Solution for End-of-Chapter Questions and Problems: Chapter Eleven

1. How do loan portfolio risks differ from individual loan risks?

Loan portfolio risks refer to the risks of a portfolio of loans as opposed to the risks of a single loan. Inherent in the distinction is the elimination of some of the borrower-specific risks of individual loans because of benefits from diversification.

2. What is migration analysis? How do FIs use it to measure credit risk concentration? What are its shortcomings?

Migration analysis uses information from the market to determine the credit risk of an individual loan or sectoral loans. With this method, FI managers track credit ratings, such as S&P and Moody's ratings, of firms in particular sectors or ratings classes for unusual declines to determine whether firms in a particular sector are experiencing repayment problems. This information can be used to either curtail lending in that sector or to reduce maturity and/or increase interest rates. A problem with migration analysis is that the information may be too late, because ratings agencies usually downgrade issues only after the firm or industry has experienced a downturn.

3. What does loan concentration risk mean?

Loan concentration risk refers to the extra risk borne by having too many loans concentrated with one firm, industry, or economic sector. To the extent that a portfolio of loans represents loans made to a diverse cross section of the economy, concentration risk is minimized.

4. A manager decides not to lend to any firm in sectors that generate losses in excess of 5 percent of capital.

   a. If the average historical losses in the automobile sector total 8 percent, what is the maximum loan a manager can lend to firms in this sector as a percentage of total capital?

Concentration limit = (Maximum loss as a percent of capital) x (1/Loss rate) = 0.05 x 1/0.08 = 62.5 percent of capital is the maximum amount that can be lent to firms in the automobile sector.

   b. If the average historical losses in the mining sector total 15 percent, what is the maximum loan a manager can lend to firms in this sector as a percentage of total capital?

Concentration limit = (Maximum loss as a percent of capital) x (1/Loss rate) = 0.05 x 1/0.15 = 33.3 percent of capital is the maximum amount that can be lent to firms in the mining sector.

5. An FI has set a maximum loss of 2 percent of total capital as a basis for setting concentration limits on loans to individual firms. If it has set a concentration limit of 25 percent to a firm, what is the expected loss rate for that firm?

Concentration limit = (Maximum loss as a percent of capital) x (1/Loss rate)
25 percent = 2 percent x 1/Loss rate => Loss rate = 0.02/0.25 = 8 percent

6. Explain how modern portfolio theory can be applied to lower the credit risk of an FI's portfolio.

The fundamental lesson of modern portfolio theory is that, to the extent that an FI manager holds widely traded loans and bonds as assets, or can calculate loan or bond returns, portfolio diversification models can be used to measure and control the FI's aggregate credit risk exposure. By taking advantage of its size, an FI can diversify considerable amounts of credit risk as long as the returns on different assets are imperfectly correlated with respect to their default risk adjusted returns. By fully exploiting diversification potential with bonds or loans whose returns are negatively correlated or that have a low positive correlations with those in the existing portfolio, the FI manager can produce a set of efficient frontier portfolios, defined as those portfolios that provide the maximum returns for a given level of risk or the lowest risk for a given level of returns. By choosing portfolios on the efficient frontier, an FI manager may be able to reduce credit risk to the fullest extent. As shown in Figure 11-1, a manager's selection of a particular portfolio on the efficient frontier is determined by the risk-return trade-off.

7. Suppose that an FI holds two loans with the following characteristics:

Loan i	X_i	R_i	σ_i	σ_i^2	
1	0.55	8%	8.55%	73.1025%	$\rho_{12} = 0.24$
2	0.45	10	9.15	83.7225	$\sigma_{12} = 18.7758$

Calculate the return and risk of the portfolio.

