

## Solutions for End-of-Chapter Questions and Problems: Chapter Fifteen

1. What is meant by *market risk*?

Market risk is the risk related to the uncertainty of an FI's earnings on its trading portfolio. Market risk is caused by changes in market conditions such as interest rate risk and foreign exchange risk. Market risk emphasizes the risks to FIs that actively trade assets and liabilities rather than hold them for longer term investment, funding, or hedging purposes.

2. Why is the measurement of market risk important to the manager of a financial institution?

Measurement of market risk can help an FI manager in the following ways:

- a. Provide information on the risk positions taken by individual traders.
  - b. Establish limit positions on each trader based on the market risk of their portfolios.
  - c. Help allocate resources to departments with lower market risks and appropriate returns.
  - d. Evaluate performance based on risks undertaken by traders in determining optimal bonuses.
  - e. Help develop more efficient internal models so as to avoid using standardized regulatory models.
3. What is meant by *daily earnings at risk* (DEAR)? What are the three measurable components? What is the price volatility component?

DEAR, or daily earnings at risk, is a measure of market risk over the next 24 hours. It is defined as the estimated potential loss of a portfolio's value over a one-day period as a result of adverse moves in market conditions, such as changes in interest rates, foreign exchange rates, and market volatility. DEAR is comprised of (a) the dollar value of the position, (b) the price sensitivity of the asset to changes in the risk factor, and (c) the adverse move in the yield. The product of the price sensitivity of the asset and the adverse move in the yield provides the price volatility component.

4. Follow Bank has a \$1 million position in a five-year, zero-coupon bond with a face value of \$1,402,552. The bond is trading at a yield to maturity of 7.00 percent. The historical mean change in daily yields is 0.0 percent and the standard deviation is 12 basis points.
  - a. What is the modified duration of the bond?

$$MD = D/(1 + R) = 5/(1.07) = 4.6729 \text{ years}$$

- b. What is the maximum adverse daily yield move given that we desire no more than a 1 percent chance that yield changes will be greater than this maximum?

Potential adverse move in yield at 1 percent =  $2.33\sigma = 2.33 \times 0.0012 = 0.002796$

- c. What is the price volatility of this bond?

Price volatility = MD x potential adverse move in yield  
=  $4.6729 \times 0.002796 = 0.013065$  or 1.3065 percent

- d. What is the daily earnings at risk for this bond?

DEAR = (\$ market value of position) x (price volatility)  
=  $\$1,000,000 \times 0.013065 = \$13,065$

5. How can DEAR be adjusted to account for potential losses over multiple days? What would be the VAR for the bond in problem 4 for a 10-day period? What statistical assumption is needed for this calculation? Could this treatment be critical?

The DEAR can be adjusted to account for losses over multiple days using the formula N-day VAR = DEAR x  $[N]^{1/2}$ , where N is the number of days over which potential loss is estimated. N-day VAR is a more realistic measure when it requires a longer period for an FI to unwind a position, that is, if markets are less liquid. The value for the 10-day VAR in problem 4 above is  $\$13,065 \times [10]^{1/2} = \$41,315$ .

According to the above formula, the relationship assumes that yield changes are independent and daily volatility is approximately constant. This means that losses incurred one day are not related to losses incurred the next day. Recent studies have indicated that this is not the case, but that shocks are autocorrelated in many markets over long periods of time.

6. The DEAR for a bank is \$8,500. What is the VAR for a 10-day period? A 20-day period? Why is the VAR for a 20-day period not twice as much as that for a 10-day period?

For the 10-day period: VAR =  $8,500 \times [10]^{1/2} = 8,500 \times 3.1623 = \$26,879$

For the 20-day period: VAR =  $8,500 \times [20]^{1/2} = 8,500 \times 4.4721 = \$38,013$

The reason that 20-day VAR  $\neq$  (2 x 10-day VAR) is because  $[20]^{1/2} \neq (2 \times [10]^{1/2})$ . The interpretation is that the daily effects of an adverse event become less as time moves farther away from the event.

7. The mean change in the daily yields of a 15-year, zero-coupon bond has been five basis points (bp) over the past year with a standard deviation of 15 bp. Use these data and assume that the yield changes are normally distributed.

- a. What is the highest yield change expected if a 99 percent confidence limit is required; that is, adverse moves will not occur more than 1 day in 100?

If yield changes are normally distributed, 98 percent of the area of a normal distribution will be 2.33 standard deviations ( $2.33\sigma$ ) from the mean – that is,  $2.33\sigma$  – and 2 percent of the area under the normal distribution is found beyond  $\pm 2.33$  (1 percent under each tail,  $-2.33\sigma$  and  $+2.33\sigma$ , respectively). Thus, for a one-tailed distribution, the 99 percent confidence level will represent adverse moves that not occur more than 1 day in 100. In this example, it means  $2.33 \times 15 \text{ bp} = 34.95 \text{ bp}$ . Thus, the maximum adverse yield change expected for this zero-coupon bond is an increase of 34.95 basis points, or 0.3495 percent, in interest rates.

- b. What is the highest yield change expected if a 95 percent confidence limit is required; adverse moves will not occur more than 1 day in 20?

If yield changes are normally distributed, 90 percent of the area of a normal distribution will be 1.65 standard deviations ( $1.65\sigma$ ) from the mean – that is,  $1.65\sigma$  – and 10 percent of the area under the normal distribution is found beyond  $\pm 1.65$  (5 percent under each tail,  $-1.65\sigma$  and  $+1.65\sigma$ , respectively). Thus, for a one-tailed distribution, the 95 percent confidence level will represent adverse moves that not occur more than 1 day in 20. Thus, the maximum adverse yield change expected for this zero-coupon bond is an increase of  $(1.65 \times 15 =) 24.75$  basis points, or 0.2475 percent, in interest rates.

8. In what sense is duration a measure of market risk?

Market risk calculations are typically based on the trading portion of an FI's fixed-rate asset portfolio because these assets must reflect changes in value as market interest rates change. As such, duration or modified duration provides an easily measured and usable link between changes in the market interest rates and changes in the market value of fixed-income assets.

9. Bank Alpha has an inventory of AAA-rated, 15-year zero-coupon bonds with a face value of \$400 million. The bonds currently are yielding 9.5 percent in the over-the-counter market.

- a. What is the modified duration of these bonds?

$$MD = D/(1 + R) = 15/(1.095) = 13.6986$$

- b. What is the price volatility if the potential adverse move in yields is 25 basis points?

$$\begin{aligned}\text{Price volatility} &= (\text{MD}) \times (\text{potential adverse move in yield}) \\ &= (13.6986) \times (0.0025) = 0.03425 \text{ or } 3.425 \text{ percent.}\end{aligned}$$

- c. What is the DEAR?

$$\text{Daily earnings at risk (DEAR)} = (\$ \text{ market value of position}) \times (\text{Price volatility})$$

$$\begin{aligned}\text{Dollar value of position} &= \$400\text{m}/(1 + 0.095)^{15} = \$102,529,350. \text{ Therefore,} \\ \text{DEAR} &= \$102,529,350 \times 0.03425 = \$3,511,279.\end{aligned}$$

- d. If the price volatility is based on a 99 percent confidence limit and a mean historical change in daily yields of 0.0 percent, what is the implied standard deviation of daily yield changes?

The potential adverse move in yields = confidence limit value  $\times$  standard deviation value.  
Therefore, 25 basis points =  $2.33 \times \sigma$ , and  $\sigma = 0.0025/2.33 = 0.001073$  or 10.73 basis points.

