

Solutions for End-of-Chapter Questions and Problems: Chapter Ten

1. Why is credit risk analysis an important component of FI risk management? What recent activities by FIs have made the task of credit risk assessment more difficult for both FI managers and regulators?

Credit risk management is important for FI managers because it determines several features of a loan: interest rate, maturity, collateral and other covenants. Riskier projects require more analysis before loans are approved. If credit risk analysis is inadequate, default rates could be higher and push a bank into insolvency, especially if the markets are competitive and the margins are low.

Credit risk management has become more complicated over time because of the increase in off-balance-sheet activities that create implicit contracts and obligations between prospective lenders and buyers. Credit risks of some off-balance-sheet products, such as loan commitments, credit guarantees, and interest rate swaps, are difficult to assess because the contingent payoffs are not deterministic, making the pricing of these products complicated.

2. Differentiate between a secured and an unsecured loan. Who bears most of the risk in a fixed-rate loan? Why would FI managers prefer to charge floating rates, especially for longer-maturity loans?

A secured loan is backed by some of the collateral that is pledged to the lender in the event of default. A lender has rights to the collateral, which can be liquidated to pay all or part of the loan. With a fixed-rate loan, the lender bears the risk of interest rate changes. If interest rates rise, the opportunity cost of lending is higher, while if interest rates fall the lender benefits. Since it is harder to predict longer-term rates, FIs prefer to charge floating rates for longer-term loans and pass the interest rate risk on to the borrower.

3. How does a spot loan differ from a loan commitment? What are the advantages and disadvantages of borrowing through a loan commitment?

A spot loan involves the immediate takedown of the loan amount by the borrower, while a loan commitment allows a borrower the option to take down the loan any time during a fixed period at a predetermined rate. This can be advantageous during periods of rising rates in that the borrower can borrow as needed at a predetermined rate. If rates decline, the borrower can borrow from other sources. The disadvantage is the cost: often an up-front fee is required in addition to a back-end fee for the unused portion of the commitment.

4. Why is commercial lending declining in importance in the U.S.? What effect does this decline have on overall commercial lending activities?

Commercial bank lending has been declining in importance because of disintermediation, a process in which customers are able to access financial markets directly such as by issuing commercial paper. The total amount of commercial paper outstanding in the U.S. has grown dramatically over the last thirty years. Historically, only the most creditworthy borrowers had access the commercial paper market, but more middle-market firms and financial institutions now have access to this market. As a consequence of this growth, the pool of borrowers available to banks has become smaller and riskier. This makes the credit assessment and monitoring of loans more difficult, yet important.

5. What are the primary characteristics of residential mortgage loans? Why does the ratio of adjustable-rate mortgages to fixed-rate mortgages in the economy vary over an interest rate cycle? When would the ratio be highest?

Residential mortgage contracts differ in size, the ratio of the loan amount to the value of the property, the maturity of the loan, the rate of interest of the loan, and whether the interest rate is fixed or adjustable. In addition, mortgage agreements differ in the amount of fees, commissions, discounts, and points that are paid by the borrower.

The ratio of adjustable-rate mortgages to fixed-rate mortgages is lowest when interest rates are low because borrowers prefer to lock in the low market rates for long periods of time. When rates are high, adjustable-rate mortgages allow borrowers the potential to realize relief from high interest rates in the future when rates decline.

6. What are the two major classes of consumer loans at U.S. banks? How do revolving loans differ from nonrevolving loans?

Consumer loans can be classified as either nonrevolving or revolving loans. Automobile loans and fixed-term personal loans usually have a maturity date at which time the loan is expected to have a zero balance, and thus they are considered to be nonrevolving loans. Revolving loans usually involve credit card debt, or similar lines of credit, and as a result the balance will rise and fall as borrowers make payments and utilize the accounts. Many banks often recognize high rates of return on these loans, even though in recent years, banks have faced chargeoff rates in the range of four to eight percent.

7. Why are rates on credit card loans generally higher than rates on car loans?

Car loans are backed by collateral (the car), while credit card loans are not. Thus, in the event of default on a car loan, the FI can take possession of the car to recoup at least some the lost interest and principal payments. In the event of a default on a credit card loan, the FI has no such

collateral available with which to recover lost interest and principal payments. Accordingly, the FI charges a higher rate on the credit card loan.

8. What are compensating balances? What is the relationship between the amount of compensating balance requirement and the return on the loan to the FI?

A compensating balance is the portion of a loan that a borrower must keep on deposit with the credit-granting FI. Thus, the funds are not available for use by the borrower. As the amount of compensating balance for a given loan size increases, the effective return on the loan increases for the lending institution.

9. Suppose that a bank does the following:

- a. Sets a loan rate on a prospective loan at 8 percent (where $BR = 5\%$ and $\phi = 3\%$).
- b. Charges a $\frac{1}{10}$ percent (or 0.10 percent) loan origination fee to the borrower.
- c. Imposes a 5 percent compensating balance requirement to be held as noninterest-bearing demand deposits.
- d. Holds reserve requirements of 10 percent imposed by the Federal Reserve on the bank's demand deposits.

Calculate the bank's ROA on this loan.

$$1 + k = 1 + \frac{0.0010 + (0.05 + 0.03)}{1 - [(0.05)(1 - 0.10)]} = 1 + \frac{0.081}{0.955} = 1.0848 \text{ or } k = 8.48\%$$

10. County Bank offers one-year loans with a stated rate of 9 percent, but requires a compensating balance of 10 percent. What is the true cost of this loan to the borrower? How does the cost change if the compensating balance is 15 percent? If the compensating balance is 20 percent? In each case, assume origination fees and the reserve requirement are zero.

The true cost is the loan rate \div (1 - compensating balance rate) = $9\% \div (1.0 - 0.1) = 10$ percent. For compensating balance rates of 15 percent and 20 percent, the true cost of the loan would be 10.59 percent ($= 9\% \div (1.0 - 0.15)$) and 11.25 percent ($= 9\% \div (1.0 - 0.2)$), respectively. Note that as the compensating balance rate increases by a constant amount, the true cost of the loan increases at an increasing rate.

11. Metrobank offers one-year loans with a 9 percent stated or base rate, charges a 0.25 percent loan origination fee, imposes a 10 percent compensating balance requirement, and must

hold a 6 percent reserve requirement at the Federal Reserve. The loans typically are repaid at maturity.

- a. If the risk premium for a given customer is 2.5 percent, what is the simple promised interest return on the loan?

The simple promised interest return on the loan is $BR + \phi = 0.09 + 0.025 = 0.115$ or 11.5%.

- b. What is the contractually promised gross return on the loan per dollar lent?

$$1 + k = 1 + \frac{\text{of} + (BR + \Phi)}{1 - [b(1 - RR)]} = 1 + \frac{0.0025 + (0.09 + 0.025)}{1 - [0.1(1 - 0.06)]} = 1 + \frac{0.1175}{0.906} = 1.1297 \text{ or } k = 0.1297 = 12.97\%$$

- c. Which of the fee items has the greatest impact on the gross return?

The compensating balance has the strongest effect on the gross return on the loan. Without the compensating balance, the gross return would equal 11.75 percent, a reduction of 1.22 percent. Without the origination fee, the gross return would be 12.69 percent, a reduction of only 0.28 percent. Eliminating the reserve requirement would cause the gross return to increase to 13.06 percent, an increase of 0.09 percent.

