed.ratemortgages} = -18.25 \times -0.0035/1.0540 \times 355m = \underline{\$2,046,096}$

$\Rightarrow \Delta MVA = \$40,851,889$

$\Delta MV_{savings} = -1.25 \times -0.005/1.0050 \times 50m = \$310,945$

$\Delta MV_{MMDAs} = -0.50 \times -0.005/1.0350 \times 460m = \$1,111,111$

$\Delta MV_{CDs} = -0.20 \times -0.005/1.0320 \times 175m = \$169,574$

$\Delta MV_{CDs} = -0.95 \times -0.005/1.0350 \times 375m = \$1,721,015$

$\Delta MV_{CDs} = -4.85 \times -0.005/1.0500 \times 350m = \$8,083,333$

$\Delta MV_{fedfunds} = -0.02 \times -0.005/1.0200 \times 225m = \$22,059$

$\Delta MV_{repos} = -0.05 \times -0.005/1.0200 \times 290m = \$71,078$

$\Delta MV_{commericalpaper} = -0.55 \times -0.005/1.0405 \times 300m = \$792,888$

$\Delta MV_{subordinatedebt} = -0.92 \times -0.005/1.0555 \times 200m = \$871,625$

$\Delta MV_{subordinatedebt} = -6.65 \times -0.005/1.0625 \times 100m = \underline{\$3,129,412}$

$\Rightarrow \Delta MVL = \$16,283,040$

f. What is the change in equity value forecasted from the duration values for decrease of 0.35 percent in interest rates on assets and 0.50 percent on liabilities?

$\Delta MVE = \Delta MVA - \Delta MVL = \$40,851,889 - (\$16,283,040) = \$24,568,849$

g. Use the duration gap model to calculate the change in equity value if the relative change in all market interest rates is a decrease of 50 basis points.

$\Delta MVE = -DGAP \times A \times \Delta R/(1 + R) = -2.81939 \text{ years} \times 3,156m \times (-0.0050) = \$44.49m$

Additional Example for Chapter 9

This example is to estimate both the duration and convexity of a 6-year bond paying 5 percent coupon annually and the annual yield to maturity is 6 percent.

6-year Coupon Bond

Par value = $1,000 Coupon = 5%

R = 6% Maturity = 6 years

t	CF	PV of CF	PV of CF x t	x(1+t)	$x(1+R)^2$
1	$50.00	$47.17	$47.17	$94.34	
2	$50.00	$44.50	$89.00	$267.00	
3	$50.00	$41.98	$125.94	$503.77	
4	$50.00	$39.60	$158.42	$792.09	
5	$50.00	$37.36	$186.81	1,120.89	
6	$1,050.00	$740.21	$4,441.25	31,088.76	
		$950.83	$5,048.60	33,866.85	1068.3

Duration = 5.3097

Convexity = 31.7

Using the textbook method:

$$CX = 10^8 [(950.3506-950.8268)/950.8268 + (951.3032-950.8268)/950.8268]$$
$$= 10^8[-0.0005007559 + 0.0005501073] = 31.70$$

What is the effect of a 2 percent increase in interest rates, from 6 percent to 8 percent?

Using Present Values, the percentage change is:

$$= (\$950.8268 - \$861.3136)/ \$950.8268 = -9.41\%$$

Using the duration formula: $\Delta MVA = -D \times \Delta R/(1 + R) + 0.5CX(R)^2$

$$= -5.3097 \times [(0.02)/1.06] + 0.5(31.7)(0.02)^2$$
$$= -0.1002 + .0063 = -9.38\%$$

Adding convexity adds more precision. Duration alone would have given the answer of -10.02%.