SM **6.** T. Haavelmo devised a model of the US economy for the years 1929–1941 based on the following equations:

$$\text{(i)}\ c = 0.712y + 95.05 \qquad \text{(ii)}\ s = 0.158(c + x) - 34.30$$
$$\text{(iii)}\ y = c + x - s \qquad \text{(iv)}\ x = 93.53$$

Here x denotes total investment, y is disposable income, s is the total saving by firms, and c is total consumption. Write the system of equations in the form (1) when the variables appear in the order x, y, s, and c. Then find the solution of the system.

5. Consider the three matrices $\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 1 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$, and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) Find a matrix $\mathbf{C}$ satisfying $(\mathbf{A} - 2\mathbf{I})\mathbf{C} = \mathbf{I}$.

(b) Is there a matrix $\mathbf{D}$ satisfying $(\mathbf{B} - 2\mathbf{I})\mathbf{D} = \mathbf{I}$?

SM **9.** (a) Consider the following linked macroeconomic model of two nations, $i = 1, 2$, that trade only with each other:

$$Y_1 = C_1 + A_1 + X_1 - M_1; \qquad C_1 = c_1 Y_1; \qquad M_1 = m_1 Y_1 = X_2$$
$$Y_2 = C_2 + A_2 + X_2 - M_2; \qquad C_2 = c_2 Y_2; \qquad M_2 = m_2 Y_2 = X_1$$

Here, for $i = 1, 2$, Y_i is income, C_i is consumption, A_i is (exogenous) autonomous expenditure, X_i denotes exports, and M_i denotes imports of country i. Interpret the two equations $M_1 = X_2$ and $M_2 = X_1$.

(b) Given the system of 8 equations in 8 unknowns in part (a), use substitution to reduce it to a pair of simultaneous equations in the endogenous variables Y_1 and Y_2. Then solve for the equilibrium values of Y_1, Y_2 as functions of the exogenous variables A_1, A_2.

(c) How does an increase in A_1 affect Y_2? Interpret your answer.

SM **8.** Consider the simple macro model described by the three equations

$$\text{(i)}\ Y = C + A_0 \qquad \text{(ii)}\ C = a + b(Y - T) \qquad \text{(iii)}\ T = d + tY$$

where Y is income, C is consumption, T is tax revenue, A_0 is the constant (exogenous) autonomous expenditure, and a, b, d, and t are all positive parameters. Find the equilibrium values of the endogenous variables Y, C, and T by: (A) successive elimination or substitution; (B) writing the equations in matrix form and applying Cramer's rule.

2. Calculate the following determinants:

(a) $\begin{vmatrix} 0 & 0 & a \\ 0 & b & 0 \\ c & 0 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 3 & 1 & 2 \\ 0 & 4 & 0 & 3 & 4 \\ 6 & 2 & 3 & 1 & 2 \end{vmatrix}$

3. Let the IS equation be

$$Y = \frac{A}{1-b} - \frac{g}{1-b}i$$

where $1 - b$ is the marginal propensity to save, g is the investment sensitivity to interest rates, and A is an aggregate of exogenous variables. Let the LM equation be

$$Y = \frac{M_0}{k} + \frac{l}{k}i$$

where k and l are income and interest sensitivity of money demand, respectively, and M_0 is real money balances.

If $b = 0.7$, $g = 100$, $A = 252$, $k = 0.25$, $l = 200$, and $M_0 = 176$, then

(a) Write the IS-LM system in matrix form.

(b) Solve for Y and i by matrix inversion.

Question 3

Assume that we have three markets in the economy. Each market can be characterized by demand and supply equations as given below.

Demand for goods A : $q_A^d = 3 - P_A + P_B$ Supply for goods A : $q_A^s = P_A - 2$
Demand for goods B : $q_B^d = 3 - 2P_B + P_C$ Supply for goods B : $q_B^s = P_B - 1$
Demand for goods C : $q_C^d = 6 + 2P_A - P_C$ Supply for goods C : $q_C^s = 2P_C - 2$

a. State the equilibrium conditions for this multi-market economy, and write the model in the form of matrix.
b. Solve for the equilibrium solution by using the *Cramer's rule*.

Question 5: *Tax and revenue.*

Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by $t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

a. (1 points) Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.
b. (1 points) State the condition that links between consumer's and producer's price.
c. (2 points) Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)

d. (2 points) Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?
e. (2 points) Find the expression of the revenue that the government can collect from the market under the equilibrium.
f. (2 points) If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?