12. Why are most retail borrowers charged the same rate of interest, implying the same risk premium or class? What is credit rationing? How is it used to control credit risks with respect to retail and wholesale loans?

Most retail loans are small in size relative to the overall investment portfolio of an FI, and the cost of collecting information on household borrowers is high. As a result, most retail borrowers are charged the same rate of interest that implies the same level of risk.

Credit rationing involves restricting the amount of loans that are available to individual borrowers. On the retail side, the amount of loans provided to borrowers may be determined solely by the proportion of loans desired in this category rather than price or interest rate differences, thus the actual credit quality of the individual borrowers. On the wholesale side, the FI may use both credit quantity and interest rates to control credit risk. Typically, more risky borrowers are charged a higher risk premium to control credit risk. However, the expected returns from increasingly higher interest rates that reflect higher credit risk at some point will be offset by higher default rates. Thus, rationing credit through quantity limits will occur at some interest rate level even though positive loan demand exists at even higher risk premiums.

13. Why could a lender's expected return be lower when the risk premium is increased on a loan? In addition to the risk premium, how can a lender increase the expected return on a wholesale loan? A retail loan?

An increase in risk premiums indicates a riskier pool of clients who are more likely to default by taking on riskier projects. This reduces the repayment probability and lowers the expected return to the lender. The lender often is able to charge fees that increase the return on the loan. However, the fees may become sufficiently high as to increase the risk of nonpayment or default on the loan.

14. What are covenants in a loan agreement? What are the objectives of covenants? How can these covenants be negative? Positive?

Covenants are restrictions that are written into loan or bond contracts that affect the actions of the borrower. Negative covenants in effect restrict actions, that is, they are "thou shall not..." conditions. Common examples include the nonincrease of dividend payments without permission of the lender, or the maintenance of net working capital above some minimum level. Positive covenants encourage actions such as the submission of quarterly financial statements. In effect both types of covenants are designed and implemented to assist the lender in the monitoring and control of credit risk.

15. Identify and define the borrower-specific and market-specific factors that enter into the credit decision. What is the impact of each type of factor on the risk premium?

The borrower-specific factors are:

- Reputation: Based on the lending history of the borrower; better reputation implies a lower risk premium.
- Leverage: A measure of the existing debt of the borrower; the larger the debt, the higher the risk premium.
- Volatility of earnings: The more stable the earnings, the lower the risk premium.
- Collateral: If collateral is offered, the risk premium is lower.

Market-specific factors include:

- Business cycle: Lenders are less likely to lend if a recession is forecasted.
- Level of interest rates: A higher level of interest rates may lead to higher default rates, so lenders are more reluctant to lend under such conditions.

- a. Which of these factors is more likely to adversely affect small businesses rather than large businesses in the credit assessment process by lenders?

Because reputation involves a history of performance over an extended time period, small businesses that are fairly young in operating time may suffer.

- b. How does the existence of a high debt ratio typically affect the risk of the borrower?

Increasing amounts of debt increase the interest charges that must be paid by the borrower, and thus, decrease the amount of cash flows available to repay the debt principal.

- c. Why is the volatility of the earnings stream of a borrower important to a lender?

A highly volatile earnings stream increases the probability that the borrower cannot meet the fixed interest and principal payments for any given capital structure.

16. Why is the degree of collateral as specified in the loan agreement of importance to a lender? If the book value of the collateral is greater than or equal to the amount of the loan, is the credit risk of a lender fully covered? Why, or why not?

Collateral provides the lender with some assets that can be used against the amount of the loan in the case of default. However, collateral has value only to the extent of its market value, and thus a loan fully collateralized at book value may not be fully collateralized at market value. Further, errors in the recording of collateralized positions may limit or severely reduce the protected positions of a lender.

17. Why are FIs consistently interested in the expected level of economic activity in the markets in which they operate? Why is monetary policy of the Federal Reserve System important to FIs?

During recessions firms in certain industries are much more likely to suffer financial distress because of the slowdown in economic activity. Specifically, the consumer durables industries are particularly hard hit because of cutbacks in spending by consumers. Further, Fed monetary policy actions that increase interest rates cause FIs to sustain a higher cost of funds and cause borrowers to increase the risk of investments. The higher cost of funds to the FI can be passed along to the borrower, but the increased risk in the investment portfolio necessary to generate returns to cover the higher funding cost to the borrower may lead to increased default risk realization. Thus, actions by the Fed often are signals of future economic activity.

18. What are the purposes of credit scoring models? How do these models assist an FI manager to better administer credit?

Credit scoring models are used to calculate the probability of default or to sort borrowers into different default risk classes. The primary benefit of credit scoring models is to improve the accuracy of predicting borrower's performance without using additional resources. This benefit results in fewer defaults and charge-offs to the FI.

The models use data on observed economic and financial borrower characteristics to assist an FI manager in (a) identifying factors of importance in explaining default risk, (b) evaluating the relative degree of importance of these factors, (c) improving the pricing of default risk, (d) screening bad loan applicants, and (e) more efficiently calculating the necessary reserves to protect against future loan losses.

19. Suppose there were two factors influencing the past default behavior of borrowers: the leverage or debt-assets ratio (D/A) and the profit margin ratio (PM). Based on past default (repayment) experience, the linear probability model is estimated as:

$$PD_i = 0.105(D/A_i) - 0.35(PM_i)$$

Prospective borrower A has a $D/A = 0.65$ and a $PM = 5\%$, and prospective borrower B has a $D/A = 0.45$ and $PM = 1\%$. Calculate the prospective borrowers' expected probabilities of default (PD_i). Which borrower is the better loan candidate? Explain your answer.

$$PD_A = 0.105(0.65) - 0.35(0.05) = 0.05075 \text{ or } 5.075\%$$

$$PD_B = 0.105(0.45) - 0.35(0.01) = 0.04375 \text{ or } 4.375\%$$

Prospective borrower B is the better loan candidate. Even though B's profit margin is lower than A's, A's higher debt-asset ratio increases the firm's probability of default to be higher than firm B's.

20. Suppose the estimated linear probability model used by an FI to predict business loan applicant default probabilities is $PD = 0.03X_1 + 0.02X_2 - 0.05X_3 + \text{error}$, where X_1 is the borrower's debt/equity ratio, X_2 is the volatility of borrower earnings, and X_3 is the borrower's profit ratio. For a particular loan applicant, $X_1 = 0.75$, $X_2 = 0.25$, and $X_3 = 0.10$.

- a. What is the projected probability of default for the borrower?

$$PD = 0.03(0.75) + 0.02(0.25) - 0.05(0.10) = 0.0225$$

- b. What is the projected probability of repayment if the debt/equity ratio is 2.5?

$$PD = 0.03(2.5) + 0.02(0.25) - 0.05(0.10) = 0.075$$

The expected probability of repayment is $1 - 0.075 = 0.925$.

c. What is a major weakness of the linear probability model?

A major weakness of this model is that the estimated probabilities can be below 0 percent or above 100 percent, an occurrence that does not make economic or statistical sense.

21. Describe how a linear discriminant analysis model works. Identify and discuss the criticisms which have been made regarding the use of this type of model to make credit risk evaluations.

Linear discriminant models divide borrowers into high or low default classes contingent on their observed characteristics. The overall measure of default risk classification (Z) depends on the values of various financial ratios and the weighted importance of these ratios based on the past or observed experience of borrowers. These weights are derived from a discriminant analysis model.