10. Bank Beta has an inventory of AAA-rated, zero-coupon bonds with a maturity of 13.42 years and a face value of \$127,503,041. The modified duration of these bonds is 12.5 years, the DEAR is \$2,150,000, and the potential adverse move in yields is 35 basis points. What is the market value of the bonds, the yield on the bonds, and the duration of the bonds?

$$\begin{aligned}\text{Price volatility} &= (\text{MD}) \times (\text{potential adverse move in yield}) \\ &= (12.5) \times (0.0035) = 0.04375 \text{ or } 4.375 \text{ percent}\end{aligned}$$

$$\text{DEAR} = (\$ \text{ market value of position}) \times (\text{Price volatility})$$

$$\begin{aligned}\text{DEAR} &= \$2,150,000 = (\$ \text{ value of position}) \times 0.04375 \\ \Rightarrow (\$ \text{ value of position}) &= \$2,150,000/0.04375 = \$49,142,857 = \text{market value}\end{aligned}$$

$$\begin{aligned}\text{Dollar value of position} &= \$127,503,041/(1 + \text{yield})^{13.42} = \$49,142,857. \\ \Rightarrow \text{yield} &= (\$127,503,041/\$49,142,857)^{1/13.42} - 1 = 7.36\%\end{aligned}$$

Therefore, the bonds currently are yielding 7.36 percent in the over-the-counter market.

$$\text{MD} = D/(1 + R) = 12.5 = D/(1.0736) \Rightarrow D = 12.5 \times 1.0736 = 13.42 \text{ years}$$

11. Bank Two has a portfolio of bonds with a market value of \$200 million. The bonds have an estimated price volatility of 0.95 percent. What are the DEAR and the 10-day VAR for these bonds?

$$\begin{aligned}
 \text{DEAR} &= (\$ \text{ market value of position}) \times (\text{Price volatility}) \\
 &= \$200 \text{ million} \times 0.0095 \\
 &= \$1,900,000
 \end{aligned}$$

$$\begin{aligned}
 \text{10-day VAR} &= \text{DEAR} \times \sqrt{N} = \$1,900,000 \times \sqrt{10} \\
 &= \$1,900,000 \times 3.1623 = \$6,008,328
 \end{aligned}$$

12. Suppose that an FI has a €1.6 million long trading position in spot euros at the close of business on a particular day. Looking back at the daily percentage changes in the exchange rate of the €/ \$ for the past year, the volatility or standard deviation ( $\sigma$ ) of daily percentage changes in the €/ \$ spot exchange rate was 62.5 basis points (bp). Calculate the FI's daily earnings at risk from this position (i.e., adverse moves in the FX markets with respect to the value of the euro against the dollar will not occur more than 1 percent of the time, or 1 day in every 100 days) if the spot exchange rate is €0.80/\$1, or \$1.25/€, at the daily close.

The first step is to calculate the dollar-equivalent amount of the position.

$$\begin{aligned}
 \text{Dollar equivalent value of position} &= \text{FX position} \times (\$ \text{ per unit of foreign currency}) \\
 &= €1.6 \text{ million} \times \$1.25/€ \\
 &= \$2 \text{ million}
 \end{aligned}$$

If changes in exchange rates are historically normally distributed, the exchange rate must change in the adverse direction by  $2.33\sigma$ , or

$$\text{FX volatility} = 2.33 \times 62.5 \text{ bp} = 145.625 \text{ bp or } 1.45625\%$$

As a result,

$$\begin{aligned}
 \text{DEAR} &= \text{Dollar value of position} \times \text{FX volatility} \\
 &= \$2 \text{ million} \times 0.0145625 \\
 &= \$29,125
 \end{aligned}$$

This is the potential daily earnings at risk exposure to adverse euro to dollar exchange rate changes for the bank from the €1.6 million spot currency holding.

13. Bank of Southern Vermont has determined that its inventory of 20 million euros (€) and 25 million British pounds (£) is subject to market risk. The spot exchange rates are \$1.20/€ and \$1.60/£, respectively. The  $\sigma$ 's of the spot exchange rates of the € and £, based on the daily changes of spot rates over the past six months, are 65 bp and 45 bp, respectively. Determine the bank's 10-day VAR for both currencies. Use adverse rate changes in the 99<sup>th</sup> percentile.

$$\begin{aligned}\text{FX position of €} &= €20\text{m} \times 1.20 = \$24 \text{ million} \\ \text{FX position of £} &= £25\text{m} \times 1.60 = \$40 \text{ million} \\ \\ \text{FX volatility €} &= 2.33 \times 65\text{bp} = 151.45\text{bp, or } 1.5145\% \\ \text{FX volatility £} &= 2.33 \times 45\text{bp} = 104.85\text{bp, or } 1.0485\% \\ \\ \text{DEAR} &= (\$ \text{ market value of position}) \times (\text{Price volatility})\end{aligned}$$

$$\begin{aligned}\text{DEAR of €} &= \$24\text{m} \times 0.015145 = \$363,480 \\ \text{DEAR of £} &= \$40\text{m} \times 0.010485 = \$419,400\end{aligned}$$

$$\begin{aligned}\text{10-day VAR of €} &= \$363,480 \times \sqrt{10} = \$363,480 \times 3.1623 = \$1,149,425 \\ \text{10-day VAR of £} &= \$419,400 \times \sqrt{10} = \$419,400 \times 3.1623 = \$1,326,259\end{aligned}$$

14. Bank of Bentley has determined that its inventory of yen (¥) and Swiss franc (SFr) denominated securities is subject to market risk. The spot exchange rates are ¥120.00/\$ and SFr0.9500/\$, respectively. The  $\sigma$ 's of the spot exchange rates of the ¥ and SFr, based on the daily changes of spot rates over the past six months, are 75 bp and 55 bp, respectively. Using adverse rate changes in the 99<sup>th</sup> percentile, the 10-day VARs for the two currencies, ¥ and SFr, are \$350,000 and \$500,000, respectively. Calculate the yen and Swiss franc-denominated value positions for Bank of Bentley.

$$\begin{aligned}\text{10-day VAR} &= \text{DEAR} \times \sqrt{N} \Rightarrow \\ \text{10-day VAR of ¥} &= \$350,000 = \text{DEAR} \times \sqrt{10} \Rightarrow \text{DEAR} = \$350,000 / \sqrt{10} = \$110,680 \\ \text{10-day VAR of SFr} &= \$500,000 = \text{DEAR} \times \sqrt{10} \Rightarrow \text{DEAR} = \$500,000 / \sqrt{10} = \$158,114\end{aligned}$$

$$\begin{aligned}\text{FX volatility} &= 2.33 \times \text{daily changes of spot rates over the past six months} \Rightarrow \\ \text{FX volatility ¥} &= 2.33 \times 75\text{bp} = 0.017475, \text{ or } 1.7475\% \\ \text{FX volatility SFr} &= 2.33 \times 55\text{bp} = 0.012815, \text{ or } 1.2815\%\end{aligned}$$

$$\begin{aligned}\text{DEAR} &= (\$ \text{ market value of position}) \times (\text{Price volatility}) \\ \text{DEAR of ¥} &= \$110,680 = (\$ \text{ market value of position}) \times 0.017475 \\ &\Rightarrow (\$ \text{ value of position}) = \$110,680 / 0.017475 = \$6,333,603 \\ \text{DEAR of SFr} &= \$158,114 = (\$ \text{ market value of position}) \times 0.012815 \\ &\Rightarrow (\$ \text{ value of position}) = \$158,114 / 0.012815 = \$12,338,188\end{aligned}$$

$$\begin{aligned}\text{FX position in ¥} &= \text{Yen position} / 120.00 = \$6,333,603 \\ &\Rightarrow \text{Yen position} = 120.00 \times \$6,333,603 = ¥760,032,399\end{aligned}$$

$$\begin{aligned}\text{FX position in SFr} &= \text{SFr position} / 0.9500 = \$12,338,188 \\ \Rightarrow \text{SFr position} &= 0.9500 \times \$12,338,188 = \text{SFr}11,721,279\end{aligned}$$

15. Suppose that an FI holds a \$15 million trading position in stocks that reflect the U.S. stock market index (e.g., the S&P 500). Over the last year, the  $\sigma_m$  of the daily returns on the stock market index was 156 bp. Calculate the DEAR for this portfolio of stocks using a 99 percent confidence limit.