The return on the loan portfolio is:

$$R_p = 0.55\ (8\%) + 0.45\ (10\%) = 8.90\%$$

The risk of the portfolio is:

$\sigma_p^2 = (0.55)^2 (73.1025\%) + (0.45)^2 (83722.5\%) + 2 (0.55) (0.45) (18.7758\%) = 48.36133\%$

or $\sigma_p^2 = (0.55)^2 (73.1025\%) + (0.45)^2 (83722.5\%) + 2 (0.55) (0.45) (0.24)(8.55\%)(9.15\%) = 48.36133\%$

and $\sigma_p = \sqrt{48.36133\%} = 6.95\%$

Notice that the risk (or standard deviation of returns) of the portfolio, σ_p (6.95 percent), is less than the risk of either individual asset (8.55 percent and 9.15 percent, respectively). The low correlation of the returns of the two loans (0.24) results in an overall reduction of risk when they are put together in an FI's portfolio.

8. The Bank of Tinytown has two $20,000 loans with the following characteristics: Loan A has an expected return of 10 percent and a standard deviation of returns of 10 percent. The expected return and standard deviation of returns for loan B are 12 percent and 20 percent, respectively.

   a. If the correlation coefficient between the returns on loans A and B is 0.15, what are the expected return and standard deviation of this portfolio?

$X_A = X_B = \$20{,}000/\$40{,}000 = 0.5$

Expected return $= 0.5(10\%) + 0.5(12\%) = 11$ percent

Standard deviation $= [0.5^2(0.10)^2 + 0.5^2(0.20)^2 + 2(0.5)(0.5)(0.15)(0.10)(0.20)]^{1/2} = 11.83\%$

   b. What is the standard deviation of the portfolio if the correlation is -0.15?

Standard deviation $= [0.5^2(0.10)^2 + 0.5^2(0.20)^2 + 2(0.5)(0.5)(-0.15)(0.10)(0.20)]^{1/2} = 10.49\%$

   c. What role does the covariance, or correlation, play in the risk reduction attributes of modern portfolio theory?

The risk of the portfolio as measured by the standard deviation is reduced when the covariance is reduced. If the correlation is less than +1.0, the standard deviation of the portfolio will always be less than the weighted average of the standard deviations of the individual assets.

9. Why is it difficult for small banks and thrifts to measure credit risk using modern portfolio theory?

The basic premise behind modern portfolio theory is the ability to diversify and reduce risk by eliminating diversifiable risk. Small banks and thrifts may not have the ability to diversify their asset base, especially if the local markets which they serve have a limited number of industries. The ability to diversify is even more acute if these loans cannot be traded easily.

10. What is the minimum risk portfolio? Why is this portfolio usually not the portfolio chosen by FIs to optimize the return-risk tradeoff?

The minimum risk portfolio is the combination of assets that reduces portfolio risk as measured by the standard deviation of returns to the lowest possible level. This portfolio usually is not the optimal portfolio choice because the returns on this portfolio are low relative to other alternative portfolio selections. By accepting some additional risk, portfolio managers are able to realize a higher level of return relative to the risk of the portfolio.

11. The obvious benefit to holding a diversified portfolio of loans is to spread risk exposures so that a single event does not result in a great loss to an FI. Are there any benefits to not being diversified?

One benefit to not being diversified is that an FI that lends to a certain industrial or geographic sector is likely to gain expertise about that sector. Being diversified requires that the FI becomes familiar with many more areas of business. This may not always be possible, particularly for small FIs.

12. A bank vice president is attempting to rank, in terms of the risk-reward trade-off, the loan portfolios of three loan officers. Information on the portfolios is noted below. How would you rank the three portfolios?

Portfolio	Expected Return	Standard Deviation
A	10%	8%
B	12%	9%
C	11%	10%

Portfolio B dominates portfolio C because B has a higher expected return and a lower standard deviation. Thus, C is clearly inferior. A comparison of portfolios A and B represents a risk-return trade-off in that B has a higher expected return, but B also has higher risk. A crude comparison may use the coefficient of variation or the Sharpe measure, but a judgment regarding which portfolio is "better" would be based on the risk preference of the vice president.