Several criticisms have been levied against these types of models. First, the models identify only two extreme categories of risk: default or no default. The real world considers several categories of default severity. Second, the relative weights of the variables may change over time. Further, the actual variables to be included in the model may change over time. Third, hard to define, but potentially important, qualitative variables are omitted from the analysis. Fourth, no centralized database on defaulted business loans for proprietary and other reasons exists. This constrains the ability of many FIs to use traditional credit scoring models (and quantitative models in general) for larger business loans.

22. Suppose that the financial ratios of a potential borrowing firm take the following values:

Working capital/total assets ratio (X_1) = 0.75

Retained earnings/total assets ratio (X_2) = 0.10

Earnings before interest and taxes/total assets ratio (X_3) = 0.05

Market value of equity/book value of total liabilities ratio (X_4) = 0.10

Sales/total assets ratio (X_5) = 0.65

Calculate the Altman's Z score for the borrower in question. How is this number a sign of the borrower's default risk?

$$Z = 1.2(0.75) + 1.4(0.10) + 3.3(0.05) + 0.6(0.10) + 1.0(0.65) = 0.90 + 0.14 + 0.165 + 0.06 + 0.65 = 1.915$$

With a Z score between 1.81 and 2.99, the firm is in the indeterminant default risk region. The ratios X_2 and X_3 are small, indicating that the firm has low earnings or even losses in recent periods. The ratio X_5 indicates that the firm may be unable to produce sales efficiently. Also, X_4 indicates that the borrower is highly leveraged. Finally, the working capital ratio (X_1) is high, indicating that the firm is investing the large majority of its funding in zero or low earning assets. The FI should not make a loan to this borrower until it improves its earnings.

23. MNO Inc., a publicly traded manufacturing firm in the United States, has provided the following financial information in its application for a loan. All numbers are in thousands of dollars.

<u>Assets</u>		<u>Liabilities and Equity</u>	
Cash	\$ 20	Accounts payable	\$ 30
Accounts receivables	90	Notes payable	40
Inventory	90	Accruals	30
		Long-term debt	150
Plant and equipment	<u>500</u>	Equity (ret. earnings = \$300)	<u>450</u>
Total assets	<u>\$700</u>	Total liabilities and equity	<u>\$700</u>

Also assume sales = \$500,000; cost of goods sold = \$360,000; and the market value of equity is equal to the book value.

- a. What is the Altman discriminant function value for MNO Inc.? Recall that:

Net working capital = Current assets - Current liabilities.

Current assets = Cash + Accounts receivable + Inventories.

Current liabilities = Accounts payable + Accruals + Notes payable.

EBIT = Revenues - Cost of goods sold.

Altman's discriminant function is given by: $Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5$

All numbers are in \$000s.

$$X_1 = (20 + 90 + 90 - 30 - 40 - 30) / 700 = 0.1429 \quad X_1 = \text{Working capital/total assets (TA)}$$

$$X_2 = 300 / 700 = 0.4286 \quad X_2 = \text{Retained earnings/TA}$$

$$X_3 = (500 - 360) / 700 = 0.20 \quad X_3 = \text{EBIT/TA}$$

$$X_4 = 450 / (30 + 40 + 30 + 150) = 1.80 \quad X_4 = \text{Market value of equity/Book value of long-term debt}$$

$$X_5 = 500 / 700 = 0.7143 \quad X_5 = \text{Sales/TA}$$

$$Z = 1.2(0.1429) + 1.4(0.4286) + 3.3(0.20) + 0.6(1.80) + 1.0(0.7143) = 3.2257$$

$$= 0.1714 + 0.6000 + 0.6600 + 1.0800 + 0.7143 = 3.2257$$

- b. Based on the Altman's Z score only, should you approve MNO Inc.'s application to your bank for a \$500,000 capital expansion loan?

Since the Z score of 3.2257 is greater than 2.99, ABC Inc.'s application for a capital expansion loan should be approved.

- c. If sales for MNO were \$250,000, the market value of equity was only half of book value, and all other values are unchanged, would your credit decision change?

ABC's EBIT would be \$300,000 - \$360,000 = -\$60,000.

$$X_1 = (20 + 90 + 90 - 30 - 40 - 30) / 700 = 0.1429$$

$$X_2 = 300 / 700 = 0.4286$$

$$X_3 = -110 / 700 = -0.1571$$

$$X_4 = 225 / (30 + 40 + 30 + 150) = 0.9000$$

$$X_5 = 250 / 700 = 0.3571$$

$$Z = 1.2(0.1429) + 1.4(0.4286) + 3.3(-0.1571) + 0.6(0.9000) + 1.0(0.3571) = 1.1500$$

Since ABC's Z-score falls to $1.1500 < 1.81$, credit should be denied.

- d. Would the discriminant function change for firms in different industries? Would the function be different for manufacturing firms in different geographic sections of the country? What are the implications for the use of these types of models by FIs?

Discriminant function models are very sensitive to the weights for the different variables. Since different industries have different operating characteristics, a reasonable answer would be yes with the condition that there is no reason that the functions could not be similar for different industries. In the retail market, the demographics of the market play a big role in the value of the weights. For example, credit card companies often evaluate different models for different areas of the country. Because of the sensitivity of the models, extreme care should be taken in the process of selecting the correct sample to validate the model for use.

24. Consider the coefficients of Altman's Z score. Can you tell by the size of the coefficients which ratio appears most important in assessing the creditworthiness of a loan applicant? Explain.

Although X_3 , or EBIT/Total assets, has the highest coefficient (3.3), it is not necessarily the most important variable. Since the value of X_3 is likely to be small, the product of 3.3 and X_3 may be quite small. For some firms, particularly those in the retail business, the asset turnover ratio, X_5 may be quite large and the product of the X_5 coefficient (1.0) and X_5 may be substantially larger than the corresponding number for X_3 . Generally, the factor that adds most to the Z-score varies from firm to firm and industry to industry.

25. If the rate on one-year Treasury strips currently is 6 percent, what is the repayment probability for each of the following two securities? Assume that if the loan is defaulted, no payments are expected. What is the market-determined risk premium for the corresponding probability of default for each security?

- a. One-year AA-rated zero coupon bond yielding 9.5 percent.

$$\begin{aligned}\text{Probability of repayment} &= p = (1 + i)/(1 + k) \\ \text{For an AA-rated bond} &= (1 + 0.06)/(1 + 0.095) = 0.9680, \text{ or } 96.80 \text{ percent} \\ \Rightarrow \text{probability of default} &= 1 - 0.9680 = 0.0320, \text{ or } 3.20\%\end{aligned}$$

The market determined risk premium is $0.095 - 0.060 = 0.035$ or 3.5 percent. This implies a probability of default of 3.2 percent on an AA-rated corporate bond requires an FI to set a risk premium of 3.5 percent.

- b. One-year BB-rated zero coupon bond yielding 13.5 percent.

$$\begin{aligned}\text{Probability of repayment} &= p = (1 + i)/(1 + k) \\ \text{For BB-rated bond} &= (1 + 0.06)/(1 + 0.135) = 93.39 \text{ percent} \\ \Rightarrow \text{probability of default} &= 1 - 0.9339 = 0.0661, \text{ or } 6.61\%\end{aligned}$$

The market determined risk premium is $0.135 - 0.060 = 0.075$ or 7.50 percent. This implies a probability of default of 6.61 percent on a BB-rated corporate bond requires an FI to set a risk premium of 7.5 percent.