Since the portfolio of stocks reflect the U.S. stock market index, the  $\beta = 1$ . Thus, the DEAR for equities is:

$$\begin{aligned}\text{DEAR} &= \text{Dollar market value of position} \times \text{Stock market return volatility} \\ &= \$15,000,000 \times 2.33 \sigma_m\end{aligned}$$

The  $\sigma_m$  of the daily returns on the stock market index was 156 bp over the last year. So,

$$\text{Stock market return volatility} = 2.33 \times 1.56\% = 3.6348\%$$

and

$$\text{DEAR} = \$15,000,000 \times 0.036348 = \$545,220$$

That is, the FI stands to lose at least \$545,220 in value if adverse stock market returns materialize tomorrow.

16. Bank of Alaska's stock portfolio has a market value of \$10 million. The beta of the portfolio approximates the market portfolio, whose standard deviation ( $\sigma_m$ ) has been estimated at 1.5 percent. What is the five-day VAR of this portfolio using adverse rate changes in the 99<sup>th</sup> percentile?

$$\begin{aligned}\text{DEAR} &= (\$ \text{ market value of portfolio}) \times (2.33 \times \sigma_m) = \$10\text{m} \times (2.33 \times 0.015) \\ &= \$10\text{m} \times 0.03495 = \$349,500\end{aligned}$$

$$5\text{-day VAR} = \$349,500 \times \sqrt{5} = \$349,500 \times 2.2361 = \$781,506$$

17. Jeff Resnick, vice president of operations at Choice Bank, is estimating the aggregate daily DEAR of the bank's portfolio of assets consisting of loans (L), foreign currencies (FX), and common stock (EQ). The individual DEARs are \$300,700; \$274,000; and \$126,700 respectively. If the correlation coefficients ( $\rho_{ij}$ ) between L and FX, L and EQ, and FX and EQ are 0.3, 0.7, and 0.0, respectively, what is the DEAR of the aggregate portfolio?

$$\begin{aligned}
 \text{DEAR portfolio} &= \left[ \begin{aligned} &(\text{DEAR}_L)^2 + (\text{DEAR}_{FX})^2 + (\text{DEAR}_{EQ})^2 \\ &+ (2\rho_{L,FX} \times \text{DEAR}_L \times \text{DEAR}_{FX}) \\ &+ (2\rho_{L,EQ} \times \text{DEAR}_L \times \text{DEAR}_{EQ}) \\ &+ (2\rho_{FX,EQ} \times \text{DEAR}_{FX} \times \text{DEAR}_{EQ}) \end{aligned} \right]^{0.5} \\
 &= \left[ \begin{aligned} &\$300,700^2 + \$274,000^2 + \$126,700^2 + 2(0.3)(\$300,700)(\$274,000) \\ &+ 2(0.7)(\$300,700)(\$126,700) + 2(0.0)(\$274,000)(\$126,700) \end{aligned} \right]^{0.5} \\
 &= [\$284,322,626,000]^{0.5} = \$533,219
 \end{aligned}$$

18. Calculate the DEAR for the following portfolio with the correlation coefficients and then with perfect positive correlation between various asset groups.

<u>Assets</u>	<u>Estimated DEAR</u>	<u>(<math>\rho_{S,FX}</math>)</u>	<u>(<math>\rho_{S,B}</math>)</u>	<u>(<math>\rho_{FX,B}</math>)</u>
Stocks (S)	\$300,000	-0.10	0.75	0.20
Foreign Exchange (FX)	200,000			
Bonds (B)	250,000			

$$\begin{aligned}
 \text{DEAR portfolio} &= \left[ \begin{aligned} &(\text{DEAR}_S)^2 + (\text{DEAR}_{FX})^2 + (\text{DEAR}_B)^2 \\ &+ (2\rho_{S,FX} \times \text{DEAR}_S \times \text{DEAR}_{FX}) \\ &+ (2\rho_{S,B} \times \text{DEAR}_S \times \text{DEAR}_B) \\ &+ (2\rho_{FX,B} \times \text{DEAR}_{FX} \times \text{DEAR}_B) \end{aligned} \right]^{0.5} \\
 &= \left[ \begin{aligned} &\$300,000^2 + \$200,000^2 + \$250,000^2 + 2(-0.1)(\$300,000)(\$200,000) \\ &+ 2(0.75)(\$300,000)(\$250,000) + 2(0.20)(\$200,000)(\$250,000) \end{aligned} \right]^{0.5} \\
 &= [\$313,000,000,000]^{0.5} = \$559,464
 \end{aligned}$$

What is the amount of risk reduction resulting from the lack of perfect positive correlation between the various assets groups?



DEAR portfolio(correlation coefficients = 1) =

$$= \left[ \begin{aligned} &\$300,000^2 + \$200,000^2 + \$250,000^2 + 2(1.0)(\$300,000)(\$200,000) \\ &+ 2(1.0)(\$300,000)(\$250,000) + 2(1.0)(\$200,000)(\$250,000) \end{aligned} \right]^{0.5}$$

$$= [\$562,500,000,000]^{0.5} = \$750,000$$

The DEAR for a portfolio with perfect correlation would be \$750,000. Therefore, the risk reduction is \$750,000 - \$559,464 = \$190,536.

19. What are the advantages of using the back simulation approach to estimate market risk? Explain how this approach would be implemented.

The advantages of the back simulation approach to estimating market risk are that (a) it is a simple process, (b) it does not require that asset returns be normally distributed, and (c) it does not require the calculation of correlations or standard deviations of returns. Implementation requires the calculation of the value of the current portfolio of assets based on the prices or yields that were in place on each of the preceding 500 days (or some large sample of days). These data are rank-ordered from worst to best case and percentile limits are determined. For example, the one percent worst case scenario provides an estimate with 99 percent confidence that the value of the portfolio will not fall more than this amount.

20. Export Bank has a trading position in Japanese yen (¥) and Swiss francs (SFr). At the close of business on February 4, the bank had ¥450 million and SFr10 million. The exchange rates for the most recent six days are given below:

	<b><u>Exchange Rates per U.S. Dollar at the Close of Business</u></b>					
	<u>2/4</u>	<u>2/3</u>	<u>2/2</u>	<u>2/1</u>	<u>1/29</u>	<u>1/28</u>
Japanese yen	120.13	120.84	120.14	123.05	124.35	124.32
Swiss francs	0.9540	0.9575	0.9533	0.9617	0.9557	0.9523

- a. What is the foreign exchange (FX) position in dollar equivalents using the FX rates on February 4?

Japanese yen: ¥450,000,000/¥120.13 = \$3,745,942

Swiss francs: SFr10,000,000/SFr0.9540 = \$10,482,180

- b. What is the definition of delta as it relates to the FX position?

