EE312 Macroeconomic Theory

Chapter 7-8 (Williamson)

The Long-term Economic Growth

From business cycles *to* long-term prosperity

Overview

- The standards of living in the long term depend on ***economic growth***.
  - *Short-run fluctuations (boom and bust) tend to cancel out in the long run;* average long-term growth rate matter a lot!
- Growth outcome has markedly differed across countries; **what determines the long-term economic growth, then?**
- Differences in growth outcome have also contributed to the disparity of income-per-capita across countries: **rich v.s. poor**
  - Distribution of income distribution within country and across countries

Overview

- In this section, we focus on academic explanations towards the determinants and process of long-term growth
  - **Models of economic growth**

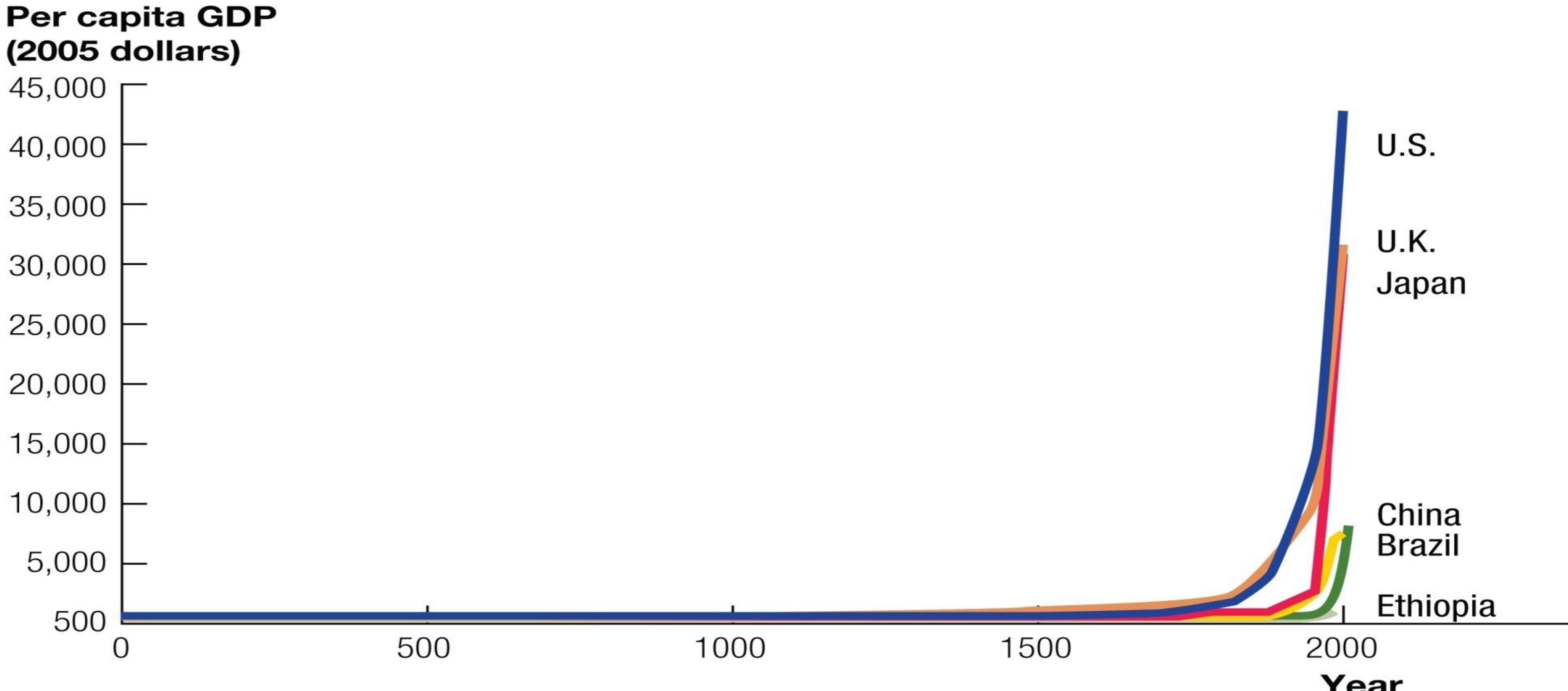
- We begin with documenting some stylized facts of historical growth over several past decades, and draw some issues that economists have aimed to explain

Agenda

- **Long-term growth stylized-facts**
- **Solow growth model**
- **How well does the Solow growth model explain the stylized-facts?**
- **Measuring source of growth**

Some stylized-facts about growth

1. Before the Industrial Revolution (1800), standards of living *differed little over time* and across regions.
2. Since *the Industrial Revolution*, per capita income growth has been sustained in *developed countries.*



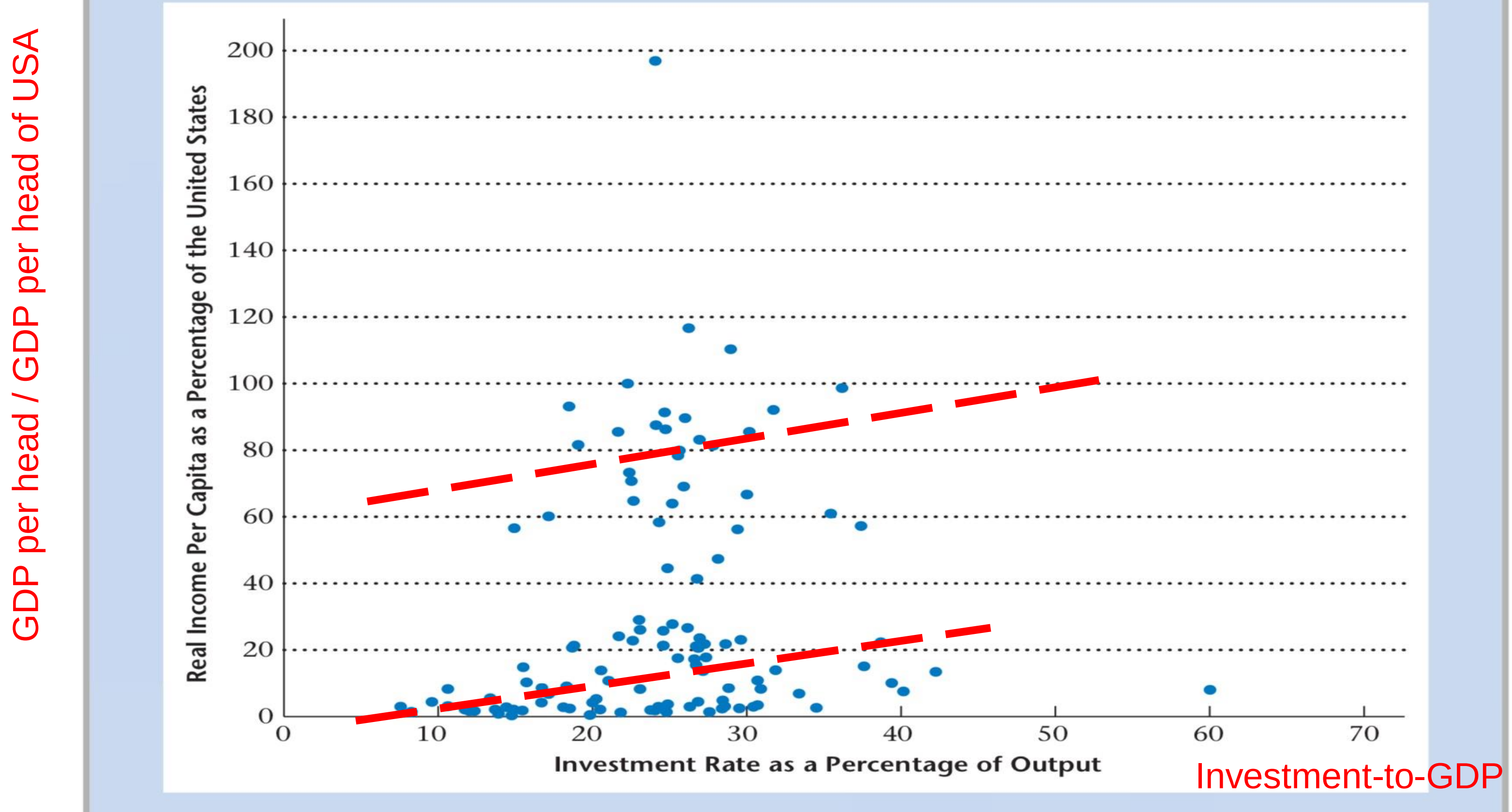
Some stylized-facts about growth

3. The higher the rate of investment, the higher output per worker.

Figure 7.2 Real Income Per Capita vs. Investment Rate

The figure shows a positive correlation across the countries of the world, between the output per capita and the investment rate.

Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.

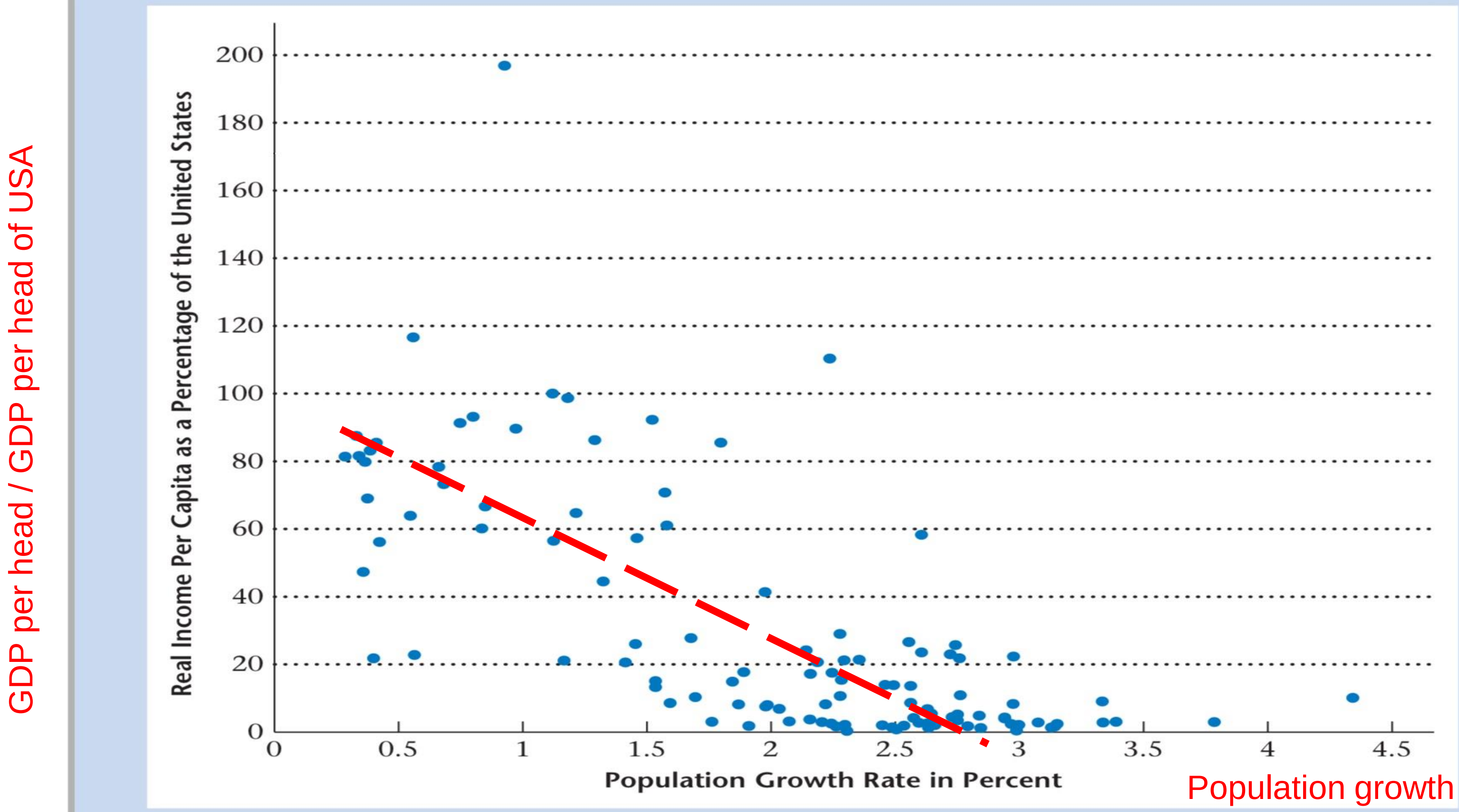


Some stylized-facts about growth

4. High population growth corresponds with low living standards.

Figure 7.3 Real Per Capita Income vs. the Population Growth Rate

Across the countries in the world, real per capita income and the population growth rate are negatively correlated.



Some stylized-facts about growth

5. Huge international ***differences in living standards increasingly and persistently widen*** between rich and poor countries

Poverty in India

Affluent society in developed countries.

Group	Income range	Example
High income (79)	> $12,235	North America, Western Europe, East Asia, oil-rich Middle East.
Upper middle income (55)	$3,956 - $12,235	South America, Eastern Europe, Russia, South-East Asia.
Lower middle income (52)	$1,006 - $3,955	South America, Central Asia, Africa.
Low income (31)	< $1,006	Central and East Africa
World average	10,302	

Some stylized-facts about growth

5. **Differences in living standards** ***have been increasingly and persistently widen*** between rich and poor countries