26. A bank has made a loan charging a base lending rate of 10 percent. It expects a probability of default of 5 percent. If the loan is defaulted, the bank expects to recover 50 percent of its money through the sale of its collateral. What is the expected return on this loan?

$$\begin{aligned}E(r) &= p(1 + k) + (1 - p)(1 + k)(\gamma) \text{ where } \gamma \text{ is the percentage generated when the loan is defaulted. } E(r) = \\ &0.95(1 + 0.10) + 0.05(1 + 0.10)(0.50) = 1.0450 + 0.0275 = 1.0725 - 1.0 = 7.25\%\end{aligned}$$

27. Assume a one-year Treasury strip is currently yielding 5.5 percent and an AAA-rated discount bond with similar maturity is yielding 8.5 percent.
- a. If the expected recovery from collateral in the event of default is 50 percent of principal and interest, what is the probability of repayment of the AAA-rated bond? What is the probability of default?

$p(1 + k) + \gamma(1 - p)(1 + k) = 1 + i$. Solving for the probability of repayment (p):

$$p = \frac{\frac{1+i}{1+k} - \gamma}{1 - \gamma} = \frac{\frac{1.055}{1.085} - 0.5}{1 - 0.5} = 0.9447 \text{ or } 94.47 \text{ percent}$$

Therefore the probability of default is $1.0 - 0.9447 = 0.0553$ or 5.53 percent.

- b. What is the probability of repayment of the AAA-rated bond if the expected recovery from collateral in the case of default is 94.47 percent of principal and interest? What is the probability of default?

$$p = \frac{\frac{1+i}{1+k} - \gamma}{1 - \gamma} = \frac{\frac{1.055}{1.085} - 0.9447}{1 - 0.9447} = 0.5000 \text{ or } 50.00 \text{ percent}$$

Therefore the probability of default is $1.0 - 0.5000 = 0.5000$ or 50.00 percent.

- c. What is the relationship between the probability of default and the proportion of principal and interest that may be recovered in case of default on the loan?

The proportion of the loan's principal and interest that is collectible on default is a perfect substitute for the probability of repayment should such defaults occur.

28. What is meant by the phrase *marginal default probability*? How does this term differ from *cumulative default probability*? How are the two terms related?

Marginal default probability is the probability of default in a given year, whereas cumulative default probability is the probability of default across several years. For example, the cumulative default probability across two years is given below, where (p_0) is the probability of nondefault in a given year.

$$Cp = 1 - (p_1)(p_2)$$

29. Suppose an FI manager wants to find the probability of default on a two-year loan. For the one-year loan, $1 - p_1 = 0.03$ is the marginal and total or cumulative probability (Cp) of default in year 1. For the second year, suppose that $1 - p_2 = 0.05$. Calculate the cumulative probability of default over the next two years.

$1 - p_1 = 0.03$ = marginal probability of default in year 1

$1 - p_2 = 0.05$ = marginal probability of default in year 2

The probability of the borrower surviving—not defaulting at any time between now (time 0) and the end of year 2—is: $p_1 \times p_2 = (0.97)(0.95) = 0.9215$. Thus,

$$Cp = 1 - [(0.97)(0.95)] = 0.785, \text{ or } 7.85\%$$

There is an 7.85 percent probability of default over this period.

30. From the Treasury strip yield curve, the current required yields on one- and two-year Treasuries are $i_1 = 4.65$ percent and $i_2 = 5.50$ percent, respectively. Further, the current yield curve indicates that appropriate one-year discount bonds are yielding $k_1 = 8.5$ percent, and two-year bonds are yielding $k_2 = 10.25$ percent.

- a. Calculate the one-year forward rate on the Treasuries and the corporate bond.

The one-year forward rate, f_1 , on the Treasury is:

$$f_1 = (1.0550)^2 / 1.0465 = 1.06357 - 1, \text{ or } f_1 = 6.357\%$$

The one-year forward rate, c_1 , on the corporate bond is:

$$c_1 = (1.1025)^2 / 1.0850 = 1.12028 - 1, \text{ or } c_1 = 12.028\%$$

- b. Using the current and forward one-year rates, calculate the marginal probability of repayment on the corporate bond in years 1 and 2, respectively.

The probability of repayment in year 1 is:

$$p_1 = 1.0465 / 1.085 = 0.9645, \text{ or probability of default} = 1 - 0.9645 = 5.35\%$$

The marginal probability of repayment in year 2 is:

$$p_2 = (1.06357) / 1.12028 = 0.9494, \text{ or probability of default} = 1 - 0.9494 = 5.06\%$$

- c. Calculate the cumulative probability of default on the corporate bond over the next two years.

The probability of the borrower surviving—not defaulting at any time between now (time 0) and the end of year 2—is: $p_1 \times p_2 = (0.9645)(0.9494) = 0.9157$. Thus,

$$Cp = 1 - [(0.9645)(0.9494)] = 0.843, \text{ or } 8.43\%$$

31. Calculate the term structure of default probabilities over three years using the following spot rates from the Treasury strip and corporate bond (pure discount) yield curves. Be sure to calculate both the annual marginal and the cumulative default probabilities.

	Spot 1 Year	Spot 2 Year	Spot 3 Year
Treasury strips	5.0%	6.1%	7.0%
BBB-rated bonds	7.0	8.2	9.3

The notation used for implied forward rates on Treasuries is f_1 = forward rate from period 1 to period 2 and on corporate bonds is c_1 = forward rate from period 1 to period 2.

<u>Treasury strips</u>	<u>BBB-rated debt</u>
$(1.061)^2 = (1.05)(1 + f_1)$	$(1.082)^2 = (1.07)(1 + c_1)$
$f_1 = 7.21\%$	$c_1 = 9.41\%$
$(1.07)^3 = (1.061)^2(1 + f_2)$	$(1.093)^3 = (1.082)^2(1 + c_2)$
$f_2 = 8.82\%$	$c_2 = 11.53\%$

Using the implied forward rates, estimate the annual marginal probability of repayment:

$$\begin{aligned} p_1(1.07) &= 1.05 & \Rightarrow p_1 &= 98.13 \text{ percent} \\ p_2(1.0941) &= 1.0721 & \Rightarrow p_2 &= 97.99 \text{ percent} \\ p_3(1.1153) &= 1.0882 & \Rightarrow p_3 &= 97.57 \text{ percent} \end{aligned}$$

Using marginal probabilities, estimate the cumulative probability of default:

$$\begin{aligned} Cp_2 &= 1 - (p_1)(p_2) \\ &= 1 - (0.9813)(0.9799) = 3.84 \text{ percent} \\ Cp_3 &= 1 - (p_1)(p_2)(p_3) \\ &= 1 - (0.9813)(0.9799)(0.9757) = 6.18 \text{ percent} \end{aligned}$$

32. The bond equivalent yields for U.S. Treasury and A-rated corporate bonds with maturities of 93 and 175 days are given below:

	<u>93 Days</u>	<u>175 Days</u>
U.S. Treasury	8.07%	8.11%
A-rated corporate	8.42%	8.66%
Spread	0.35%	0.55%

- a. What are the implied forward rates for both an 82-day Treasury and an 82-day A-rated bond beginning in 93 days? Use daily compounding on a 365-day year basis.

The forward rate, f , for the period 93 days to 175 days, or 82 days, for the Treasury is:

$$(1 + 0.0811)^{175/365} = (1 + 0.0807)^{93/365} (1 + f)^{82/365} \Rightarrow f = 8.16 \text{ percent}$$

The forward rate, c , for the corporate bond for the 82-day period is:

$$(1 + 0.0866)^{175/365} = (1 + 0.0842)^{93/365} (1 + c)^{82/365} \Rightarrow c = 8.933\%$$

- b. What is the implied probability of default on A-rated bonds over the next 93 days? Over 175 days?