Delta measures the change in the dollar value of each FX position if the foreign currency depreciates by 1 percent against the dollar.

- c. What is the sensitivity of each FX position; that is, what is the value of delta for each currency on February 4?

Japanese yen:  $1.01 \times \text{current exchange rate} = 1.01 \times ¥120.13 = ¥121.3313/\$$   
 Revalued position in \$s  $= ¥450,000,000/121.3313 = \$3,708,853$   
 Delta of \$ position to Yen  $= \$3,708,853 - \$3,745,942$   
 $= -\$37,089$

Swiss francs:  $1.01 \times \text{current exchange rate} = 1.01 \times \text{SFr}0.9540 = \text{SFr}0.96354$   
 Revalued position in \$s  $= \text{SFr}10,000,000/0.96354 = \$10,378,396$   
 Delta of \$ position to CHF  $= \$10,378,396 - \$10,482,180$   
 $= -\$103,784$

- d. What is the daily percentage change in exchange rates for each currency over the five-day period?

Day	Japanese yen:	Swiss franc	
2/4	-0.58755	-0.36554	% Change = $(\text{Rate}_t/\text{Rate}_{t-1}) - 1 \times 100$
2/3	0.58265	0.44057	
2/2	-2.36489	-0.87345	
2/1	-1.04544	0.62781	
1/29	0.02413	0.35703	

- e. What is the total risk faced by the bank on each day? What is the worst-case day? What is the best-case day?

Day	Japanese yen			Swiss francs			Total
	Delta	%Rate $\Delta$	Risk	Delta	%Rate $\Delta$	Risk	Risk
2/4	-\$37,089	-0.58755	\$21,792	-\$103,784	-0.36554	\$37,937	\$59,729
2/3	-\$37,089	0.58265	-\$21,610	-\$103,784	0.44057	-\$45,724	-\$67,334
2/2	-\$37,089	-2.36489	\$87,710	-\$103,784	-0.87345	\$90,650	\$178,360
2/1	-\$37,089	-1.04544	\$38,774	-\$103,784	0.62781	-\$65,157	-\$26,383
1/29	-\$37,089	0.02413	-\$895	-\$103,784	0.35703	-\$37,054	-\$37,949

The worst-case day is February 3, and the best-case day is February 2.

- f. Assume that you have data for the 500 trading days preceding February 4. Explain how you would identify the worst-case scenario with a 99 percent degree of confidence?

The appropriate procedure would be to repeat the process illustrated in part (e) above for all 500 days. The 500 days would be ranked on the basis of total risk from the worst-case to the best-case. The one percent from the absolute worst-case situation would be day 5 in the ranking.

- g. Explain how the 1 percent value at risk (VAR) position would be interpreted for business on February 5.

Management would expect with a confidence level of 99 percent that the total risk on February 5 would be no worse than the total risk value for the 5<sup>th</sup> worst day in the previous 500 days. This value represents the VAR for the portfolio.

- h. How would the simulation change at the end of the day on February 5? What variables and/or processes in the analysis may change? What variables and/or processes will not change?

The analysis can be upgraded at the end of the each day. The values for delta may change for each of the assets in the analysis. As such, the value for VAR may also change. Any historical data used in the analysis will not change.

21. Export Bank has a trading position in euros and Australian dollars. At the close of business on October 20, the bank had €22 million and A\$40 million. The exchange rates for the most recent six days are given below:

**Exchange Rates per U.S. Dollar at the Close of Business**

	<u>10/20</u>	<u>10/19</u>	<u>10/18</u>	<u>10/17</u>	<u>10/16</u>	<u>10/15</u>
Euros	0.8800	0.8770	0.8575	0.8675	0.8750	0.8915
Australian \$s	1.2900	1.2750	1.3000	1.2855	1.2705	1.2660

- a. What is the foreign exchange (FX) position in dollar equivalents using the FX rates on October 20?

Euros: €22 million/€0.8800 = \$25,000,000

Australian \$s: A\$40 million/A\$1.2900 = \$31,007,752

- b. What is the sensitivity of each FX position; that is, what is the value of delta for each currency on October 20?

Euros:  $1.01 \times \text{current exchange rate} = 1.01 \times \text{€}0.8800 = \text{€}0.8888/\text{\$}$   
Revalued position in  $\text{\$}$   $= \text{€}22 \text{ million} / \text{€}0.8888 = \$24,752,475$   
Delta of  $\text{\$}$  position to  $\text{€}$   $= \$24,752,475 - \$25,000,000$   
 $= -\$247,525$

Australian  $\text{\$}$ s:  $1.01 \times \text{current exchange rate} = 1.01 \times \text{A\$}1.2900 = \text{A\$}1.3029$   
Revalued position in  $\text{\$}$   $= \text{A\$}40 \text{ million} / 1.3029 = \$30,700,744$   
Delta of  $\text{\$}$  position to  $\text{A\$}$   $= \$30,700,744 - \$31,007,752$   
 $= -\$307,007$

- c. What is the daily percentage change in exchange rates for each currency over the five-day period?

<u>Day</u>	<u>Euro:</u>	<u>Australian <math>\text{\\$}</math>s</u>	
10/20	0.34208	1.17647	$\% \text{ Change} = (\text{Rate}_t / \text{Rate}_{t-1}) - 1 * 100$
10/19	2.27405	-1.92308	
10/18	-1.15274	1.12797	
10/17	-0.85714	1.18064	
10/16	-1.85081	0.35545	

- d. What is the total risk faced by the bank on each day? What is the worst-case day? What is the best-case day?

<u>Day</u>	<u>Euro</u>			<u>Australian <math>\text{\\$}</math>s</u>			<u>Total</u>
	<u>Delta</u>	<u>% Rate <math>\Delta</math></u>	<u>Risk</u>	<u>Delta</u>	<u>% Rate <math>\Delta</math></u>	<u>Risk</u>	<u>Risk</u>
10/20	-\$247,525	0.34208	-\$84,672	-\$307,007	1.17647	-\$361,185	-\$445,857
10/19	-\$247,525	2.27405	-\$562,884	-\$307,007	-1.92308	\$590,399	\$27,515
10/18	-\$247,525	-1.15274	\$285,331	-\$307,007	1.12797	-\$346,294	-\$60,963
10/17	-\$247,525	-0.85714	\$212,164	\$307,007	1.18064	-\$362,465	-\$150,301
10/16	-\$247,525	-1.85081	\$458,122	-\$307,007	0.35545	-\$109,126	\$348,996

The worst-case day is October 20, and the best-case day is October 16.

22. What is the primary disadvantage to the back simulation approach in measuring market risk? What effect does the inclusion of more observation days have as a remedy for this disadvantage? What other remedies can be used to deal with the disadvantage?

The primary disadvantage of the back simulation approach is the confidence level contained in the number of days over which the analysis is performed. One possible solution to the problem is to go back further in time and use more days and to estimate the 1 percent VAR inclusive of the

extra daily observations. The problem is that as one goes back farther in time, past observations may become decreasingly relevant in predicting VAR in the future.

23. How is Monte Carlo simulation useful in addressing the disadvantages of back simulation? What is the primary statistical assumption underlying its use?

Monte Carlo simulation can be used to generate additional observations that more closely capture the statistical characteristics of recent experience. The generating process is based on the historical variance-covariance matrix of value changes. The values in this matrix are multiplied by random numbers that produce results that pattern closely the actual observations of recent historic experience.