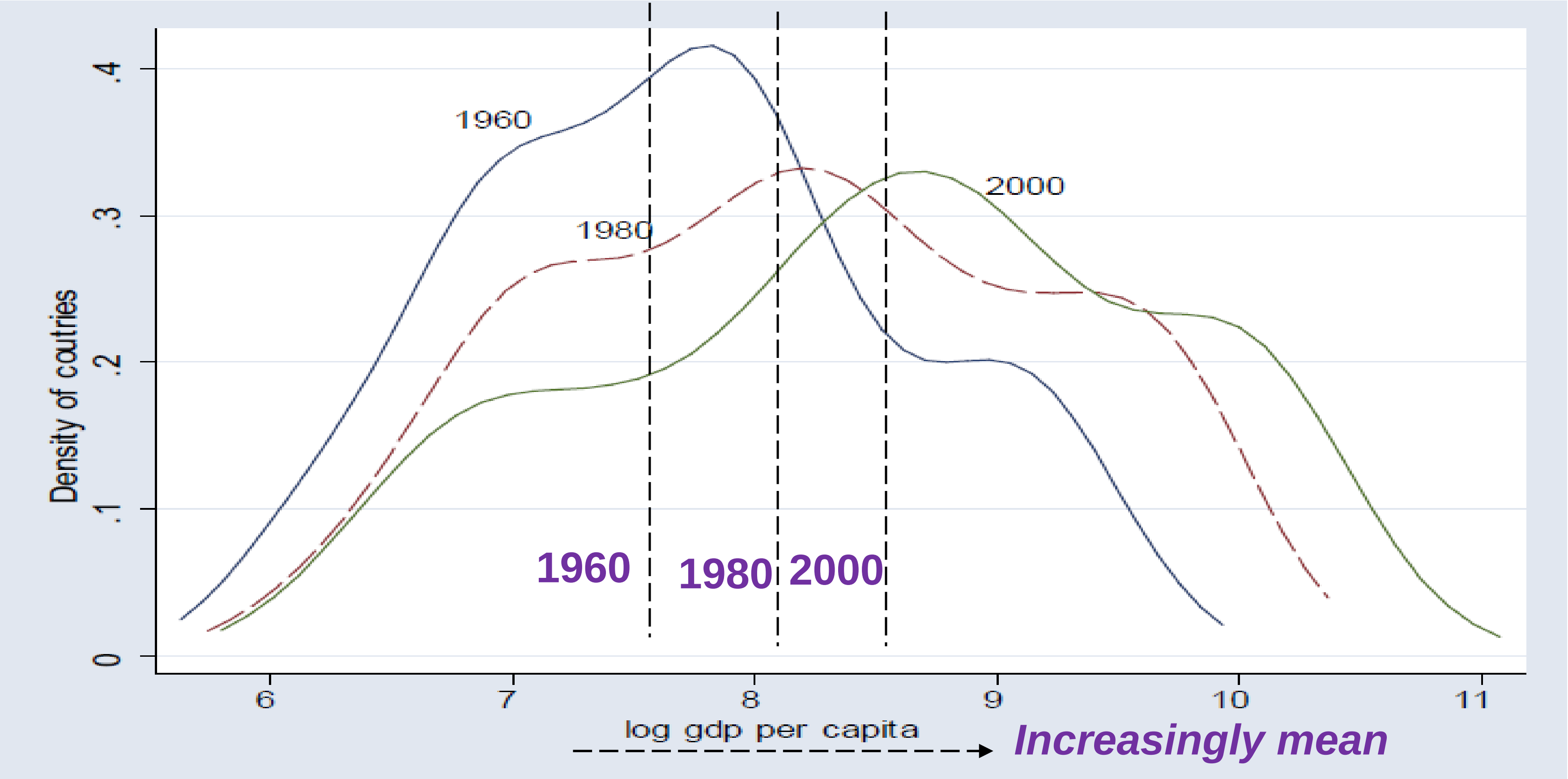
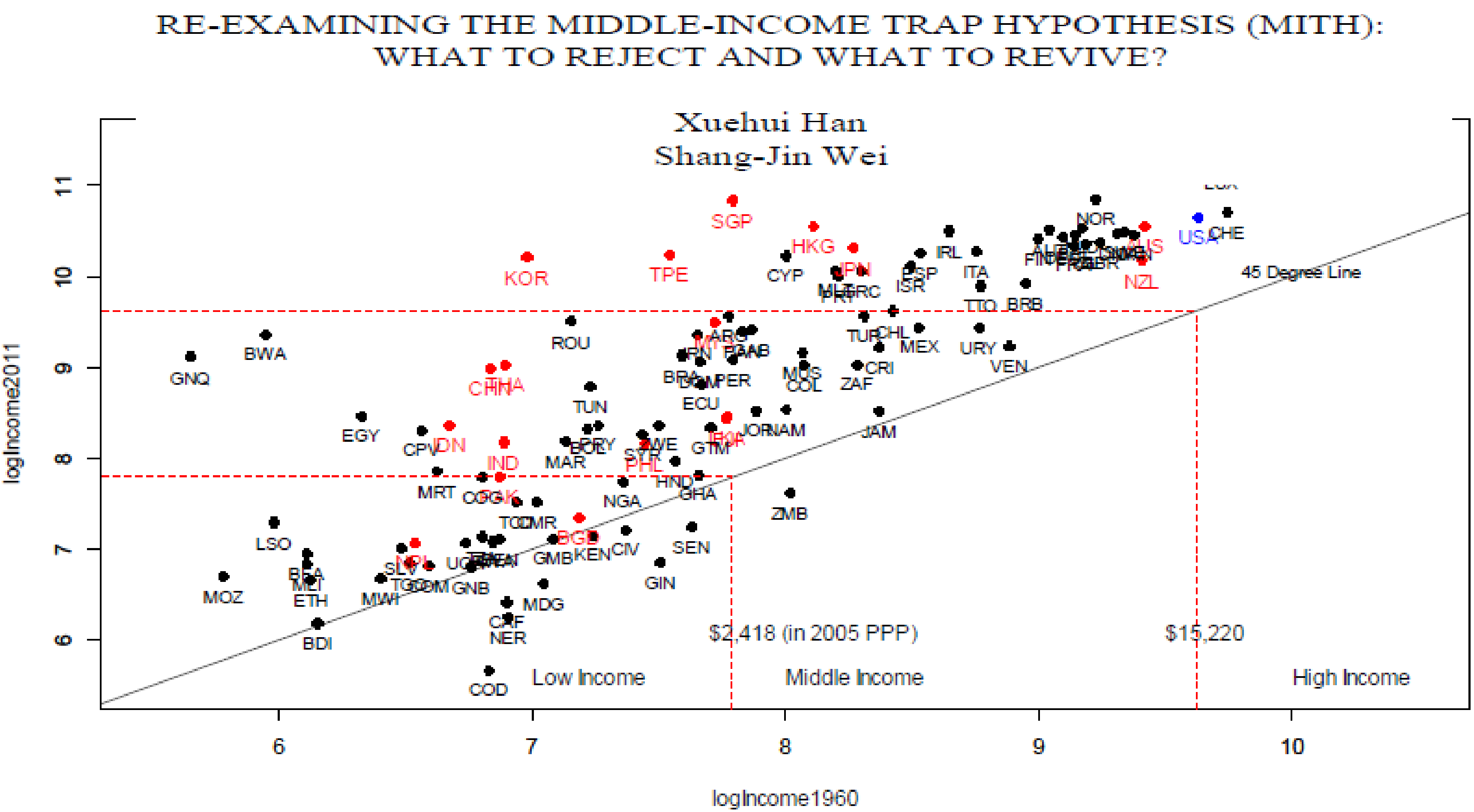


Figure: Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

Some stylized-facts about growth

6. Historical transition suggests that some have been successful in closing up the gap, and hence achieving **a convergence**. Those countries had experienced a miraculous growth path over the past several decades



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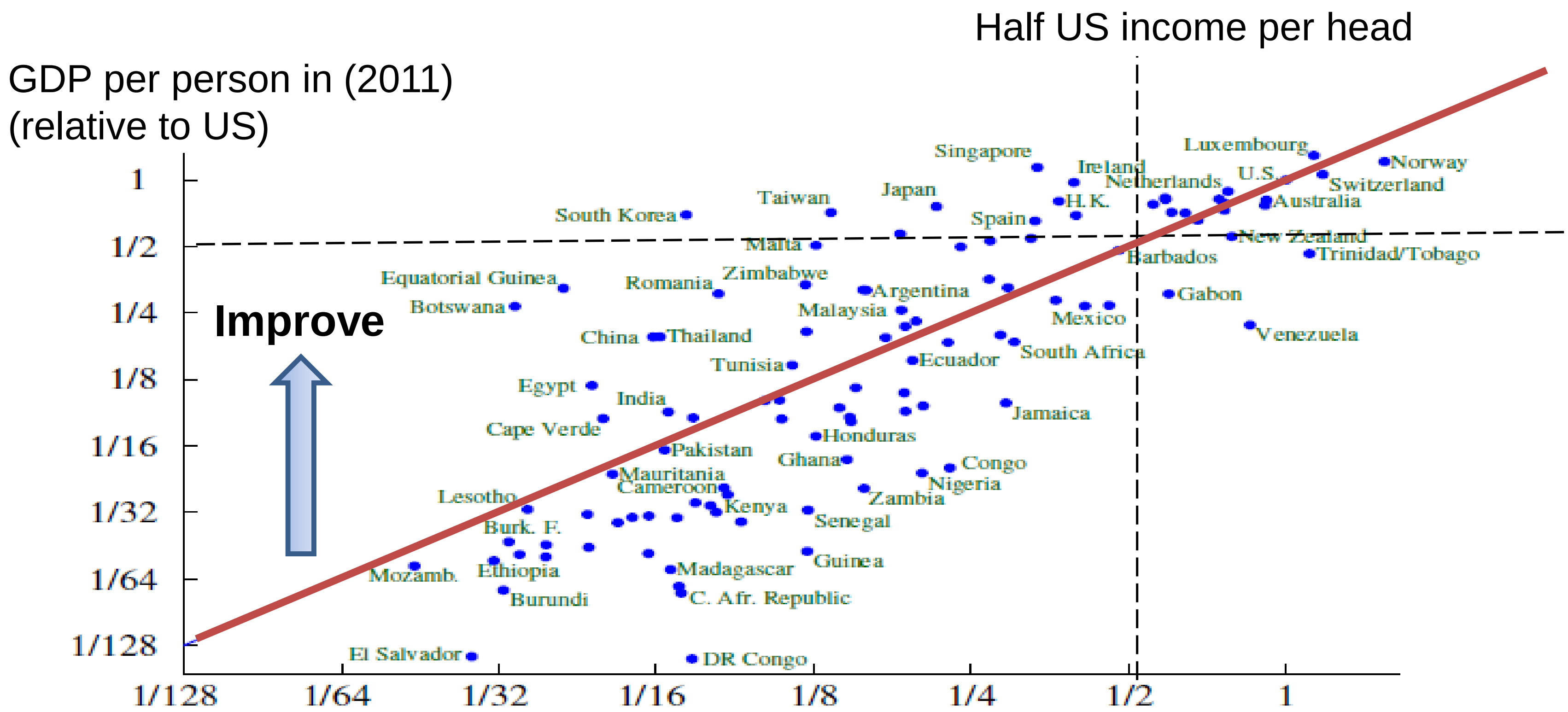

Source: The Penn World Tables 8.0.