The probability of repayment of the 93-day A-rated bond is:

$$p(1 + 0.0842)^{93/365} = (1 + 0.0807)^{93/365} \Rightarrow p = 99.92 \text{ percent}$$

Therefore, the probability of default is $(1 - p) = (1 - 0.9992) = 0.0008$ or 0.08 percent.

The probability of repayment of the 175-day A-rated bond is:

$$p(1 + 0.0866)^{175/365} = (1 + 0.0811)^{175/365} \Rightarrow p = 99.76 \text{ percent}$$

Therefore, the probability of default is $(1 - p) = (1 - 0.9976) = 0.0024$ or 0.24 percent.

- c. What is the implied default probability on an 82-day A-rated bond to be issued in 93 days?

The probability of repayment of the A-rated bond for the period 93 days to 175 days, p , is:

$$p(1.08933)^{82/365} = (1 + 0.0816)^{82/365} \Rightarrow p = 0.9984, \text{ or } 99.84 \text{ percent}$$

Therefore, the probability of default is $(1 - p)$ or 0.0016 or 0.16 percent.

33. What is the mortality rate of a bond or loan? What are some of the problems with using a mortality rate approach to determine the probability of default of a given bond issue?

Mortality rates reflect the historic default risk experience of a bond or a loan. One major problem is that the approach looks backward rather than forward in determining probabilities of default. Further, the estimates are sensitive to the time period of the analysis, the number of bond issues, and the sizes of the issues.

34. The following is a schedule of historical defaults (yearly and cumulative) experienced by an FI manager on a portfolio of commercial and mortgage loans.

Loan Type	Years after Issuance				
	1 Year	2 Years	3 Years	4 Years	5 Years
Commercial:					
Annual default	0.00%	_____	0.50%	_____	0.30%
Cumulative default	_____	0.10%	_____	0.80%	_____
Mortgage:					
Annual default	0.10%	0.25%	0.60%	_____	0.80%
Cumulative default	_____	_____	_____	1.64%	_____

- a. Complete the blank spaces in the table.

Commercial: Annual default 0.00%, 0.10%, 0.50%, 0.20%, and 0.30%
 Cumulative default: 0.00%, 0.10%, 0.60%, 0.80%, and 1.10%
 Mortgage: Annual default 0.10%, 0.25%, 0.60%, 0.70%, and 0.80%
 Cumulative default 0.10%, 0.35%, 0.95%, 1.64%, and 2.43%

The annual survival rate is $p_t = 1 - \text{annual default rate}$, and the cumulative default rate for:

$n = 3$ of commercial loans is $1 - (p_1 \times p_2 \times p_3) = 1 - (1 \times 0.999 \times 0.995) = 0.005995 = 0.60\%$

$n = 4$ of commercial loans is $1 - (p_1 \times p_2 \times p_3 \times p_4) = 1 - (1 \times 0.999 \times 0.995 \times p_4) = 0.008$

$$\Rightarrow p_4 = (1 - 0.008) / (1 \times 0.999 \times 0.995) = 0.99798$$

$$\Rightarrow \text{annual default rate} = 1 - 0.99798 = 0.0020 = 0.20\%$$

$n = 5$ of commercial loans is $1 - (p_1 \times p_2 \times p_3 \times p_4 \times p_5)$

$$= 1 - (1 \times 0.999 \times 0.995 \times 0.99798 \times 0.997) = 0.0110 = 1.10\%$$

$n = 2$ of mortgage loans is $1 - (p_1 \times p_2) = 1 - (0.999 \times 0.9975) = 0.0035 = 0.35\%$

$n = 3$ of mortgage loans is $1 - (p_1 \times p_2 \times p_3) = 1 - (0.999 \times 0.9975 \times 0.9940) = 0.0095 = 0.95\%$

$n = 4$ of mortgage loans is $1 - (p_1 \times p_2 \times p_3 \times p_4) = 1 - (0.999 \times 0.9975 \times 0.9940 \times p_4) = 0.0164$

$$\Rightarrow p_4 = (1 - 0.0164) / (0.999 \times 0.9975 \times 0.9940) = 0.9930$$

$$\Rightarrow \text{annual default rate} = 1 - 0.9930 = 0.0070 = 0.70\%$$

n = 5 of commercial loans is $1 - (p_1 \times p_2 \times p_3 \times p_4 \times p_5)$

$$= 1 - (0.999 \times 0.9975 \times 0.9940 \times 0.9930 \times 0.992) = 0.0243 = 2.43\%$$

b. What are the probabilities that each type of loan will not be in default after five years?

The cumulative survival rate is $= (1 - \text{MMR}_1) \times (1 - \text{MMR}_2) \times (1 - \text{MMR}_3) \times (1 - \text{MMR}_4) \times (1 - \text{MMR}_5)$ where MMR = marginal mortality rate

$$\text{Commercial loan} = (1 - 0.00) \times (1 - 0.001) \times (1 - 0.005) \times (1 - 0.002) \times (1 - 0.003) = 0.989 \text{ or } 98.9\%.$$

$$\text{Mortgage loan} = (1 - 0.001) \times (1 - 0.0025) \times (1 - 0.006) \times (1 - 0.007) \times (1 - 0.008) = 0.9757 \text{ or } 97.57\%.$$

c. What is the measured difference between the cumulative default (mortality) rates for commercial and mortgage loans after four years?

Looking at the table, the cumulative rates of default in year 4 are 0.80% and 1.64%, respectively, for the commercial and mortgage loans. Another way of estimation is:

$$\text{Cumulative mortality rate (CMR)} = 1 - (1 - \text{MMR}_1)(1 - \text{MMR}_2)(1 - \text{MMR}_3)(1 - \text{MMR}_4)$$

$$\begin{aligned} \text{For commercial loan} &= 1 - (1 - 0.000)(1 - 0.0010)(1 - 0.0050)(1 - 0.0020) \\ &= 1 - 0.9920 = 0.0080 \text{ or } 0.80 \text{ percent.} \end{aligned}$$

$$\begin{aligned} \text{For mortgage loan} &= 1 - (1 - 0.0010)(1 - 0.0025)(1 - 0.0060)(1 - 0.0070) \\ &= 1 - 0.98359 = 0.01641 \text{ or } 1.64 \text{ percent.} \end{aligned}$$

The difference in cumulative default rates is $1.64 - 0.80 = 0.84$ percent.

35. The table below shows the dollar amounts of outstanding bonds and corresponding default amounts for every year over the past five years. Note that the default figures are in millions, while those outstanding are in billions. The outstanding figures reflect default amounts and bond redemptions.

Loan Type	Years after Issuance				
	1 Year	2 Years	3 Years	4 Years	5 Years
A-rated: Annual default (millions)	0	0	0	\$ 1	\$ 2
Outstanding (billions)	\$100	\$95	\$93	\$91	\$88
B-rated: Annual default (millions)	0	\$ 1	\$ 2	\$ 3	\$ 4
Outstanding (billions)	\$100	\$94	\$92	\$89	\$85
C-rated: Annual default (millions)	\$ 1	\$ 3	\$ 5	\$ 5	\$ 6

Outstanding (billions) \$100 \$97 \$90 \$85 \$79

a. What are the annual and cumulative default rates of the above bonds?