24. What is the difference between VAR and expected shortfall (ES) as measure of market risk?

VAR corresponds to a specific point of loss on the probability distribution. It does not provide information about the potential size of the loss that exceeds it, i.e., VAR completely ignores the patterns and the severity of the losses in the extreme tail. Thus, VAR gives only partial information about the extent of possible losses, particularly when probability distributions are non-normal. The drawbacks of VAR became painfully evident during the financial crisis as asset returns plummeted into the “fat tail” region of non-normally shaped distributions. FI's managers and regulators were forced to recognize that VAR projections of possible losses far underestimated actual losses on extreme bad days. Expected shortfall (ES), also referred to as conditional VAR and expected tail loss, is a measure of market risk that estimates the expected value of losses beyond a given confidence level, i.e., it is the average of VARs beyond a given confidence level. ES, which incorporates points to the left of VAR, is larger when the probability distribution exhibits fat tail losses. Accordingly, ES provides more information about possible market risk losses than VAR. For situations in which probability distributions exhibit fat tail losses, VAR may look relatively small, but ES may be very large.

25. Consider the following discrete probability distribution of payoffs for two securities, A and B, held in the trading portfolio of an FI:

Probability	A	Probability	B
50.00%	\$80m	50.00%	\$80m
49.00	60m	49.00	68m
1.00	-740m	0.40	-740m
		0.60	-1,393m

Which of the two securities will add more market risk to the FI's trading portfolio according to the VAR and ES measures?

The expected return on security A =  $0.50(\$80m) + 0.49(\$60m) + 0.01(-\$740m) = \$62m$

The expected return on security B =  $0.50(\$80m) + 0.49(\$68m) + 0.0040(-\$740m) + 0.0060(-\$1,393m) = \$62m$

For a 99% confidence level,  $VAR_A = VAR_B = -\$740m$

For a 99% confidence level,  $ES_A = -\$740m$ , while  $ES_B = 0.40(-\$740m) + 0.60(-\$1,393m) = -\$1,131.8m$

While the VAR is identical for both securities, the ES finds that security B has the potential to subject the FI to much greater losses than security A. Specifically, if tomorrow is a bad day, VAR finds that there is a 1 percent probability that the FI's losses will exceed \$740 million on either security. However, if tomorrow is a bad day, ES finds that there is a 1 percent probability that the FI's losses will exceed \$740 million if security A is in its trading portfolio, but losses will exceed \$1,131.8 million if security B is in its trading portfolio.

26. Consider the following discrete probability distribution of payoffs for two securities, A and B, held in the trading portfolio of an FI:

Probability	A	Probability	B
55.00%	\$120m	55.00%	\$120m
44.00	95m	44.00	100m
1.00	-1,100m	0.30	-1,100m
		0.70	-1,414m

Which of the two securities will add more market risk to the FI's trading portfolio according to the VAR and ES measures?

The expected return on security A =  $0.55(\$120m) + 0.44(\$95m) + 0.01(-\$1,100m) = \$96.8m$

The expected return on security B =  $0.55(\$120m) + 0.44(\$100m) + 0.0030(-\$1,100m) + 0.0070(-\$1,414m) = \$96.8m$

For a 99% confidence level,  $VAR_A = VAR_B = -\$1,100m$

For a 99% confidence level,  $ES_A = -\$1,100m$ , while  $ES_B = 0.30(-\$1,100m) + 0.70(-\$1,414m) = -\$1,319.8m$

Thus, while the VAR is identical for both securities, the ES finds that security B has the potential to subject the FI to much greater losses than security A. Specifically, if tomorrow is a bad day, VAR finds that there is a 1 percent probability that the FI's losses will exceed \$1,100 million on either security. However, if tomorrow is a bad day, ES finds that there is a 1 percent probability

that the FI's losses will exceed \$1,100 million if security A is in its trading portfolio, but losses will exceed \$1,319.8m if security B is in its trading portfolio.

27. An FI has £5 million in its trading portfolio on the close of business on a particular day. The current exchange rate of pounds for dollars is £0.6400/\$, or dollars for pounds is \$1.5625, at the daily close. The volatility, or standard deviation ( $\sigma$ ), of daily percentage changes in the spot £/\$ exchange rate over the past year was 58.5 bp. The FI is interested in adverse moves – bad moves that will not occur more than 1 percent of the time, or 1 day in every 100. Calculate the one-day VAR and ES from this position.

The first step is to calculate the dollar value position:

$$\begin{aligned}\text{Dollar value of position} &= \text{pound value of position} \times \text{dollar for pound exchange rate} \\ &= £5 \text{ million} \times 1.5625 = \$7,812,500\end{aligned}$$

Using VAR, which assumes that changes in exchange rates are normally distributed, the exchange rate must change in the adverse direction by  $2.33\sigma$  ( $2.33 \times 58.5$  bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.33 \times 58.5 \text{ bp} = 136.305 \text{ bp}$$

In other words, using VAR during the last year the pound declined in value against the dollar by 136.305 bp 1 percent of the time. As a result, the one-day VAR is:

$$\text{VAR} = \$7,812,500 \times 0.0136305 = \$106,488$$

Using ES, which assumes that changes in exchange rates are normally distributed but with fat tails, the exchange rate must change in the adverse direction by  $2.665\sigma$  ( $2.665 \times 58.5$  bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.665 \times 58.5 \text{ bp} = 155.9025 \text{ bp}$$

In other words, using ES during the last year the pound declined in value against the dollar by 155.9025 bp 1 percent of the time. As a result, the one-day ES is:

$$\text{ES} = \$7,812,500 \times 0.01559025 = \$121,799$$

The potential loss exposure to adverse pound to dollar exchange rate changes for the FI from the £5 million spot currency holdings are higher using the ES measure of market risk. ES estimates potential losses that are \$15,311 higher than VAR. This is because VAR focuses on the location

of the extreme tail of the probability distribution. ES also considers the shape of the probability distribution once VAR is exceeded.

28. An FI has ¥750 million in its trading portfolio on the close of business on a particular day. The current exchange rate of yen for dollars is ¥120.00/\$, or dollars for yen is \$0.00833, at the daily close. The volatility, or standard deviation ( $\sigma$ ), of daily percentage changes in the spot ¥/\$ exchange rate over the past year was 121.6 bp. The FI is interested in adverse moves – bad moves that will not occur more than 1 percent of the time, or 1 day in every 100. Calculate the one-day VAR and ES from this position.

The first step is to calculate the dollar value position:

$$\begin{aligned}\text{Dollar value of position} &= \text{yen value of position} \times \text{dollar for pound exchange rate} \\ &= ¥750 \text{ million} / 120.00 = \$6,250,000\end{aligned}$$

Using VAR, which assumes that changes in exchange rates are normally distributed, the exchange rate must change in the adverse direction by  $2.33\sigma$  ( $2.33 \times 121.6$  bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.33 \times 121.6 \text{ bp} = 283.328 \text{ bp}$$

In other words, using VAR during the last year the yen declined in value against the dollar by 283.328 bp 1 percent of the time. As a result, the one-day VAR is:

$$\text{VAR} = \$6,250,000 \times 0.0283328 = \$177,080$$

Using ES, which assumes that changes in exchange rates are normally distributed but with fat tails, the exchange rate must change in the adverse direction by  $2.665\sigma$  ( $2.665 \times 121.6$  bp) for this change to be viewed as likely to occur only 1 day in every 100 days:

$$\text{FX volatility} = 2.665 \times 121.6 \text{ bp} = 324.064 \text{ bp}$$

In other words, using ES during the last year the yen declined in value against the dollar by 324.064 bp 1 percent of the time. As a result, the one-day ES is:

$$\text{ES} = \$6,250,000 \times 0.0324064 = \$202,540$$

The potential loss exposure to adverse yen to dollar exchange rate changes for the FI from the ¥750 million spot currency holdings are higher using the ES measure of market risk. ES estimates potential losses that are \$25,460 higher than VAR. This is because VAR focuses on the location



of the extreme tail of the probability distribution. ES also considers the shape of the probability distribution once VAR is exceeded.