13. Suppose that an FI holds two loans with the following characteristics.

Loan	X_i	Annual Spread between Loan Rate and FI's Cost of Funds	Annual Fees	Loss to FI Given Default	Expected Default Frequency	
1	0.45	5.5%	2.25%	30%	3.5%	$\rho_{12} = -0.15$
2	0.55	3.5	1.75	20	1.0	

Calculate of the return and risk on the two-asset portfolio using Moody's Analytics Portfolio Manager.

The return and risk on loan 1 are:

$$R_1 = (0.055 + 0.0225) - [0.035 \times 0.30] = 0.0670 \text{ or } 6.70\%$$
$$\sigma_1 = [0.035 \times (1 - 0.035)]^{1/2} \times 0.30 = 0.05513 \text{ or } 5.513\%$$

The return and risk on loan 2 are:

$$R_2 = (0.035 + 0.0175) - [0.01 \times 0.20] = 0.0505 \text{ or } 5.05\%$$
$$\sigma_2 = [0.01 \times (1 - 0.01)]^{1/2} \times 0.20 = 0.01990 \text{ or } 1.990\%$$

The return and risk of the portfolio is then:

$$R_p = 0.45\ (6.70\%) + 0.55\ (5.05\%) = 5.7925\%$$

$$\sigma_p^2 = (0.45)^2\ (5.513\%)^2 + (0.55)^2\ (1.990\%)^2 + 2\ (0.45)\ (0.55)(-0.15)(5.513\%)(1.990\%) = 6.53876\%$$

and, $\sigma_p = (6.53876\%)^{1/2} = 2.56\%$

14. CountrySide Bank uses Moody's Analytics Portfolio Manager to evaluate the risk-return characteristics of the loans in its portfolio. A specific $10 million loan earns 2 percent per year in fees and the loan is priced at a 4 percent spread over the cost of funds for the bank. Because of collateral considerations, the loss to the bank if the borrower defaults will be 20 percent of the loan's face value. The expected probability of default is 3 percent. What is the anticipated return on this loan? What is the risk of the loan?

Expected return $= AIS_i - E(L_i) = (0.02 + 0.04) - (0.03 \times 0.20) = 0.054$ or 5.4 percent
Risk of the loan $= \sigma_{Di} \times LGD_i = [0.03(0.97)]^{½} \times 0.20 = 0.0341$ or 3.41 percent

15. Suppose that an FI holds two loans with the following characteristics.

	Annual Spread between Loan Rate and FI's	Annual	Loss to FI Given	Expected Default

Loan	X_i	Cost of Funds	Fees	Default	Frequency	
1	?	4.0%	1.50%	?%	4.0%	$\rho_{12} = -0.10$
2	?	2.5	1.15	?	1.5	

The return on loan 1 is $R_1 = 6.25\%$, the risk on loan 2 is $\sigma_2 = 1.8233\%$, and the return of the portfolio is $R_p = 4.555\%$. Calculate of the loss given default on loans 1 and 2, the proportions of loans 1 and 2 in the portfolio, and the risk of the portfolio, σ_p, using Moody's Analytics Portfolio Manager.

$R_1 = 0.0625 = (0.04 + 0.015) - [0.040 \text{ x } LGD_1] \Rightarrow LGD_1 = (0.0625 - (0.04 + 0.015))/(-0.04) = 0.1875$

$\Rightarrow \sigma_1 = [0.04(1 - 0.04)]^{1/2} \text{ x } 0.1875 = 0.03674 \text{ or } 3.674\%$

$\sigma_2 = 0.018233 = [(0.015(1 - 0.015)]^{1/2} \text{ x } LGD_2 \Rightarrow LGD_2 = 0.018233/[(0.015(1 - 0.015)]^{1/2} = 0.15$

$\Rightarrow R_2 = (0.025 + 0.0115) - [0.015 \text{ x } 0.15] = 0.03425 \text{ or } 3.425\%$

$\Rightarrow R_p = X_1 (0.0625) + (1 - X_1) (0.03425) = 0.04555$

$\Rightarrow X_1 = (0.04555 - 0.03425)/(0.0625 - 0.03425) = 0.40 \text{ and } X_2 = 1 - 0.40 = 0.60$

$\sigma_p^2 = (0.40)^2 (0.03674)^2 + (0.60)^2 (0.018233)^2 + 2 (0.40)(0.60)(-0.10)(0.03674)(0.018233) = 0.000303523$

Thus, $\sigma_p = (0.000303523)^{1/2} = 0.0174 = 1.74\%$.