A-rated Bonds

	Millions	Millions	Annual	Survival =	Cumulative	% Cumulative
<u>Year</u>	<u>Default</u>	<u>Balance</u>	<u>Default</u>	<u>1 - An. Def.</u>	<u>Default Rate</u>	<u>Default Rate</u>
1	0	100,000	0.000000	1.000000	0.000000	0.0000%
2	0	95,000	0.000000	1.000000	0.000000	0.0000%
3	0	93,000	0.000000	1.000000	0.000000	0.0000%
4	1	91,000	0.000011	0.999989	0.000011	0.0011%
5	2	88,000	0.000023	0.999977	0.000034	0.0034%

Where cumulative default for nth year = 1 - product of survival rates to that year.

B-rated Bonds

	Millions	Millions	Annual	Survival =	Cumulative	% Cumulative
<u>Year</u>	<u>Default</u>	<u>Balance</u>	<u>Default</u>	<u>1 - An. Def.</u>	<u>Default Rate</u>	<u>Default Rate</u>
1	0	100,000	0.000000	1.000000	0.000000	0.0000%
2	1	94,000	0.000011	0.999989	0.000011	0.0011%
3	2	92,000	0.000022	0.999978	0.000032	0.0032%
4	3	89,000	0.000034	0.999966	0.000066	0.0066%
5	4	85,000	0.000047	0.999953	0.000113	0.0113%

C-rated Bonds

	Millions	Millions	Annual	Survival =	Cumulative	% Cumulative
<u>Year</u>	<u>Default</u>	<u>Balance</u>	<u>Default</u>	<u>1 - An. Def.</u>	<u>Default Rate</u>	<u>Default Rate</u>
1	1	100,000	0.000010	0.999990	0.000010	0.0010%
2	3	97,000	0.000031	0.999969	0.000041	0.0041%
3	5	90,000	0.000056	0.999944	0.000096	0.0096%
4	5	85,000	0.000059	0.999941	0.000155	0.0155%
5	6	79,000	0.000076	0.999924	0.000231	0.0231%

		<u>Years after Issuance</u>				
<u>Bond Type</u>		<u>1 Year</u>	<u>2 Years</u>	<u>3 Years</u>	<u>4 Years</u>	<u>5 Years</u>
A-rated:	Annual default	0%	0%	0%	0.0011%	0.0023%
	Cumulative default	0%	0%	0%	0.0011%	0.0034%
B-rated:	Annual default	0%	0.0011%	0.0022%	0.0034%	0.0047%
	Cumulative default	0%	0.0011%	0.0032%	0.0066%	0.0113%

C-rated: Annual default	0.0010%	0.0031%	0.0056%	0.0059%	0.0076%
Cumulative default	0.0010%	0.0041%	0.0096%	0.0155%	0.0231%

Note: These percentage values seem very small. More reasonable values can be obtained by increasing the default dollar values by a factor of ten, or by decreasing the outstanding balance values by a factor of 0.10. Either case will give the same answers that are shown below. While the percentage numbers seem somewhat more reasonable, the true values of the problem are (a) that default rates are higher on lower rated assets, and (b) that the cumulative default rate involves more than the sum of the annual default rates.

36. What is RAROC? How does this model use the concept of duration to measure the risk exposure of a loan? How is the expected change in the credit risk premium measured? What precisely is ΔLN in the RAROC equation?

RAROC is a measure of expected loan net income in the form of interest plus fees less cost of funding relative to some measure of asset risk. One version of the RAROC model uses the duration model to measure the change in the value of the loan for given changes or shocks in credit quality. The change in credit quality (ΔR) is measured by finding the change in the spread in yields between Treasury bonds and corporate bonds of the same risk class on the loan. The actual value chosen is the highest change in yield spread for the same maturity or duration value assets. In this case, ΔLN represents the change in loan value or the change in capital for the largest reasonable adverse changes in yield spreads. The actual equation for ΔLN looks very similar to the duration equation.

$$RAROC = \frac{\text{Net Income}}{\text{Loan risk (or } \Delta LN)} \text{ where } \Delta LN = -D_{LN} \times LN \times \frac{\Delta R}{1+R} \text{ where } R \text{ is the change in yield spread.}$$

37. An FI wants to evaluate the credit risk of a \$5 million loan with a duration of 4.3 years to a AAA borrower. There are currently 500 publicly traded bonds in that class (i.e., bonds issued by firms with a AAA rating). The current average level of rates (R) on AAA bonds is 8 percent. The largest increase in credit risk premiums on AAA loans, the 99 percent worst-case scenario, over the last year was equal to 1.2 percent (i.e., only 6 bonds out of 500 had risk premium increases exceeding the 99 percent worst case). The projected (one-year) spread on the loan is 0.3 percent and the FI charges 0.25 percent of the face value of the loan in fees. Calculate the capital at risk and the RAROC on this loan.

The estimate of loan (or capital) risk is:

$$\Delta LN = -D_{LN} \times LN \times (\Delta R / (1 + R)) = -4.3 \times \$5m \times (0.012 / (1 + 0.08)) = \$238,889$$

While the market value of the loan amount is \$5 million, the risk amount, or change in the loan's market value due to a decline in its credit quality, is \$238,889. Thus, the denominator of the RAROC equation is this possible loss, or \$238,889. To determine whether the loan is worth making, the estimated loan risk is compared with the loan's income (spread over the FI's cost of funds plus fees on the loan).

$$\begin{aligned}\text{Spread} &= 0.003 \times \$5 \text{ million} = \$15,000 \\ \text{Fees} &= 0.0025 \times \$5 \text{ million} = \underline{\$12,500} \\ &\qquad\qquad\qquad \$27,500\end{aligned}$$

The loan's RAROC is:

$$\text{RAROC} = \$27,500 / \$238,889 = 11.51\%$$

38. A bank is planning to make a loan of \$5,000,000 to a firm in the steel industry. It expects to charge a servicing fee of 50 basis points. The loan has a maturity of 8 years with a duration of 7.5 years. The cost of funds (the RAROC benchmark) for the bank is 10 percent. The bank has estimated the maximum change in the risk premium on the steel manufacturing sector to be approximately 4.2 percent, based on two years of historical data. The current market interest rate for loans in this sector is 12 percent.

- a. Using the RAROC model, determine whether the bank should make the loan?

RAROC = Fees and interest earned on loan / Loan or capital risk

Loan risk, or $\Delta LN = -D_{LN} \times LN \times (\Delta R / (1 + R))$	$= -7.5 \times \$5\text{m} \times (0.042 / 1.12)$	$= -\$1,406,250$
Expected interest	$= 0.12 \times \$5,000,000$	$= \$600,000$
Servicing fees	$= 0.0050 \times \$5,000,000$	$= \$25,000$
Less cost of funds	$= 0.10 \times \$5,000,000$	$= \underline{-\$500,000}$
Net interest and fee income		$= \underline{\$125,000}$

RAROC = $\$125,000 / \$1,406,250 = 8.89$ percent. Since RAROC is lower than the cost of funds to the bank, the bank should not make the loan.

- b. What should be the duration in order for this loan to be approved?

For RAROC to be 10 percent, loan risk should be:

$$\$125,000 / \Delta LN = 0.10 \quad \Rightarrow \quad \Delta LN = \$125,000 / 0.10 = \$1,250,000$$

$$\Rightarrow -D_{LN} \times LN \times (\Delta R / (1 + R)) = 1,250,000$$

$$D_{LN} = 1,250,000 / (5,000,000 \times (0.042 / 1.12)) = 6.67 \text{ years.}$$

Thus, this loan can be made if the duration is reduced to 6.67 years from 7.5 years.