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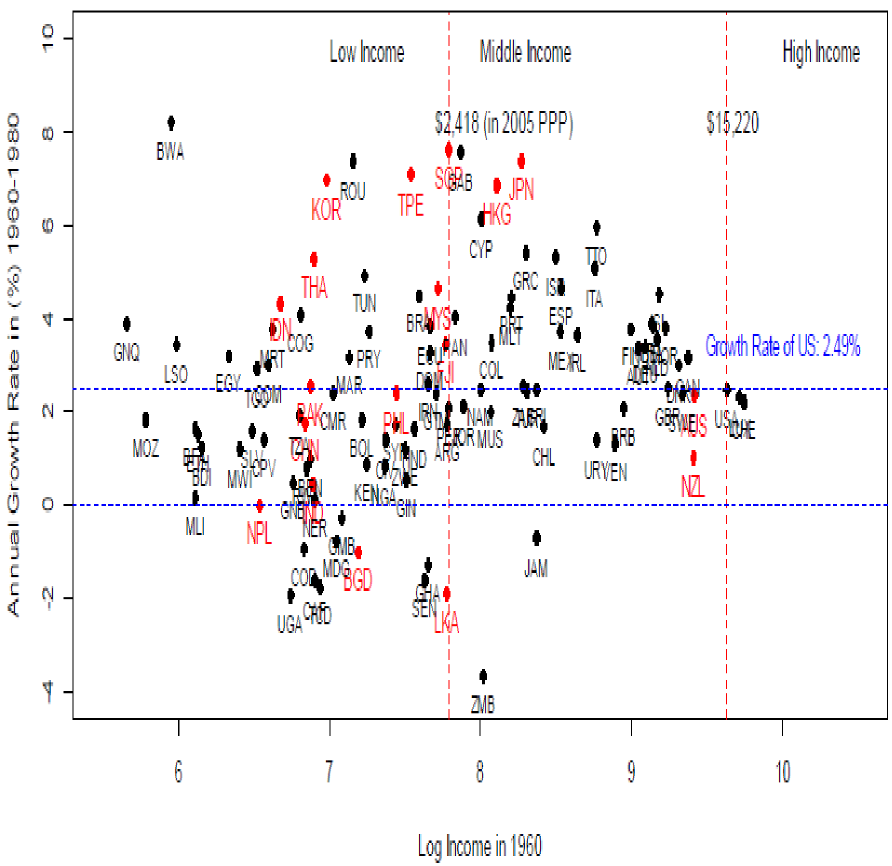

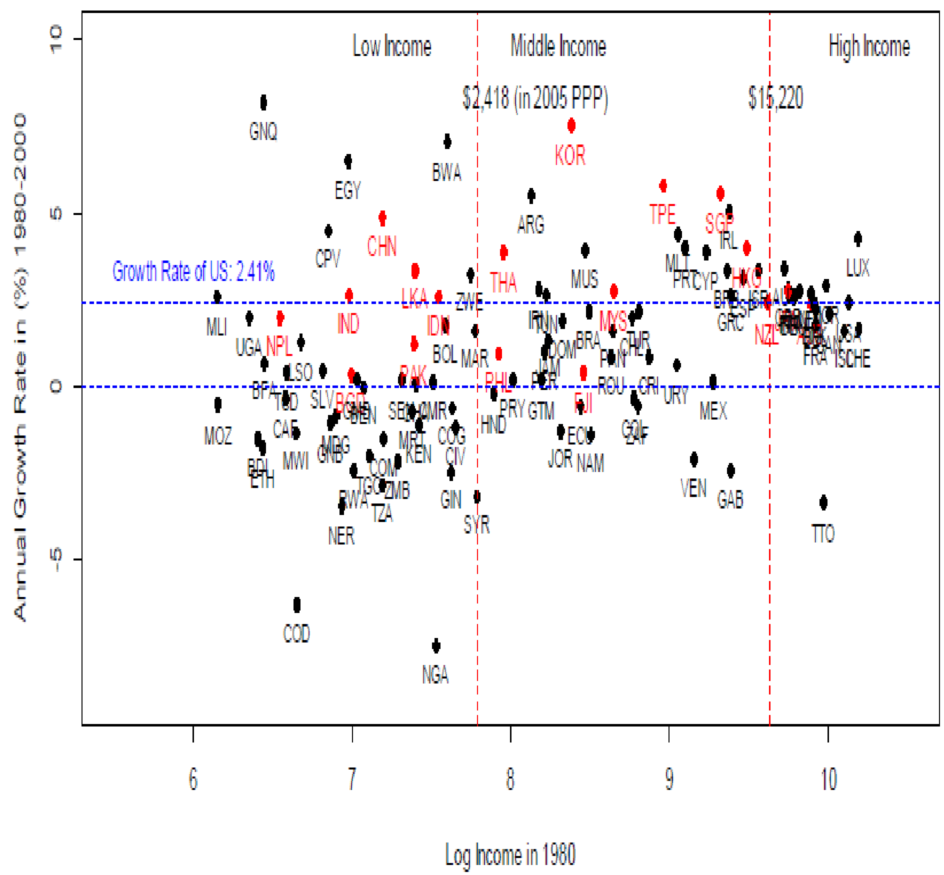

Some stylized-facts about growth

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Table 1. Decade-average transition matrix for 1960–2010 (in %)

	Extremely Low-Income	Low-Income	Lower-Middle-Income	Upper-Middle-Income	High-Income
ELI	82	18	0	0	0
LI	3	72	25	0	0
LMI	0	3	68	29	0
UMI	0	0	0	70	30
HI	0	0	0	0	100
Ergodic distribution for the average decade transition matrix					
	0	0	0	0	100

ELI = extremely low-income; HI = high-income; LI = low-income; LMI = lower-middle-income; UMI = upper-middle-income

Table 3. Decade-average transition matrix for 1960–2010 relative to US (in %)

	16% and Below (Extremely Low and Low)	16%–36% (Lower-Middle)	36%–75% (Upper-Middle)	75% and above (High)
16% and Below (LI)	92	8	0	0
16%–36% (LMI)	13	72	15	0
36%–75% (UMI)	0	4	74	22
75% and above (HI)	0	2	19	79
Ergodic distribution associated with transition matrices relative to US				
	23	13	31	33

HI = high-income; LI = low-income; LMI = lower-middle-income; UMI = upper-middle-income

Key motivated questions

- ***Positive questions***
  - What is the growth process?
  - Do we expect a convergence in growth? Pattern of convergence? All become rich at the end?
  - If not, what causes the divergence, and hence income disparities at the global scale?
- ***Normative questions***
  - Growth policies → closing the gap!

Growth models

- **Solow growth**: sustainable growth based on technological progress.
  - **Exogenous growth**: *technology is determined outside the model.*
  - Growth convergence among countries.
- **Endogenous growth**: sustainable growth based on human capital / Technology.
  - Growth engine is **endogenous**.
  - No certainty in growth convergence.

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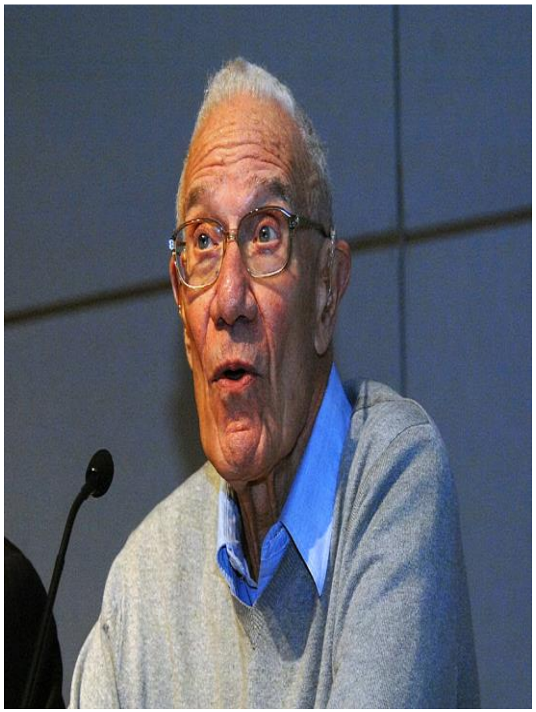
The Solow growth model



Robert M. Solow (b.1924), MIT, Nobel Prize 1987

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor --- **technological progress**.
  - Rising **total factor productivity (z)**.
  - Sustained improvement in living standards (real per capita income or output per worker).