16. What databases are available that contain loan information at national and regional levels? How can they be used to analyze credit concentration risk?

Two publicly available databases are (a) the commercial bank Call Reports of the Federal Reserve Board which contain various information supplied by banks quarterly and (b) the Shared National Credit database, which provides information on loan volumes of FIs separated by two-digit SIC (Standard Industrial Classification) codes. Such data can be used as a benchmark to determine whether an FI's asset allocation is significantly different from the national or regional average.

17. Information concerning the allocation of loan portfolios to different market sectors is given below.

Allocation of Loan Portfolios in Different Sectors (%)

Sectors	**National**	**Bank A**	**Bank B**
Commercial	30%	50%	10%
Consumer	40	30	40
Real Estate	30	20	50

Bank A and Bank B would like to estimate how much their portfolios deviate from the national average.

a. Which bank is further away from the national average?

Using Xs to represent portfolio holdings:

	Bank A	Bank B
$(X_{1j} - X_1)^2$	$(0.50 - 0.30)^2 = 0.0400$	$(0.10 - 0.30)^2 = 0.0400$
$(X_{2j} - X_2)^2$	$(0.30 - 0.40)^2 = 0.0100$	$(0.40 - 0.40)^2 = 0.0000$
$(X_{3j} - X_3)^2$	$(0.20 - 0.30)^2 = 0.0100$	$(0.50 - 0.30)^2 = 0.0400$
$\sum_{i-1}^{n} (X_{ij} - X_i)^2$	$\sum_{i=1}^{n=3} = 0.0600$	$\sum_{i=1}^{n=3} = 0.0800$
$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_{ij} - X_i)^2}{n}}$	$\sigma_A = 14.14$ percent	$\sigma_B = 16.33$ percent

Bank B deviates from the national average more than Bank A.

b. Is a large standard deviation necessarily bad for an FI using this model?

No, a higher standard deviation is not necessarily bad for an FI because the FI could have comparative advantages that are not required or available to a national well-diversified bank. For example, an FI could generate high returns by serving specialized markets or product niches that are not well diversified. Further, an FI could specialize in only one product, such as mortgages, but be well-diversified within this product line by investing in several different types of mortgages that are distributed both nationally and internationally. This would still enable it to obtain portfolio diversification benefits that are similar to the national average.

18. Assume that, on average, national banks engaged primarily in mortgage lending have their assets diversified in the following proportions: 60 percent residential, 15 percent commercial, 5 percent international, and 20 percent mortgage-backed securities. A local bank has the following distribution of mortgage loans: 50 percent residential, 30 percent commercial, and 20 percent international. How does the local bank differ from national banks?

Using Xs to represent portfolio holdings:

$(X_{1j} - X_1)^2$	$(0.50 - 0.60)^2 = 0.0100$
$(X_{2j} - X_2)^2$	$(0.30 - 0.15)^2 = 0.0225$

$(X_{3j} - X_3)^2$ $(0.20 - 0.05)^2 = 0.0225$

$(X_{4j} - X_4)^2$ $(0.00 - 0.20)^2 = 0.0400$

$\sum_{i=1}^{n} (X_{ij} - X_i)^2$ $\sum_{i=1}^{n=4} = 0.0950$

$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_{ij} - X_i)^2}{n}}$ $\sigma = 15.41$ percent

The bank's standard deviation in its loan portfolio allocation is 15.41 percent. This suggests that the bank is different from the national average. Whether it is significantly different cannot be stated without comparing it to other banks.