- c. Assuming that duration cannot be changed, how much additional interest and fee income will be necessary to make the loan acceptable?

$$\begin{aligned} \text{Necessary RAROC} &= \text{Income} / \text{Risk} \Rightarrow \text{Income} = \text{RAROC} \times \text{Risk} \\ &= \$1,406,250 \times 0.10 = \$140,625 \end{aligned}$$

Therefore, additional income = \$140,625 - \$125,000 = \$15,625, or
 $\$15,625 / \$5,000,000 = 0.003125 = 0.3125\%$.

Thus, this loan can be made if fees are increased from 50 basis points to 81.25 basis points.

- d. Given the proposed income stream and the negotiated duration, what adjustment in the loan rate would be necessary to make the loan acceptable?

Need an additional \$15,625 $\Rightarrow \$15,625 / \$5,000,000 = 0.003125$ or 0.3125%

Expected interest	= 0.123125 x \$5,000,000	=	\$615,625
Servicing fees	= 0.0050 x \$5,000,000	=	\$25,000
Less cost of funds	= 0.10 x \$5,000,000	=	<u>-\$500,000</u>
Net interest and fee income		=	<u>\$140,625</u>

RAROC = \$140,625 / 1,406,250 = 10.00 percent = cost of funds to the bank. Thus, increasing the loan rate from 12% to 12.3125% will make the loan acceptable

39. Calculate the value of and interest rate on a loan using the option model and the following information.

Face value of loan (B) = \$500,000

Length of time remaining to loan maturity (τ) = 4 years

Risk-free rate (i) = 4%

Borrower's leverage ratio (d) = 51%

Standard deviation of the rate of change in the value of the underlying assets = 15%

Substituting these values into the equations for h_1 and h_2 and solving for the areas under the standardized normal distribution, we find that:

$$h_1 = -[0.5 \times (0.15)^2 \times 4 - \ln(0.51)] / (0.15)(4)^{1/2} = -2.39448$$

$$h_2 = -[0.5 \times (0.15)^2 \times 4 + \ln(0.51)] / (0.15)(4)^{1/2} = 2.09448$$

<u>h</u>	<u>N(h)</u>	<u>h</u>	<u>N(h)</u>
-2.40	0.0082	2.00	0.9773
-2.35	0.0094	2.05	0.9798
-2.30	0.0107	2.10	0.9821
-2.25	0.0122	2.15	0.9842

$$\begin{aligned} \text{Current market value of loan} &= L(\tau) = Be^{-i\tau} [N(h_1)1/d + N(h_2)] \\ &= \$500,000 e^{-0.04(4)} [N(-2.39448) \times 1.6667 + N(2.09448)] \\ &= \$426,071.89[0.008332 \times 1.6667 + 0.981846] = \$424,254 \end{aligned}$$

$$\begin{aligned} \text{The risk premium, } \phi &= k(\tau) - i = (-1/\tau) \ln[N(h_2) + (1/d)N(h_1)] \\ &= (-1/4) \ln[0.981846 + 0.008332 \times 1.6667] = 0.001069 = 0.1069\% \end{aligned}$$

Thus, the risky loan rate $k(\tau)$ should be set at 4.1069 percent when the risk-free rate (i) is 4 percent.

40. A firm is issuing a two-year loan in the amount of \$200,000. The current market value of the borrower's assets is \$300,000. The risk-free rate is 4 percent and the standard deviation of the rate of change in the underlying assets of the borrower is 20 percent. Using an options framework, determine the following:

- The current market value of the loan.
- The risk premium to be charged on the loan.

The following need to be estimated first: d , h_1 and h_2 .

$$d = Be^{-i\tau}/A = \$200,000e^{-0.04(2)}/\$300,000 = 0.6154 \text{ or } 61.54 \text{ percent.}$$

$$h_1 = -[0.5 \times (0.20)^2 \times 2 - \ln(0.6154)] / (0.20)(2)^{1/2} = -1.8578$$

$$h_2 = -[0.5 \times (0.20)^2 \times 2 + \ln(0.6154)] / (0.20)(2)^{1/2} = 1.5750$$

$$\begin{aligned} \text{Current market value of loan} &= L(\tau) = Be^{-i\tau} [N(h_1)1/d + N(h_2)] \\ &= \$184,623.27[N(-1.8578) \times 1.62493 + N(1.5750)] \\ &= \$184,623.27[0.031654 \times 1.62493 + 0.94265] = \$183,531 \end{aligned}$$

$$\begin{aligned} \text{The risk premium, } \phi &= k(\tau) - i = (-1/\tau) \ln[N(h_2) + (1/d)N(h_1)] \\ &= (-1/2) \ln[0.94265 + 1.62493 \times 0.031654] = 0.002966 = 0.2966\% \end{aligned}$$

41. A firm has assets of \$200,000 and total debts of \$175,000. With an option pricing model, the implied volatility of the value of the firm's assets is estimated at \$10,730. Under the Moody's Analytics method, what is the expected default frequency (assuming a normal distribution for assets)?

The firm will be in technical bankruptcy if the value of the assets falls below \$175,000. If $\sigma = \$10,730$, then it takes $25,000/10,730 = 2.33$ standard deviations for the assets to fall below this value. Under the assumption that the market value of the assets are normally distributed, then 2.33 represents a 1 percent probability that the firm will become bankrupt.

42. Carman County Bank (CCB) has a \$5 million face value outstanding adjustable-rate loan to a company that has a leverage ratio of 80 percent. The current risk-free rate is 6 percent and the time to maturity on the loan is exactly $\frac{1}{2}$ year. The asset risk of the borrower, as measured by the standard deviation of the rate of change in the value of the underlying assets, is 12 percent. The normal density function values are given below.

<u>h</u>	<u>N(h)</u>	<u>h</u>	<u>N(h)</u>
-2.55	0.0054	2.50	0.9938
-2.60	0.0047	2.55	0.9946
-2.65	0.0040	2.60	0.9953
-2.70	0.0035	2.65	0.9960
-2.75	0.0030	2.70	0.9965

- a. Use the Merton option valuation model to determine the market value of the loan.

The following need to be estimated first: d , h_1 and h_2 .

$$D = 0.80$$

$$h_1 = -[0.5 \times (0.12)^2 \times 0.5 - \ln(0.8)] / (0.12) \sqrt{0.5} = -0.226744 / 0.084853 = -2.6722$$

$$h_2 = -[0.5 \times (0.12)^2 \times 0.5 + \ln(0.8)] / (0.12) \sqrt{0.5} = 0.219544 / 0.084853 = 2.5873$$

$$\begin{aligned} \text{Current market value of loan} &= L(\tau) = Be^{-i\tau} [N(h_1)1/d + N(h_2)] \\ &= \$4,852,227.67 [N(-2.6722) \times 1.25 + N(2.5873)] \\ &= \$4,852,227.67 [0.003778 \times 1.25 + 0.995123] \\ &= \$4,851,478 \end{aligned}$$

- b. What should be the interest rate for the last six months of the loan?

The risk premium $k(\tau) - i = (-1/\tau) \ln[N(h_2) + (1/d)N(h_1)]$

$$=(-1/0.5)\ln[0.995123 + 1.25 \times 0.003778] = 0.0003$$

The loan rate = risk-free rate plus risk premium = $0.06 + 0.0003 = 0.0603$ or 6.03%.

