Exercise 2

Part I (*Derivative Definition, Power/Sum/Product/Quotient Rules*) .

1. Find the derivative $f'(x)$ of the following functions $f(x)$ by using the definition of the derivative:
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$
   (a) $f(x) = 2x^2 + x + 1$
   (b) $f(x) = \cos(x)$
   Hint: Use the identity: $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, and the fact that $\lim\limits_{h\to 0} \frac{\sin(h)}{h} = 1, \lim\limits_{h\to 0} \frac{\cos(h)-1}{h} = 0$.
2. Differentiate each of the following functions to find $f'(x)$.
   (a) $f(x) = \pi^{15}$
   (b) $f(x) = x^8 - \frac{1}{3x^3} + \sqrt{x^3} + \pi$
   (c) $f(x) = \left(x^2 + x + 1\right)\left(x^8 - x^4 + \pi\right)$
   (d) $f(x) = \frac{x^8 - x^4 + \pi}{x^2 + x + 1}$
3. Determine every point x on the graph $f(x) = \frac{1}{3}x^3 - 16x$ where the tangent line is horizontal.

Part II (*Derivatives of trigonometric functions, Chain Rule, Implicit Differentiation*) .

1. Find $f'(x)$ for the following functions.
   (a) $f(x) = 3\sin(x) + \tan(x)$
   (b) $f(x) = \frac{\cos(x)\cot(x)}{x + \csc(x)}$
2. Use the Chain Rule to find the derivatives $f'(x)$ of the functions below.
   (a) $f(x) = (x^3 - 1)(x^2 + 1)^{100}$
   (b) $f(x) = \sin^3(\cos(\sqrt{x^3 - 2}))$
3. Use *implicit differentiation* to find $\frac{dy}{dx}$ for the graphs of the following equations.
   (a) $y^3 - 2y = x + \sec(3y)$
   (b) $\frac{x+y}{x-y} = x^3$
4. Find $\frac{dy}{dx}$ for $y = \sin(xy)$ at the point $(x, y) = (\pi/2, 1)$.
5. Find an equation of the tangent line to the graph of $\tan(y) = x$ at $y = \pi/4$.
6. Find an equation of the normal line to the graph of $x^4 + y^3 = 24$ at $(-2, 2)$.
7. Find the point(s) on the graph of $x^2 + y^2 = 8$ at which the slope of the tangent is 1.
8. Find the point(s) on the graph of $x^2 - xy + y^2 = 3$ where the tangent line is horizontal.
9. Find the point(s) on the graph of $x^2 - xy + y^2 = 27$ at which the tangent line is parallel to the the line $y = 1$.
10. Find $\frac{d^2y}{dx^2}$ for
   (a) $x^2 - y^2 = 25$
   (b) $x + y = \sin(y)$.