19. Over the past 10 years, a bank has experienced the following loan losses on its C&I loans, consumer loans, and total loan portfolio.

Year	C&I Loans	Consumer Loans	Total Loans
2018	0.0080	0.0165	0.0075
2017	0.0088	0.0183	0.0085
2016	0.0100	0.0210	0.0100
2015	0.0120	0.0255	0.0125
2014	0.0104	0.0219	0.0105
2013	0.0084	0.0174	0.0080
2012	0.0072	0.0147	0.0065
2011	0.0080	0.0165	0.0075
2010	0.0096	0.0201	0.0095
2009	0.0144	0.0309	0.0155

Using regression analysis on these historical loan losses, the bank has estimated the following:

$X_C = 0.002 + 0.8X_L$ and $X_h = 0.003 + 1.8X_L$

where X_C = loss rate in the commercial sector, X_h = loss rate in the consumer (household) sector, and X_L = loss rate for its total loan portfolio.

a. If the bank's total loan loss rates increase to 10 percent, what are the expected loss rate increases in the commercial and consumer sectors?

Commercial loan loss rates will increase to $0.002 + 0.8(0.10) = 8.20$ percent.
Consumer loan loss rates will increase to $0.003 + 1.8(0.10) = 18.30$ percent.

b. In which sector should the bank limit its loans and why?

The bank should limit its loans to the consumer sector because the loss rates are systematically higher than the loss rates for the total loan portfolio. Loss rates are lower for the commercial sector. For a 10 percent loss rate in the total loan portfolio, the consumer loss rate is expected to be by 18.30 percent, as opposed to only 8.2 percent for the commercial sector.

20. What reasons did the Federal Reserve Board offer for recommending the use of subjective evaluations of credit concentration risk instead of quantitative models? How did this change in 2006?

The Federal Reserve Board recommended a subjective evaluation of credit concentration risk instead of quantitative models because (a) current methods to identify credit concentrations were not reliable and (b) there was insufficient data to develop reliable quantitative models. This changed in June 2006 as the Bank for International Settlements released guidance on sound credit risk assessment and valuation for loans. The guidance addresses how common data and processes related to loans may be used for assessing credit risk, accounting for loan impairment and determining regulatory capital requirements and is structured around ten principles that fall within two broad categories: i) supervisory expectations concerning sound credit risk assessment and valuation for loans and ii) supervisory evaluation of credit risk assessment for loans, controls and capital adequacy.

21. What rules on credit concentrations has the National Association of Insurance Commissioners enacted? How are they related to modern portfolio theory?

The NAIC set a maximum limit of 3% that life insurers can hold in securities belonging to a single issuer. Similarly, the limit is 5% for property-casualty (PC) insurers. This forces life insurers to hold a minimum of 33 different securities and PC insurers to hold a minimum of 20 different securities. Modern portfolio theory shows that by holding well-diversified portfolios, investors can eliminate undiversifiable risk and be subject only to market risk. This enables investors to hold portfolios that provide either high returns for a given level of risk or low risks for a given level of returns.

22. An FI is limited to holding no more than 8 percent of its assets in securities of a single issuer. What is the minimum number of securities it should hold to meet this requirement? What if the requirements are 2 percent, 4 percent, and 7 percent?

If an FI is limited to holding a maximum of 8 percent of securities of a single issuer, it will be forced to hold 100/8 = 12.5, or 13 different securities.

For 2%, it will be 100/2, or 50 different securities.
For 4%, it will be 100/4, or 25 different securities.
For 7%, it will be 100/7, or 15 different securities.

The questions and problems that follow refer to Appendixes 11A and 11B. Refer to the information in Appendix 11A for problems 23 through 25. Refer to Appendix 11B for problem 26.