The questions and problems that follow refer to Appendix 10A.

43. Suppose you are a loan officer at Carbondale Local Bank. Joan Doe listed the following information on her mortgage application.

Characteristic	Value
Annual gross income	\$45,000
TDS	10%
Relations with FI	Checking account
Major credit cards	5
Age	27
Residence	Own/mortgage
Length of residence	2½ years
Job stability	5½ years
Credit history	Missed 2 payments 1 year ago

Use the information below to determine whether or not Joan Doe should be approved for a mortgage from your bank.

Characteristic		Characteristic Values and Weights				
Annual gross income	<\$10,000	\$10,000-\$25,000	\$25,000-\$50,000	\$50,000-\$100,000	>\$100,000	
Score	0	10	20	35	60	
TDS	>50%	35%-50%	15%-35%	5%-15%	<5%	
Score	-10	0	20	40	60	
Relations with FI	None	Checking account	Savings account	Both		
Score	0	10	10	20		
Major credit cards	None	Between 1 and 4	5 or more			
Score	0	20	10			
Age	<25	25-60	>60			
Score	5	25	35			
Residence	Rent	Own with mortgage	Own outright			
Score	5	20	50			
Length of residence	<1 year	1-5 years	>5 years			
Score	0	25	40			
Job stability	<1 year	1-5 years	>5 years			
Score	0	25	50			
Credit history	No record	Missed a payment in last 5 years	Met all payments			
Score	0	-15	40			

The loan is automatically rejected if the applicant's total score is less than or equal to 120; the loan is automatically approved if the total score is greater than or equal to 190. A score between 120 and 190 (noninclusive) is reviewed by a loan committee for a final decision.

Jane Doe=s credit score is calculated as follows:

Characteristic	Value	Score
Annual gross income	\$45,000	20
TDS	10%	40
Relations with FI	Checking account	10
Major credit cards	5	10
Age	27	25
Residence	Own/Mortgage	20
Length of residence	2½years	25
Job stability	5½ years	50
Credit history	Missed 2 payments 1 year ago	<u>-15</u>
	Score	185

The loan request will go to the credit committee for review and decision.

44. What are some of the special risks and considerations when lending to small businesses rather than large businesses?

Besides the obvious difference in the sizes of the borrowers, for a large business there is also a more well defined corporate structure and a clearer delineation of the corporate assets from the personal assets of the owners. The large business borrower is also more likely to have a track record to use as a basis for future performance. Small-business loans are also more complicated because the FI is frequently asked to assume the credit risk of an individual whose business cash flows require considerable analysis, often with incomplete accounting information available to the credit officer. The payoff for this analysis is also small, by definition, because loan principal amounts are usually small. A \$50,000 loan with a 3 percent interest spread over the cost of funds provides only \$1,500 of gross revenues before loan loss provisions, monitoring costs, and allocation of overheads. This low profitability has caused many FIs to build small business scoring models similar to, but more sophisticated than, those used for mortgages and consumer credit. These models often combine computer-based financial analysis of borrower financial statements with behavioral analysis of the owner of the small business.

45. How does ratio analysis help to answer questions about the production, management, and marketing capabilities of a prospective borrower?

Although financial ratios are normally thought to represent financial health, they also demonstrate other aspects of the company's health. Generally, a set of healthy ratios should reflect a well-managed company. A company with strong profits, an ability to pay off its debt, and an above average turnover of assets should be in a good position to meet future obligations. More specifically, the profitability and asset management ratios reflect the production efficiency of management. Receivables turnover and days sales outstanding are indicators of the company's

credit policy and collection policies and are also indicative of the marketing efficiency of the company.

46. Consider the following company balance sheet and income statement.

Balance Sheet:

Assets		Liabilities and Equity	
Cash	\$4,000	Accounts payable	\$30,000
Accounts receivable	52,000	Notes payable	<u>12,000</u>
Inventory	<u>40,000</u>	Total current liabilities	42,000
Total current assets	96,000	Long-term debt	36,000
Fixed assets	<u>44,000</u>	Equity	<u>62,000</u>
Total assets	\$140,000	Total liabilities and equity	\$140,000

Income Statement

Sales (all on credit)	\$200,000
Cost of goods sold	130,000
Gross margin	70,000
Selling and administrative expenses	20,000
Depreciation	<u>8,000</u>
EBIT	42,000
Interest expense	<u>4,800</u>
Earning before tax	37,200
Taxes	<u>11,160</u>
Net income	\$26,040

For this company, calculate the following:

a. Current ratio.

$$96,000/42,000 = 2.2857X$$

b. Number of days' sales in receivables.

$$52,000 \times 365/200,000 = 94.90 \text{ days}$$

c. Sales to total assets.

$$200,000/140,000 = 1.4286X$$

d. Number of days in inventory.

$$40,000 \times 365/130,000 = 112.31 \text{ days}$$

e. Debt to assets ratio.

$$(42,000 + 36,000)/140,000 = .5571 = 55.71\%$$

f. Cash flow to debt ratio.

$$(42,000 + 8,000)/(42,000 + 36,000) = .6410 = 64.10\%$$

g. Return on assets.

$$26,040/140,000 = 0.1860 = 18.60\%$$

h. Return on equity.

$$26,040/62,000 = 0.4200 = 42.00\%$$

47. Industrial Corporation has an income to sales (profit margin) ratio of 0.03, a sales to assets (asset utilization) ratio of 1.5, and a debt to asset ratio of 0.66. What is Industrial's return on equity?

$$\text{ROE} = \text{NI/Equity} = \text{NI/Sales} \times \text{Sales/Total assets} \times \text{Total assets/Equity} = \text{PM} \times \text{AU} \times \text{EM} \\ = 0.03 \times 1.5 \times (1/(1 - 0.66)) = 0.1324 = 13.24\%$$

Answer to Integrated Mini Case: Loan Analysis

Loan:

1. $\text{PD} = -0.08(2.15) + 0.15(0.45) + 1.25(0.13) - 0.45(0.12) = 0.004 = 0.4\% < 0.5\% \Rightarrow \text{accept the loan}$
2. $\text{Z} = 1.2((40\text{m}+120\text{m}+210\text{m}-55\text{m}-60\text{m}-70\text{m})/1470\text{m}) + 1.4(200\text{m}/1470\text{m}) + 3.3((1250\text{m}-930\text{m})/1470\text{m}) + 0.6(2.2 \times 735\text{m}/(55\text{m}+60\text{m}+70\text{m}+550\text{m})) + 1.0(1250\text{m}/1470\text{m}) = 0.1510 + 0.1905 + 0.7184 + 1.32 + 0.8503 = 3.2302 > 2.99 \Rightarrow \text{accept the loan}$
3. Cumulative default probability = 0.595% < 1.25% => accept the loan
4. $\Delta \text{LN} = -4.5 \times \$2\text{m} \times (0.055/1.10) = -\$450,000$

Expected interest	= 0.10 x \$2,000,000	= \$200,000
Servicing fees	= 0.0075 x \$2,000,000	= \$15,000
Less cost of funds	= 0.08 x \$2,000,000	= <u>-\$160,000</u>
Net interest and fee income		= <u>\$ 55,000</u>

$\text{RAROC} = \$55,000/\$450,000 = 12.22$ percent. Since RAROC is greater than the cost of funds to the bank, 8%, the bank should make the loan.

The bank should accept all four of the loans.