29. Bank of Hawaii's stock portfolio has a market value of \$250 million. The beta of the portfolio approximates the market portfolio, whose standard deviation ( $\sigma_m$ ) has been estimated at 2.25 percent. What are the five-day VAR and ES of this portfolio using adverse rate changes in the 99<sup>th</sup> percentile?

$$\begin{aligned}\text{Daily VAR} &= (\$ \text{ market value of portfolio}) \times (2.33 \times \sigma_m) = \$250\text{m} \times (2.33 \times 0.0225) \\ &= \$250\text{m} \times 0.052425 = \$13,106,250\end{aligned}$$

$$\text{5-day VAR} = \$13,106,250 \times \sqrt{5} = \$13,106,250 \times 2.2361 = \$29,306,466$$

$$\begin{aligned}\text{Daily ES} &= (\$ \text{ market value of portfolio}) \times (2.665 \times \sigma_m) = \$250\text{m} \times (2.665 \times 0.0225) \\ &= \$250\text{m} \times 0.0599625 = \$14,990,625\end{aligned}$$

$$\text{5-day ES} = \$14,990,625 \times \sqrt{5} = \$14,990,625 \times 2.2361 = \$33,520,057$$

30. Despite the fact that market risk capital requirements have been imposed on FIs since the 1990s, huge losses in value were recorded during the financial crisis from losses incurred in FIs' trading portfolios. Why did this happen? What changes to capital requirements did regulators propose to prevent such losses from reoccurring?

During the financial crisis, losses due to market risk were significantly higher than the minimum market risk capital requirements under BIS Basel I and Basel II rules. The financial crisis exposed a number of shortcomings in the way market risk was being measured in accordance with Basel II rules. Although the crisis largely exposed problems with the large-bank internal models approach to measuring market risk, the BIS also identified shortcomings with the standardized approach. These included a lack of risk sensitivity, a very limited recognition of hedging and diversification benefits, and an inability to sufficiently capture risks associated with more complex instruments.

To address shortcomings of the standardized approach to measuring market risk, Basel III proposes a "partial risk factor" approach as a revised standardized approach. To address shortcomings in the internal models approach, in addition to the risk capital charge already in place, an incremental capital charge is assessed which includes a "stressed value at risk" capital requirement taking into account a one year observation period of significant financial stress relevant to the FI's portfolio. The introduction of stressed VAR in Basel 2.5 is intended to reduce the cyclicity of the VAR measure and alleviate the problem of market stress periods dropping

out of the data period used to calculate VAR after some time. Basel 2.5 requires the following process be followed by large FIs using internal models to calculate the market risk capital charge.

Basel III proposes to replace VAR models with those based on Extreme Value Theory and Expected Shortfall (ES) (discussed above). The ES measure analyzes the size and likelihood of losses above the 99<sup>th</sup> percentile in a crisis period for a traded asset and thus measures “tail risk” more precisely. Thus, ES is a risk measure that considers a more comprehensive set of potential outcomes than VAR. The BIS change to ES highlights the importance of maintaining sufficient regulatory capital not only in stable market conditions, but also in periods of significant financial stress. Indeed, it is precisely during periods of stress that capital is vital for absorbing losses and safeguarding the stability of the banking system. Accordingly, the Committee intends to move to a framework that is calibrated to a period of significant financial stress. Two methods of identifying the stress period and calculating capital requirements under the internal models are the direct method and the indirect method. The direct method is based on the approach used in the Basel 2.5 stressed VAR. The FI would search the entire historical period and identify the period which produces the highest ES result when all risk factors are included. However Basel III would require the FI to determine the stressed period on the basis of a reduced set of risk factors. Once the FI has identified the stressed period, it must then determine the ES for the full set of risk factors for the stress period. The indirect method identifies the relevant historical period of stress by using a reduced set of risk factors. However, instead of calculating the full ES model to that period the FI calculates a loss based on the reduced set of risk factors. This loss is then scaled using the ratio of the full ES model using current market data to the full ES model using the reduced set of risk factors using current market data.

31. In its trading portfolio, an FI holds 2,000 shares of Under Armour at a share price of \$84.50, 3,000 shares of BT Group (British telecommunication firm) at a price of \$68.50, and 4,500 shares of AT&T at \$34.40. The FI also has sold 5,000 Swatch Group (a Swiss watchmaker) at \$19.00 and 2,500 of AT&T shares. Calculate the market risk capital charge on these securities.

### **Step 1: Offset equities in the same equity name**

Long and short notional positions in the same equity name are allowed offset each other. So, the 2,500 shares of AT&T sold by the FI are offset against the 4,500 shares long to leave 1,500 shares to be assigned to a bucket

### **Step 2: Place the net position in each equity name into the relevant risk bucket**

Net notional positions in each equity name are then assigned to the appropriate equity bucket based on their observable characteristics, according to the following table:

Bucket Number	Size	Region	Sector
1	Large	Emerging markets	Consumer, Utilities
2			Telecommunications, Industrials
3			Basic materials, Energy
4			Financial, Technology
5		Developed markets	Consumer, Utilities
6			Telecommunications, Industrials
7			Basic materials, Energy
8			Financial, Technology
9	Small	Emerging markets	All sectors
10		Developed markets	All sectors

Accordingly, the shares of Under Armour and Swatch Group will be assigned to bucket 5 and shares of AT&T and BT Group will fall in bucket 6. Using the BIS assigned risk weights to each notional position as follows:

Bucket number	Risk weight
1	55%
2	60%
3	45%
4	55%
5	30%
6	35%
7	40%
8	50%
9	70%
10	50%
Residual bucket	70%

So, the shares of Under Armour and Swatch Group will have a risk weight of 30%, AT&T and BT Group will have a risk weight of 35%. Further, the BIS has set these correlations for equity risk as follows:

Bucket number	Same sign	Different sign
1	20%	10%
2	20%	15%
3	25%	15%
4	30%	20%
5	20%	10%
6	30%	15%
7	35%	20%
8	35%	20%
9	15%	5%
10	25%	10%
Residual bucket	100%	0%

Thus, Under Armour (long position) and Swatch Group (short position) will have a correlation,  $\rho_{\text{US}}$ , of 10% and AT&T (long position) and BT Group (long position) will have a correlation,  $\rho_{\text{AT}}$ ,

of 30%. And the risk exposure from the two bucket 5 securities and the two bucket 6 securities is calculated as:

$$K_5 = [(0.30)^2 (2,000 \times \$84.50)^2 + (0.30)^2 (5,000 \times \$19.00)^2 + 2(0.10)(0.30)(2,000 \times \$84.50)(0.30)(5,000 \times \$19.00)]^{1/2} = \$60,594.80$$

$$K_6 = [(0.35)^2 (1,500 \times \$34.40)^2 + (0.35)^2 (3,000 \times \$68.50)^2 + 2(0.30)(0.35)(1,500 \times \$34.40)(0.35)(3,000 \times \$68.50)]^{1/2} = \$79,238.55$$