Population growth

- Assume population grows exogenously at a constant rate.
- N = population in the current period.
- N’ = population in the future period.
- n > -1; rate of population growth.

$$N' = (1 + n)N$$

Consumers

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output (Y) as (wage and dividend) income.
  - Spend on consumption goods (C) and save a constant fraction (s) of Y as savings (S).

$$Y = C + S; \quad S = sY$$

$$C = (1 - s)Y$$

The representative firm

- The firm produces output using current capital stock (K) and current labor input (N).
- Assuming **constant returns to scale**.

$$Y = zF(K, N)$$

$$\frac{Y}{N} = zF(\frac{K}{N}, 1)$$

Per worker production function

$$Let\ y = \frac{Y}{N} = output\ per\ worker$$

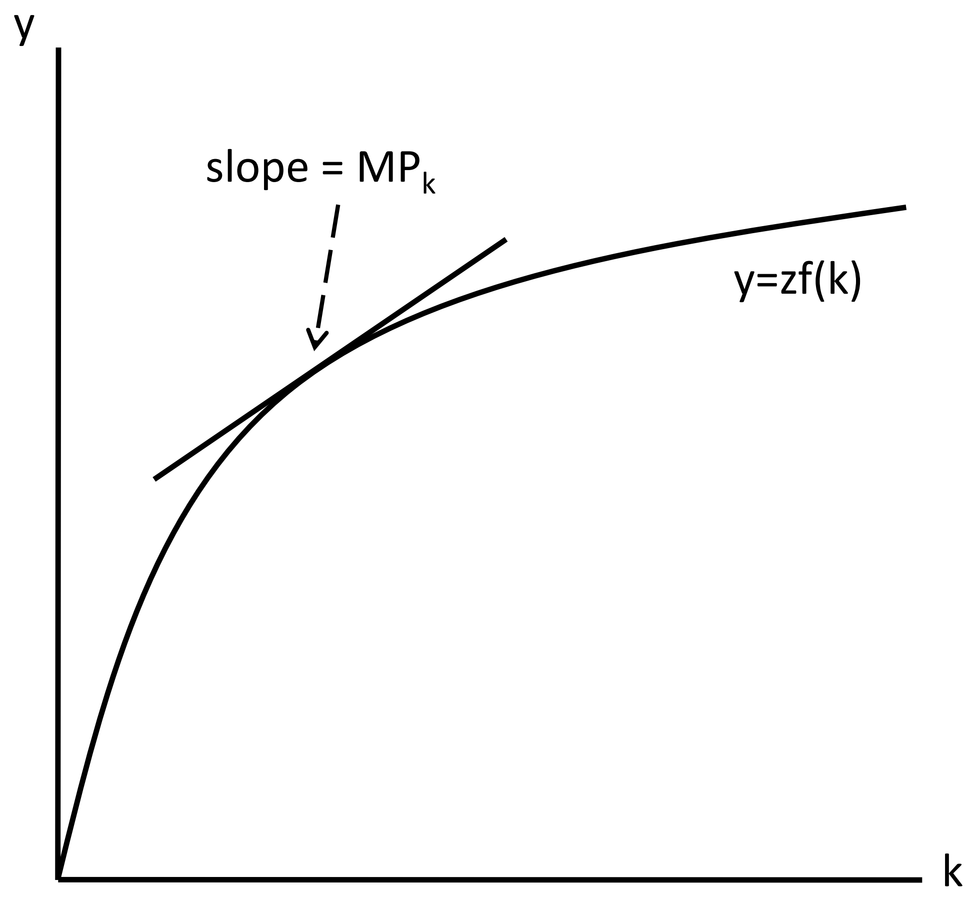
$$k = \frac{K}{N} = capital\ per\ worker$$

$$f(k) = zF\left(\frac{K}{N}, 1\right)$$

$$y = zf(k)$$

Marginal product of k

- Output per worker (y) **increases at a decreasing rate** as capital per worker (k) rises.
- Slope is the marginal product of k.

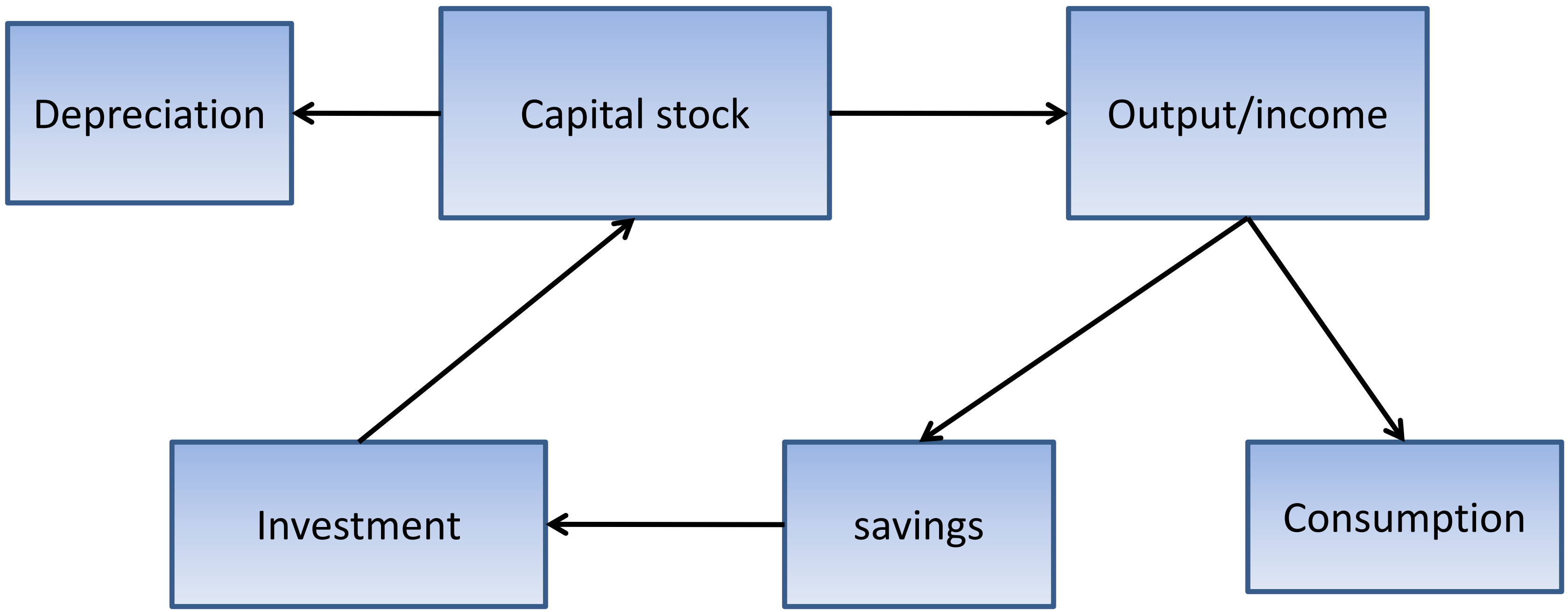


Growth of capital stock
Capital accumulation

- Assumer capital wears out over time at the rate of d (or depreciation).
  - where 0 > d > 1.
- I = investment = addition to capital stock.
- K' = capital stock in the future period.

$$K' = (1-d)K + I$$

The working of growth process: capital accumulation *drives* output



Solution of model: Equilibrium output

- At equilibrium, **savings equals investment** so that output consists of consumption and investment.

$$S = I$$

$$S = Y - C$$

$$Y = C + I$$

Equilibrium condition

- The future capital stock is current capital stock deducted by depreciation and added by investment (= savings).

$$Y = C + I$$
$$C = (1 - s)Y$$
$$I = K' - (1 - d)K$$

- Substitute C and I in the Y equation.

