



Practice problem set 8

Semester 1/2016

Integration and its application in economics.

Question 1:

In the manufacture of a product, the marginal cost of producing x units is $C'(x)$ and fixed cost are $C(0)$. Find the total cost function $C(x)$ when:

- a. $C'(x) = 3x + 4$, $C(0) = 40$.
- b. $C'(x) = ax + b$, $C(0) = C_0$.

Question 2

Let $K(t)$ denote the capital stock of an economy at time t . Then net investment at time t , denoted by $I(t)$, is given by the rate of increase $\frac{dK}{dt}$ of $K(t)$.

- a. If $I(t) = 3t^2 + 2t + 5$, $t \geq 0$, what is the total increase in the capital stock during the interval from $t = 0$ to $t = 5$?
- b. If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

Question 3:

Given the following demand and supply curves, compute the consumer and producer surplus.

a. Demand: $P = 200 - 0.2Q$; Supply: $P = 20 + 0.1Q$.

b. Demand: $P = \frac{6000}{Q+50}$; Supply: $P = Q + 10$.

Question 4

Suppose that the profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, \quad x > 0$$

- Find the level of output that maximizes profit. Sketch the graph of f .
- The actual output varies between 1000 and 3000 units. Compute the

average profit $I = \frac{1}{2000} \int_{1000}^{3000} f(x) dx$.

Question 5:

Evaluate the following integrals by using integrations by substitution:

a. $\int_0^1 x\sqrt{1+x^2} dx$

b. $\int_1^e \frac{\ln y}{y} dy$

Question 6:

Evaluate the following integrals by using integrations by parts ($r \neq 0$).

a. $\int_0^T bte^{-rt} dt$

b. $\int_0^T (a + bt)e^{-rt} dt$

Question 7:

- a. Evaluate $\int_0^1 x^p (x^q + x^r + x^s) dx$ where $p, q, r, \text{ and } s$ are positive numbers.
- b. Let $F(x) = \int_0^x (t^2 + 2) dt$ and $G(x) = \int_0^{x^2} (t^2 + 2) dt$. Find $F'(x)$ and $G'(x)$.

Question 8:

Let the demand and supply of goods Q in a perfectly competitive market be the followings;

$$\text{Demand Function : } P = 25 - Q^2$$

$$\text{Supply Function : } P = 2Q + 1$$

- a. Determine the consumer surplus as the equilibrium.
- b. If the government imposes tax on consumers for \$4 per unit of production, calculate the deadweight loss.

Question 9:

Let $P = 274 - Q^2$ be the demand function in a monopoly market. Suppose further that marginal cost of the monopolist is given by $MC = 4 + 3Q$.

- a. Determine consumer's surplus at the profit-maximizing production level.
- b. Calculate deadweight loss under monopoly.

Question 10:

A company has $MC = 80$ where the demand function is $P = 1400 - 6Q$. At zero production, the company faces loss by \$1,500. Determine the maximum profit by using integral calculus and prove your answer.