### Step 3: Aggregate the buckets

The risk exposures for each of the individual risk buckets are then aggregated to obtain the capital requirement for equity risk. The BIS has set the correlations between buckets as follows:

Buckets	1	2	3	4	5	6	7	8	9	10
1										
2	15%									
3	15%	15%								
4	15%	15%	15%							
5	10%	10%	10%	10%						
6	10%	10%	10%	10%	20%					
7	10%	10%	10%	10%	20%	20%				
8	10%	10%	10%	10%	20%	20%	20%			
9	10%	10%	10%	10%	10%	10%	10%	10%		
10	10%	10%	10%	10%	15%	15%	15%	15%	10%	

Thus, the Equity Risk Capital for this FI is:

$$\text{Equity Risk Capital} = \{ [(\$60,594.80)^2 + (\$79,238.55)^2 + 2(0.20)[(0.30)(2,000 \times \$84.50) + (0.30)(5,000 \times \$19.00)][(0.35)(1,500 \times \$34.40) + (0.35)(3,000 \times \$68.50)] \}^{1/2} = \$113,142.40$$

32. In its trading portfolio, an FI holds 5,000 shares of HSBC at a share price of \$43.50, 4,000 shares of China Construction Bank at a price of \$16.40, and 2,500 shares of Whirlpool at \$170.00. The FI also has sold 3,000 McDonald's at \$96.75 and 1,500 of China Construction Bank. Calculate the market risk capital charge on these securities.

### Step 1: Offset equities in the same equity name

Long and short notional positions in the same equity name are allowed offset each other. So, the 1,500 shares of China Construction Bank sold by the FI are offset against the 4,000 shares long to leave 2,500 shares to be assigned to a bucket

### Step 2: Place the net position in each equity name into the relevant risk bucket

Net notional positions in each equity name are then assigned to the appropriate equity bucket based on their observable characteristics, according to the following table:

Bucket Number	Size	Region	Sector
1	Large	Emerging markets	Consumer, Utilities
2			Telecommunications, Industrials
3			Basic materials, Energy
4			Financial, Technology
5		Developed markets	Consumer, Utilities
6			Telecommunications, Industrials
7			Basic materials, Energy
8			Financial, Technology
9	Small	Emerging markets	All sectors
10		Developed markets	All sectors

Accordingly, the shares of Whirlpool and McDonald's will be assigned to bucket 5 and shares of HSBC and China Construction Bank will fall in bucket 8. Using the BIS assigned risk weights to each notional position as follows:

Bucket number	Risk weight
1	55%
2	60%
3	45%
4	55%
5	30%
6	35%
7	40%
8	50%
9	70%
10	50%
Residual bucket	70%

So, the shares of Whirlpool and McDonald's will have a risk weight of 30%, HSBC and China Construction Bank will have a risk weight of 50%. Further, the BIS has set these correlations for equity risk as follows:

Bucket number	Same sign	Different sign
1	20%	10%
2	20%	15%
3	25%	15%
4	30%	20%
5	20%	10%
6	30%	15%
7	35%	20%
8	35%	20%
9	15%	5%
10	25%	10%
Residual bucket	100%	0%

Thus, Whirlpool (long position) and McDonald's (short position) will have a correlation,  $\rho_{\text{WS}}$ , of 10% and HSBC (long position) and China Construction Bank (long position) will have a

correlation,  $\rho_{56}$ , of 35%. And the risk exposure from the two bucket 5 securities and the two bucket 6 securities is calculated as:

$$K_5 = [(0.30)^2 (2,500 \times \$170.00)^2 + (0.30)^2 (3,000 \times \$96.75)^2 + 2(0.10)(0.30)(2,500 \times \$170.00)(0.30)(3,000 \times \$96.75)]^{1/2} = \$161,427.13$$

$$K_8 = [(0.50)^2 (2,500 \times \$16.40)^2 + (0.50)^2 (5,000 \times \$43.50)^2 + 2(0.35)(0.50)(2,500 \times \$16.40)(0.50)(5,000 \times \$43.50)]^{1/2} = \$117,504.79$$

### Step 3: Aggregate the buckets

The risk exposures for each of the individual risk buckets are then aggregated to obtain the capital requirement for equity risk. The BIS has set the correlations between buckets as follows:

Buckets	1	2	3	4	5	6	7	8	9	10
1										
2	15%									
3	15%	15%								
4	15%	15%	15%							
5	10%	10%	10%	10%						
6	10%	10%	10%	10%	20%					
7	10%	10%	10%	10%	20%	20%				
8	10%	10%	10%	10%	20%	20%	20%			
9	10%	10%	10%	10%	10%	10%	10%	10%		
10	10%	10%	10%	10%	15%	15%	15%	15%	10%	

Thus, the Equity Risk Capital for this FI is:

$$\text{Equity Risk Capital} = \{ [(\$161,427.13)^2 + (\$117,504.79)^2 + 2(0.20)[(0.30)(2,500 \times \$170.00) + (0.30)(3,000 \times \$96.75)][(0.50)(2,500 \times \$16.40) + (0.50)(5,000 \times \$43.50)] \}^{1/2} = \$225,742.38$$

33. Suppose an FI's portfolio VAR for the previous 60 days was \$3 million and stressed VAR for the previous 60 days was \$8 million using the 1 percent worst case (or 99<sup>th</sup> percentile). Calculate the minimum capital charge for market risk for this FI.

$$\text{Capital charge} = (\$3 \text{ million} \times \sqrt{10} \times 3) + (\$8 \text{ million} \times \sqrt{10} \times 3) = \$104.355 \text{ million}$$

### **Integrated Mini Case: Calculating DEAR on an FI's Trading Portfolio**

An FI wants to obtain the DEAR on its trading portfolio. The portfolio consists of the following securities.

#### *Fixed-income securities:*

- i) The FI has a \$1 million position in a six-year, zero bonds with a face value of \$1,543,302. The bond is trading at a yield to maturity of 7.50 percent. The historical mean change in daily yields is 0.0 percent, and the standard deviation is 22 basis points.
- ii) The FI also holds a 12-year, zero bond with a face value of \$1,000,000. The bond is trading at a yield to maturity of 6.75 percent. The price volatility of the potential adverse move in yields is 65 basis points.

#### *Foreign exchange contracts:*

The FI has a €2.0 million long trading position in spot euros at the close of business on a particular day. The exchange rate is €0.80/\$1, or \$1.25/€, at the daily close. Looking back at the daily percentage changes in the exchange rate of the euro to dollars for the past year, the FI finds that the volatility or standard deviation ( $\sigma$ ) of the spot exchange rate was 55.5 basis points (bp).

#### *Equities:*

The FI holds a \$2.5 million trading position in stocks that reflect the U.S. stock market index (e.g., the S&P 500). The  $\beta = 1$ . Over the last year, the standard deviation of the stock market index was 175 basis points.