Per worker formulation

$$Y = (1-s)Y + K' - (1-d)K$$

rearrange the terms:

$$K' = sY + (1-d)K$$

$$\text{but} \quad Y = zF(K,N)$$

$$\text{so} \quad K' = szF(K,N) + (1-d)K$$

divide it by N :

$$\frac{K'}{N} = sz\frac{F(K,N)}{N} + (1-d)\frac{K}{N}$$

Future capital per worker function

$$\frac{K'}{N}\frac{N'}{N'} = szF(\frac{K}{N},1) + (1-d)\frac{K}{N}$$

$$where \quad k' = \frac{K'}{N'} \quad and \quad \frac{N'}{N} = (1+n)$$

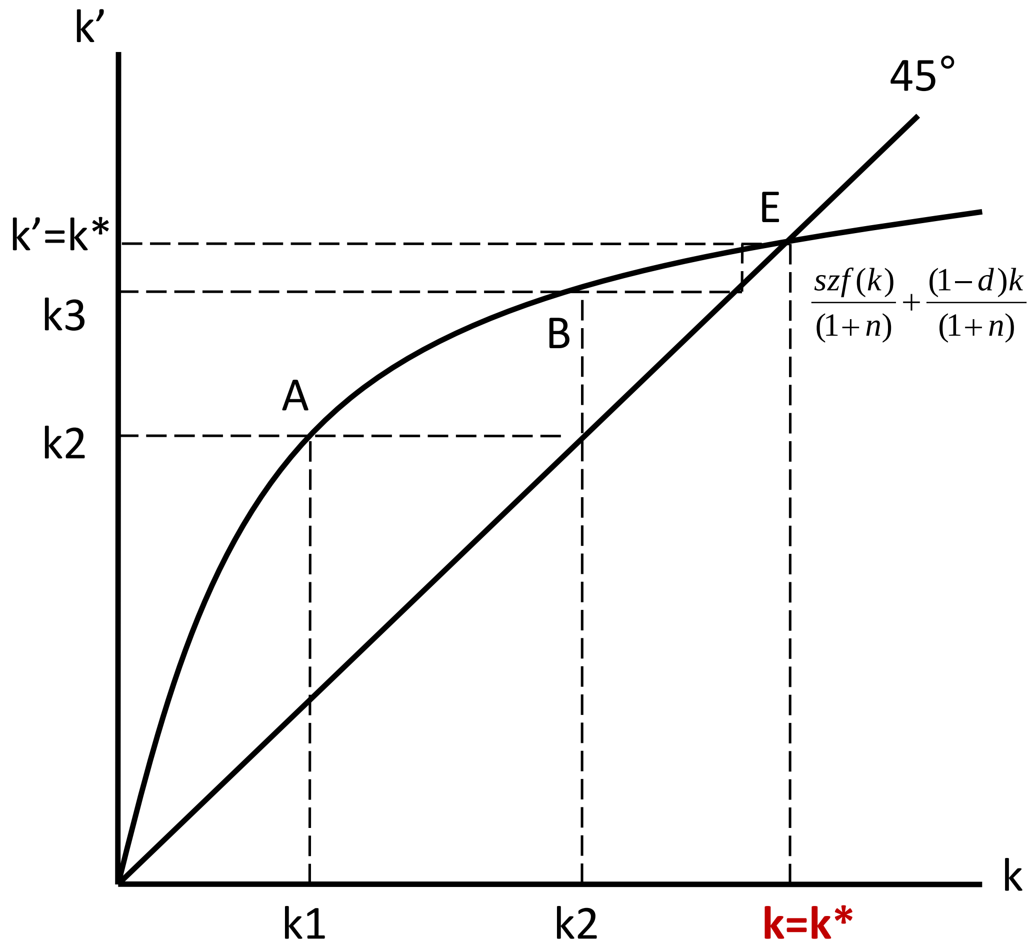
$$k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

- Future k' as a function of current k. (evolution of per-worker capital stock)

The steady-state capital per worker

- At A, k2 > k1; k is growing.
- At B, k3 > k2; k is growing.
- k = k*; steady-state capital per worker.



Why? → Diminishing returns on k

- At E, k = k'= k* so that k* is steady.
  - To the left of k*, k'> k so that k is increasing.
  - To the right of k*, k'< k so that k is decreasing.
- As k is increasing, MP_k is falling so that **y is increasing at a decreasing rate**.
- Finally, **investment** (or new capital) is just sufficient to keep up with **population growth** and **depreciation**, so that k (and y) is stagnant.

Steady-state aggregates

- With k* at the steady state, y*, c* and szf(k*) are all at **the steady-state**.
  - No further improvement in output per worker (y).
- Given population growth (n), total factor productivity (z) and the savings rate (s), the steady-state growth rate is ‘n’ for aggregate quantities:
  - Capital stock (K) and output (Y);
  - Consumption (C), savings (S) and investment (I).

Analysis of the steady-state

$$k^* = \frac{szf(k^*)}{(1+n)} + \frac{(1-d)k^*}{(1+n)}$$

Multiplying k* by (1+n)

$$szf(k^*) = (n+d)k^*$$

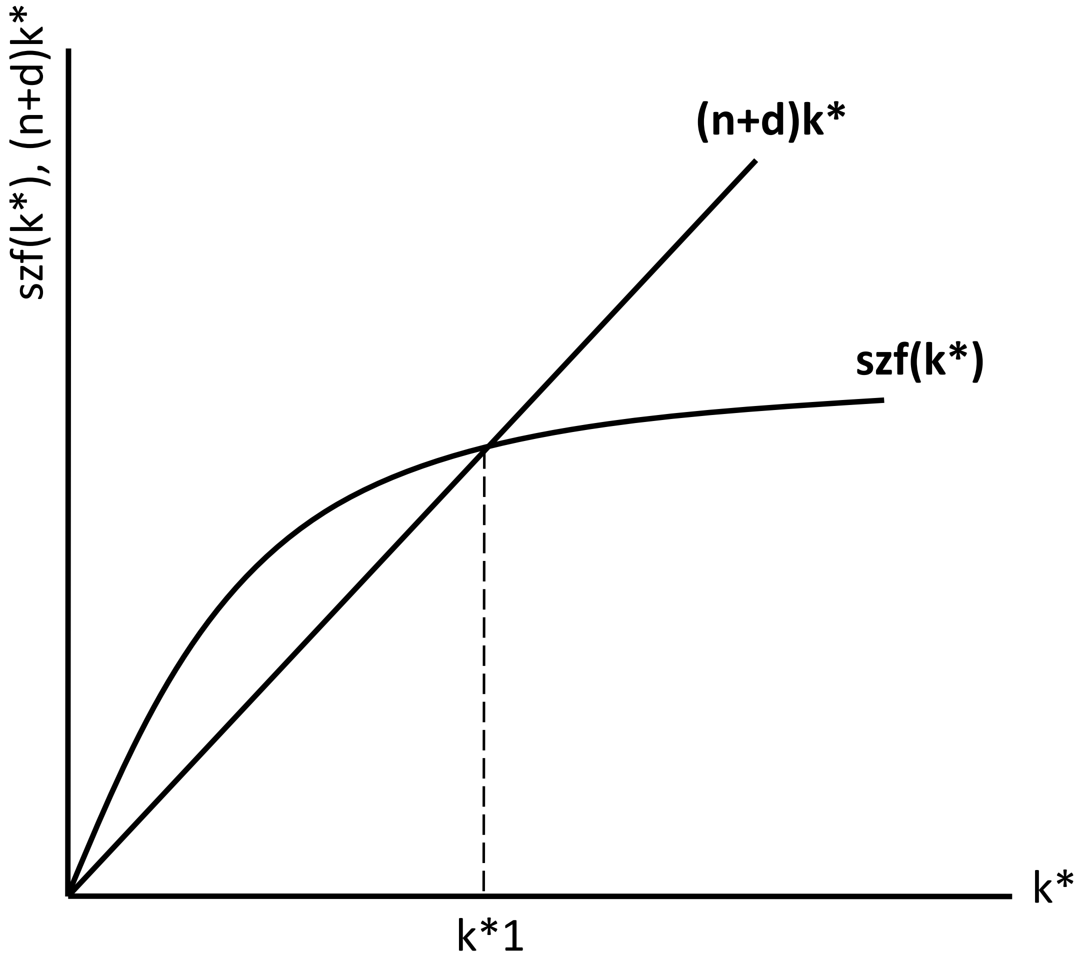
- Or steady-state savings = steady-state investment.

$$szf(k^*) = (n+d)k^*$$

- szf(k*) = savings per worker;
- (n+d)k* = investment per worker needed to keep up with population growth and depreciation.
- At k*, the capital stock is still growing, but just sufficient to equip each worker with the same k and depreciation (so k* is steady).
  - '**Capital widening**': growing K just to keep the steady k and y.