23. From Table 11A-1, what is the probability of a loan upgrade? A loan downgrade?

The probability of an upgrade is 5.95% + 0.33% + 0.02% = 6.30%. The probability of a downgrade is 5.30% + 1.17% + 0.12% = 6.59%.

a. What is the impact of a rating upgrade or downgrade?

The effect of a rating upgrade or downgrade will be reflected on the credit-risk spreads or premiums on loans and thus on the implied market value of the loan. A downgrade should cause this credit-risk spread to rise.

b. How is the discount rate determined after a credit event has occurred?

The discount rate for each year in the future in which cash flows are expected to be received includes the forward rates from the current Treasury yield curve plus the annual credit spreads for loans of a particular rating class for each year. These credit spreads are determined by observing the spreads of the corporate bond market over Treasury securities.

c. Why does the probability distribution of possible loan values have a negative skew?

The negative skew occurs because the probability distribution is non-normal. The potential downside change in a loan's value is greater than the possible upside change in value.

d. How do the capital requirements of the CreditMetrics approach differ from those of the BIS and Federal Reserve System?

The Fed and the BIS require the capital reserve to be a fixed percentage of the risk-weighted book value of the loan (e.g., 8 percent). Under CreditMetrics each loan is likely to have a different VAR and thus a different implied capital requirement. Further, this required capital is likely to be

greater than 8 percent of the risk-weighted book value because of the non-normality of the probability distributions.

24 A five-year fixed-rate loan of $100 million carries a 7 percent annual interest rate. The borrower is rated BB. Based on hypothetical historical data, the probability distribution given below has been determined for various ratings upgrades, downgrades, status quo, and default possibilities over the next year. Information also is presented reflecting the forward rates of the current Treasury yield curve and the annual credit spreads of the various maturities of BBB bonds over Treasuries.

Rating	Probability Distribution	New Loan Value plus Coupon $
AAA	0.01%	$114.82m
AA	0.31	114.60m
A	1.45	114.03m
BBB	6.05	
BB	85.48	108.55m
B	5.60	98.43m
CCC	0.90	86.82m
Default	0.20	54.12m

Forward Rate Spreads at Time t

t	r_t%	ϕ_t%
1	3.00%	0.72%
2	3.40	0.96
3	3.75	1.16
4	4.00	1.30

a. What is the present value of the loan at the end of the one-year risk horizon for the case where the borrower has been upgraded from BB to BBB?

$$PV = \$7m + \frac{\$7m}{1.0372} + \frac{\$7m}{(1.0436)^2} + \frac{\$7m}{(1.0491)^3} + \frac{\$107m}{(1.0530)^4} = \$113.27\,\text{million}$$

b. What is the mean (expected) value of the loan at the end of year 1?

Year-end Rating	Probability	Value (m of $)	Probability x Value	Deviation	Probability x Deviation Squared
AAA	0.0001	$114.82	$0.01	6.76	0.0046
AA	0.0031	114.60	0.36	6.54	0.1325
A	0.0145	114.03	1.65	5.97	0.5162
BBB	0.0605	113.27	6.85	5.21	1.6402
BB	0.8548	108.55	92.79	0.49	0.2025
B	0.056	98.43	5.51	-9.63	5.1968
CCC	0.009	86.82	0.78	-21.24	4.0615

Default	0.002	54.12	0.11	-53.94	5.8197
	1.000	Mean =	$108.06m	Variance =	17.5740
				Standard Deviation =	$4.19m

The solution table reveals a value of $108.06 million.

c. What is the volatility of the loan value at the end of year 1?

The volatility or standard deviation of the loan value is $4.19 million.

d. Calculate the 5 percent and 1 percent VARs for this loan assuming a normal distribution of values.

The 5 percent VAR is 1.65 x $4.19m = $6.91m.
The 1 percent VAR is 2.33 x $4.19m = $9.76m.

e. Estimate the approximate 5 percent and 1 percent VARs using the actual distribution of loan values and probabilities.