Correlations ( $\rho_{ij}$ ) among Assets

	Six-Year, Zero-Coupon	12-Year, Zero-Coupon	€/\$	U.S. Stock Index
Six-Year, Zero-Coupon	-	0.75	-0.2	0.40
12-Year, Zero-Coupon	-	-	-0.3	0.45
€/\$	-	-	-	0.25
U.S. Stock Index	-	-	-	-

1. Calculate the DEAR of this trading portfolio.

*Fixed-income securities:*

i)  $MD = D/(1 + R) = 6/(1.075) = 5.581395$

=> Potential adverse move in yield at 1 percent =  $2.33\sigma = 2.33 \times 0.0022 = 0.005126$

=> Price volatility =  $MD \times \text{potential adverse move in yield}$   
 $= 5.581395 \times 0.005126 = 0.02861$  or 2.861 percent

and the daily earnings at risk for this bond is:

$$\begin{aligned} \text{DEAR} &= (\$ \text{ value of position}) \times (\text{price volatility}) \\ &= \$1,000,000 \times 0.02861 = \$28,610 \end{aligned}$$

ii) Dollar value of position =  $\$1\text{m.}/(1 + 0.0675)^{12} = \$456,652$ . The modified duration of these bonds is:

$$\begin{aligned} MD &= D/(1 + R) = 12/(1.0675) = 11.24122 \\ \Rightarrow \text{Price volatility} &= (MD) \times (\text{potential adverse move in yield}) \\ &= (11.24122) \times (0.0065) = 0.073068 \text{ or } 7.3068 \text{ percent.} \\ \Rightarrow \text{DEAR} &= \$456,652 \times 0.073068 = \$33,367 \end{aligned}$$

*Foreign exchange contracts:*

$$\begin{aligned} \text{Dollar equivalent value of € position} &= \text{FX position} \times (\$/\text{€ spot exchange rate}) \\ &= \text{€2.0 million} \times \$ \text{ per unit of foreign currency} \end{aligned}$$

$$\begin{aligned} \text{Dollar equivalent value of € position} &= \text{€2.0 million} \times \$1.25/\text{€} \\ &= \$2,500,000 \end{aligned}$$



FX volatility =  $2.33 \times 55.5 \text{ bp} = 129.315 \text{ bp}$  or  $1.29315\%$   
 $\Rightarrow \text{DEAR} = \text{Dollar value of DM position} \times \text{FX volatility}$   
 $= \$2,500,000 \times 0.0129315$   
 $= \$32,329$

*Equities:*

Stock market return volatility =  $2.33 \sigma_m = 2.33 \times 175 \text{ bp} = 0.040775 = 4.0775\%$   
 $\Rightarrow \text{DEAR} = \text{Dollar market value of position} \times \text{Stock market return volatility}$   
 $= \$2,500,000 \times 0.040775 = \$101,937$

*Portfolio DEAR:*

Using the correlation matrix along with the individual asset DEARs the risk (or standard deviation) of the whole (four-asset) trading portfolio is:

$$\begin{aligned} \text{DEAR portfolio} &= (\text{DEAR}_{z6})^2 + (\text{DEAR}_{z12})^2 + (\text{DEAR}_{\epsilon})^2 + (\text{DEAR}_{US})^2 \\ &+ (2 \times \rho_{z6,z12} \times \text{DEAR}_{z6} \times \text{DEAR}_{z12}) + (2 \times \rho_{z6,\epsilon} \times \text{DEAR}_{z6} \times \text{DEAR}_{\epsilon}) \\ &+ (2 \times \rho_{z6,US} \times \text{DEAR}_{z6} \times \text{DEAR}_{US}) + (2 \times \rho_{z12,\epsilon} \times \text{DEAR}_{z12} \times \text{DEAR}_{\epsilon}) \\ &+ (2 \times \rho_{z12,US} \times \text{DEAR}_{z12} \times \text{DEAR}_{US}) + (2 \times \rho_{\epsilon,US} \times \text{DEAR}_{\epsilon} \times \text{DEAR}_{US}) \\ &= [(28,610)^2 + (33,367)^2 + (32,329)^2 + (101,937)^2 + 2(0.75)(28,610)(33,367) \\ &\quad + 2(-0.2)(28,610)(32,329) + 2(0.4)(28,610)(101,937) + 2(-0.3)(33,367)(32,329) \\ &\quad + 2(0.45)(33,367)(101,937) + 2(0.25)(32,329)(101,937)]^2 \\ &= \$144,309 \end{aligned}$$

2. If the correlation matrix changes as follows, what will the FI's DEAR be? Explain the change in DEAR.

	Six-year zero-coupon	12-year zero-coupon	€/\$	U.S. stock index
Six-year, zero-coupon	-	0.85	-0.1	0.50
12-year, zero-coupon	-	-	-0.2	0.55
€/\$	-	-	-	0.35
U.S. stock index	-	-	-	-

$$\begin{aligned} \text{DEAR portfolio} &= [(28,610)^2 + (33,367)^2 + (32,329)^2 + (101,937)^2 + 2(0.85)(28,610)(33,367) \\ &\quad + 2(-0.1)(28,610)(32,329) + 2(0.5)(28,610)(101,937) + 2(-0.2)(33,367)(32,329) \\ &\quad + 2(0.55)(33,367)(101,937) + 2(0.35)(32,329)(101,937)]^2 \end{aligned}$$

$$= \$152,772$$

The DEAR increases because the correlation coefficients all increased. Thus, less of the individual security risk was diversified through portfolio formation.

3. If the standard deviation of the stock market index increases to 325 basis points, what will the FI's DEAR be? Use the original correlation matrix. Explain the change in DEAR.

$$\begin{aligned}\text{Stock market return volatility} &= 2.33 \sigma_m = 2.33 \times 325 \text{ bp} = 0.075725 = 7.5725\% \\ \Rightarrow \text{DEAR} &= \text{Dollar market value of position} \times \text{Stock market return volatility} \\ &= \$2,500,000 \times 0.075725 = \$189,312\end{aligned}$$

*Portfolio DEAR:*

Using the correlation matrix along with the individual asset DEARs the risk (or standard deviation) of the whole (four-asset) trading portfolio is:

$$\begin{aligned}\text{DEAR portfolio} &= [(28,610)^2 + (33,367)^2 + (32,329)^2 + (189,312)^2 + 2(0.75)(28,610)(33,367) \\ &\quad + 2(-0.2)(28,610)(32,329) + 2(0.4)(28,610)(189,312) + 2(-0.3)(33,367)(32,329) \\ &\quad + 2(0.45)(33,367)(189,312) + 2(0.25)(32,329)(189,312)]^{1/2} \\ &= \$228,712\end{aligned}$$

The DEAR increases because the risk of the equity portion of the portfolio increased.

4. If the FI's FX position were changed to €4.0 million, what will the FI's DEAR be? Explain the change in DEAR. Use the original correlation matrix and the original standard deviation of the stock market index.

$$\begin{aligned}\text{Dollar equivalent value of € position} &= \text{FX position} \times (\$/\text{€ spot exchange rate}) \\ &= \text{€}4.0 \text{ million} \times \$ \text{ per unit of foreign currency} \\ \text{Dollar value of € position} &= \text{€}4.0 \text{ million} \times \$1.25/\text{€} \\ &= \$5,000,000\end{aligned}$$

$$\begin{aligned}\text{FX volatility} &= 2.33 \times 55.5 \text{ bp} = 129.315 \text{ bp or } 1.29315\% \\ \Rightarrow \text{DEAR} &= \text{Dollar value of DM position} \times \text{FX volatility} \\ &= \$5,000,000 \times 0.0129315 \\ &= \$64,658\end{aligned}$$

$$\begin{aligned}\text{DEAR portfolio} &= [(28,610)^2 + (33,367)^2 + (64,658)^2 + (101,937)^2 + 2(0.75)(28,610)(33,367) \\ &\quad + 2(-0.2)(28,610)(64,658) + 2(0.4)(28,610)(101,937) + 2(-0.3)(33,367)(64,658) \\ &\quad + 2(0.45)(33,367)(101,937) + 2(0.25)(64,658)(101,937)]^{1/2}\end{aligned}$$

= \$156,815

The DEAR increases because the risk of the FX portion of the portfolio increased.