Graphical illustration: Determination of steady-state k*

- szf(k*) is concave due to zf(k*).
- (n+d)k* has the slope = (n+d).



Transitional dynamic of per-capita

- What happen to "growth of per-capita variables" along the path towards their long-term steady states?
  - Suppose you start from an initial k to the left of k*, k' will be increasing.
  - Since k' increases, income-per-capita (y) will increase as well.
  - As y increases at a decreasing rate, **growth will decline over time.**
  - At the steady state, y = y* , i.e. zero income-per-capita growth.

Transitional dynamic of aggregates

- At the steady state, aggregates grow at the rate of “n”. (aka, balanced growth path)

- While directing towards the steady state, aggregate Y will grow at the rate above “n”, and slowly decline to consistently keep trace with the growth of income per capita (y).

Policy experiments

- Change in saving rate (s)

- Change in population growth (n)

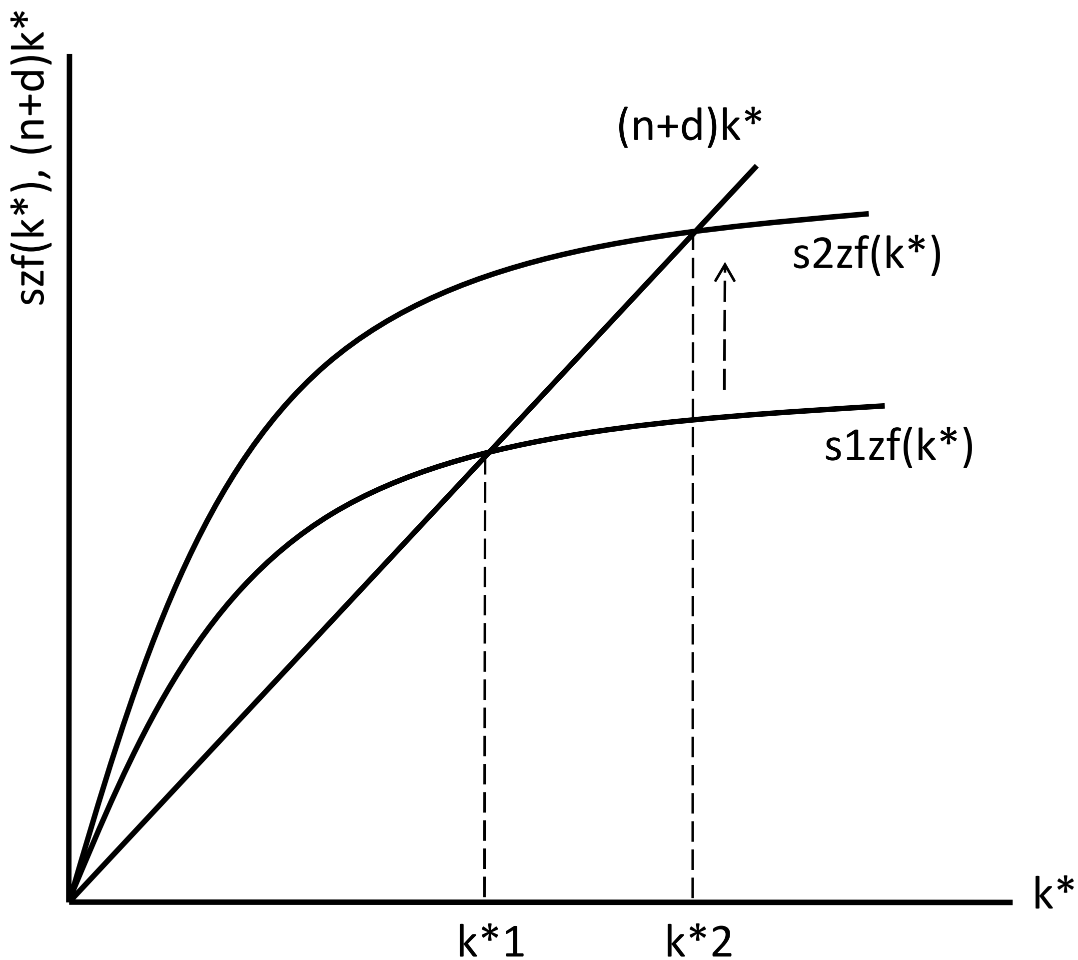
- Change in the level of technology (z)

Effect of an increase in s

- Savings rate may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in s:
  - szf(k*) rotates upwards.
  - Higher steady-state k* and y* (on a different 'growth path').
  - Higher growth of K and Y is transitional.
  - Convergence to the same steady-state growth rate of 'n'.

A rise in s raises k*, and hence y*

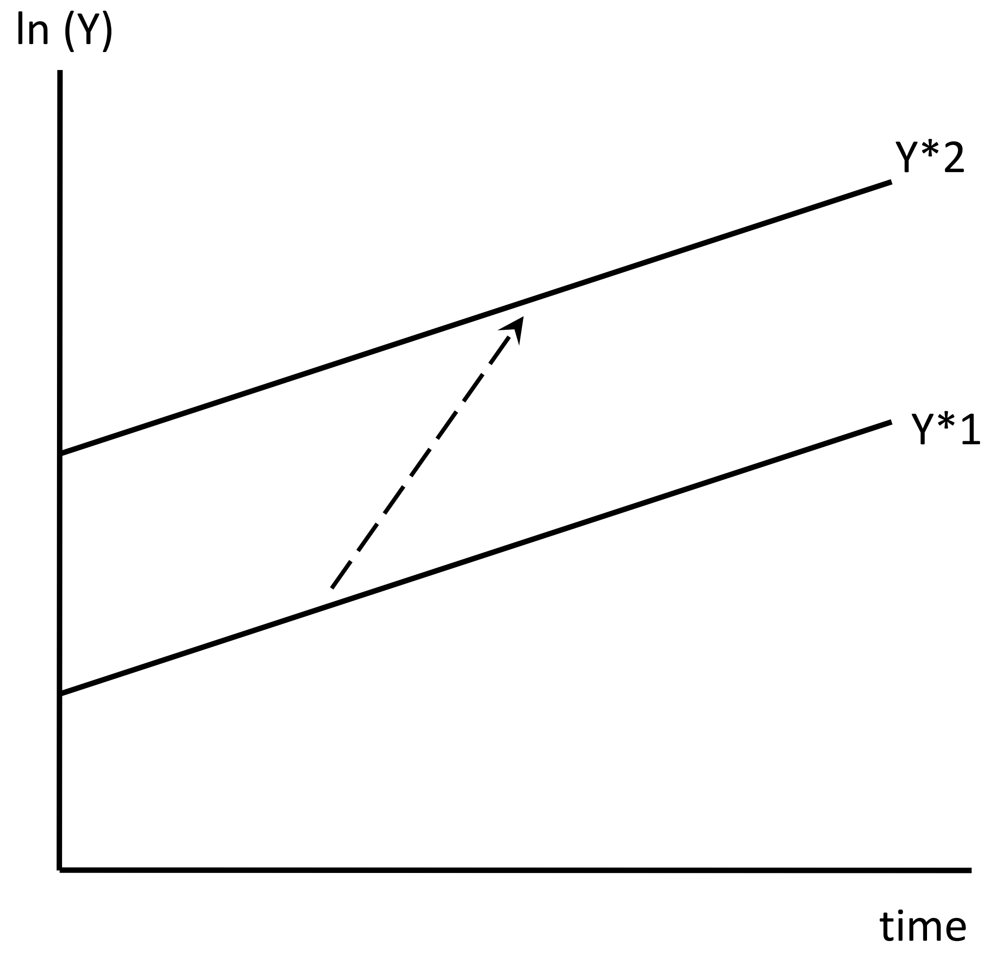
- Higher savings rate results in a higher k* and y*.
- Match with Stylized-fact #2



Temporary gain in growth rate

- K and Y move to new 'growth paths'.