5% VAR = 95% of actual distribution = $108.06m - $98.43m = $9.63m
1% VAR = 99% of actual distribution = $108.06m - $86.82m = $21.24m

where: 5% VAR is approximated by 0.056 + 0.009 + 0.002 = 0.067 or 6.7 percent, and
1% VAR is approximated by 0.009 + 0.002 = 0.011 or 1.1 percent.

Using linear interpolation, the 5% VAR = $10.65 million and the 1% VAR = $19.31 million. For the 1% VAR, $19.31m = (1 – 0.1/1.1) x $21.24m.

f. How do the capital requirements of the 1 percent VARs calculated in parts (d) and (e) above compare with the capital requirements of the BIS and Federal Reserve System?

The Fed and BIS systems would require 8 percent of the loan value, or $8 million. The 1 percent VAR would require $19.31 million under the approximate method, and $9.76 million (2.33 x $4.19m) in capital under the normal distribution assumption. In each case, the amounts exceed the Fed/BIS amount.

25. How does the Credit Risk+ model of Credit Suisse Financial Products differ from the CreditMetrics model of J.P. Morgan Chase?

CreditRisk+ attempts to estimate the expected loss of loans and the distribution of these losses with the focus on calculating the required capital reserves necessary to meet these losses. The

method assumes that the probability of any individual loan defaulting is random and that the correlation between the defaults on any pair of loan defaults is zero. CreditMetrics focuses on estimating a complete VAR framework.

26. An FI has a loan portfolio of 10,000 loans of $10,000 each. The loans have a historical average default rate of 4 percent and the severity of loss is 40 cents per dollar.

a. Over the next year, what are the probabilities of having default rates of 2, 3, 4, 5, and 8 percent?

$$\text{Probability of 2 defaults} = \frac{e^{-m}m^{n}}{n!} = \frac{(2.71828)^{-4}\text{x}4^{2}}{1\text{x}2} = \frac{0.018316\text{x}16}{2} = 0.1465 = 14.65\%$$

$$\text{Probability of 3 defaults} = \frac{e^{-m}m^{n}}{n!} = \frac{(2.71828)^{-4}\text{x}4^{3}}{1\text{x}2\text{x}3} = \frac{0.018316\text{x}16}{6} = 0.1465 = 19.54\%$$

n	2	3	4	5	8
Probability	14.65%	19.54%	19.54%	15.63%	2.98%

b. What would be the dollar loss on the portfolios with default rates of 4 and 8 percent?

Dollar loss of 4 loans defaulting = 4 x 0.40 x $10,000 = $16,000
Dollar loss of 8 loans defaulting = 8 x 0.40 x $10,000 = $32,000

c. How much capital would need to be reserved to meet the 1 percent worst-case loss scenario? What proportion of the portfolio's value would this capital reserve be?

The probability of 8 defaults is ~3 percent. The probability of 10 defaults is 0.00529 or rounded up to 1 percent. The dollar loss of 10 loans defaulting is $40,000. Thus, a 1 percent chance of losing $40,000 exists.

A capital reserve should be held to meet the difference between the unexpected 1 percent loss rate and the expected loss rate of 4 defaults. This difference is $40,000 minus $16,000 or $24,000. This amount is 0.024 percent of the total portfolio.

Integrated Mini Case: Loan Portfolio Analysis

As a senior loan officer at MC Financial Corp, you have a loan application from a firm in the biotech industry. While the loan has been approved on the basis of an individual loan, you must evaluate the loan based on its impact on the risk of the overall loan portfolio. The FI uses the following three methods to assess its loan portfolio risk.

1. Concentration Limits - The FI currently has lent an amount equal to 40 percent of its capital to the biotech industry and does not lend to a firm in any sector that generates losses in excess of 2 percent of capital. The average historical losses in the biotech industry total 5 percent.

> Concentration limit = (Maximum loss as a percent of capital) x (1/Loss rate) = 0.02 x 1/0.05 = 40 percent of capital is the maximum amount that can be lent to firms in the biotech sector.

MC Financial already has 40 percent of its capital lent out to the biotech industry. To give out this new loan would put the FI over its concentration limit. Thus, MC Financial should not grant this loan.

2. Loan Volume-Based Model - National and MC Financial's loan portfolio allocations are as follows.

Allocation of Loan Portfolios in Different Sectors (%)

Sectors	National	MC Financial
Commercial	30%	50%
Real Estate	50%	35%
Consumer	20%	15%

MC Financial does not want to deviate from the national average by more than 14.72 percent.

Using Xs to represent portfolio holdings:

$(X_{1j} - X_1)^2$ $(0.50 - 0.30)^2 = 0.0400$

$(X_{2j} - X_2)^2$ $(0.35 - 0.50)^2 = 0.0225$

$(X_{3j} - X_3)^2$ $(0.15 - 0.20)^2 = 0.0025$

$\sum_{i=1}^{n} (X_{ij} - X_i)^2$ $\sum_{i=1}^{n=3} = 0.0650$

$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_{ij} - X_i)^2}{n}}$ $\sigma = 14.72$ percent

The FI's standard deviation in its loan portfolio allocation is 14.72 percent. To issue another C&I loan would push MC Financial even further from the national average. Thus, the FI would not want to give out the loan

3. Loan Loss Ratio-Based Model - Based on regression analysis on historical loan losses, the FI estimates the following loan loss ratio models:

$X_{C\&I} = 0.001 + 0.85X_L$ and $X_{con} = 0.003 + 0.65X_L$

where $X_{C\&I}$ = loss rate in the commercial sector, X_{con} = loss rate in the consumer (household) sector, X_L = loss rate for its total loan portfolio.

MC Financial's total increase in the loan loss ratio is expected to be 12 percent next year.

Commercial loan loss rates will increase by $0.001 + 0.85(0.12) = 10.30$ percent.
Consumer loan loss rates will increase by $0.003 + 0.65(0.12) = 8.10$ percent.

MC Financial should limit its loans to the commercial sector because the loss rates are systematically higher than the loss rates for the total loan portfolio. Loss rates are lower for the consumer sector. For a 12 percent increase in the total losses in the loan portfolio, the commercial

loss rate is expected to increase by 10.30 percent, as opposed to only 8.10 percent for the consumer sector. Thus, MC Financial should not issue this loan.

Should MC Financial Corp. grant this loan?

Even though the loan was approved on an individual loan basis, from an overall loan portfolio perspective, all three models suggest that MC Financial would not want to issue this loan to a firm in the biotech sector.

Additional Example for Chapter 12

Sectors	Allocation of Loan Portfolios in Different Sectors (%) National	Bank A	Bank B
Commercial	20%	50%	30%
Consumer	40%	20%	40%
Real Estate	40%	30%	30%

How different are Banks A and B from the national benchmark? When using this example, note that there is an implied assumption that Bank A and B belong to a certain size class or have some common denominator linking them to the national benchmark. If that is the case, then the solution is to estimate the standard deviation.

We use Xs to represent the portfolio concentrations. X_1, X_2 and X_3 are the national benchmark percentages

	Bank A	Bank B
$(X_{1j} - X_1)^2$	$(50 - 20)^2 = 900$	$(30 - 20)^2 = 100$
$(X_{2j} - X_2)^2$	$(20 - 40)^2 = 400$	$(40 - 40)^2 = 0$
$(X_{3j} - X_3)^2$	$(30 - 40)^2 = 100$	$(30 - 40)^2 = 100$
$\sum_{i=1}^{n} (X_{ij} - X_i)^2$	$\sum_{i=1}^{n=3} = 1{,}400$	$\sum_{i=1}^{n=3} = 200$
$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_{ij} - X_i)^2}{n}}$	$\sigma = 37.42$ percent	$\sigma = 14.14$ percent

Thus we can see here that Bank A is significantly different from the national benchmark