- Higher growth rates of K and Y are transitional, converging to n.



Steady-state consumption per worker

- A broad measure of aggregate welfare.

$$y^* = zf(k^*)$$

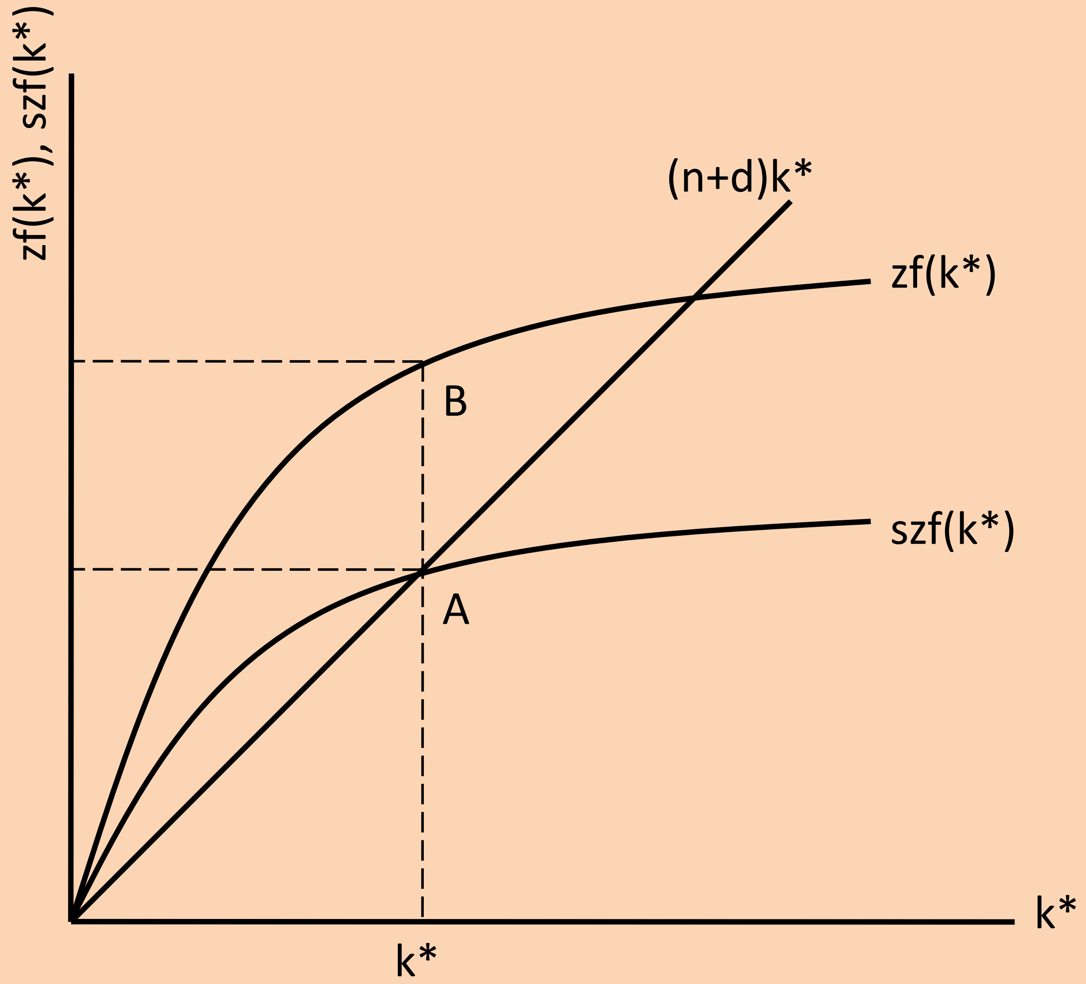
$$\frac{S}{N} = szf(k^*)$$

$$c^* = zf(k^*) - szf(k^*)$$

$$c^* = zf(k^*) - (n+d)k^*$$

$$c^* = (1-s)zf(k^*)$$

- AB = c*.
- Each savings rate is associated with a value of steady-state consumption per worker (c**).



Maximized c**

$$c^* = zf(k^*) - (n+d)k^*$$

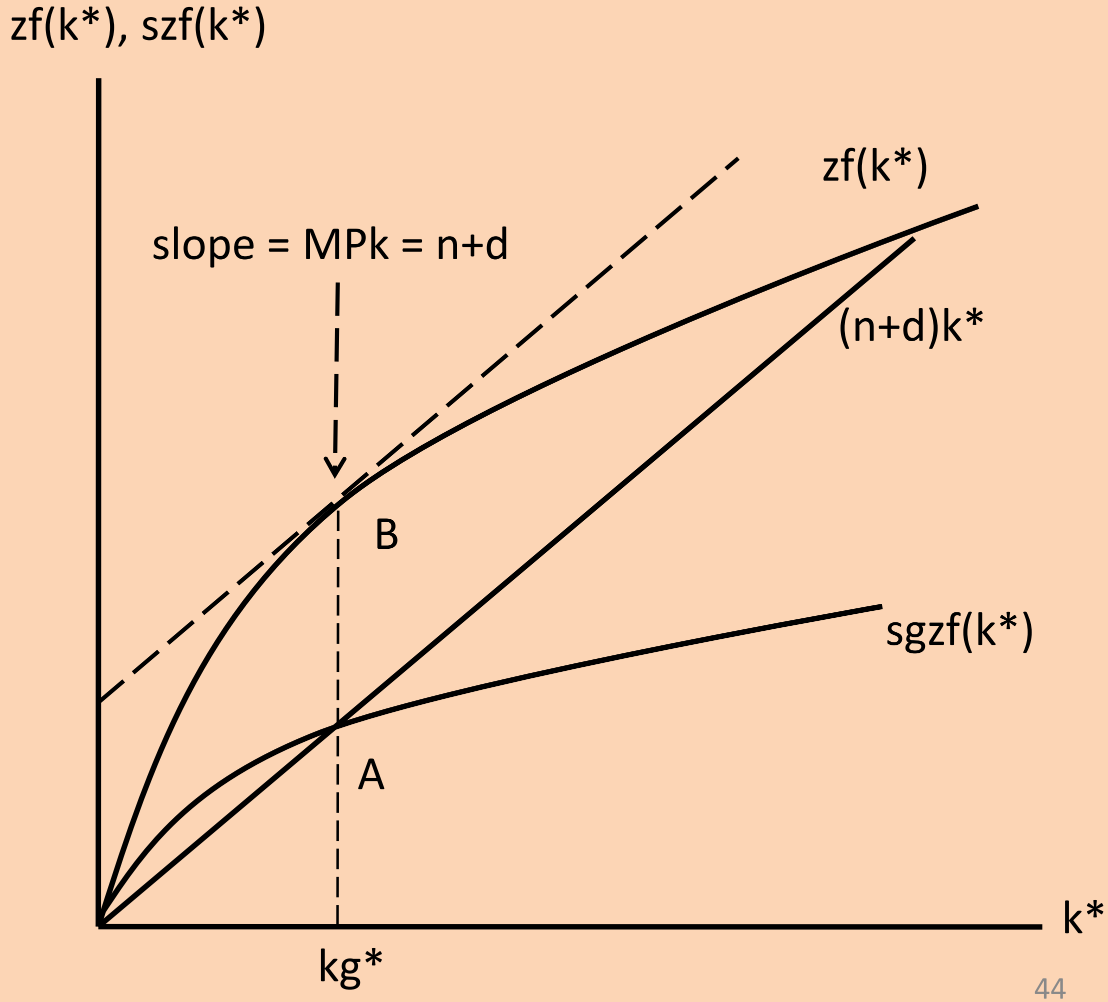
$$Set\ \frac{dc^*}{dk^*} = \frac{d(zf(k^*))}{dk^*} - (n+d) = 0$$

$$\frac{d(zf(k^*))}{dk^*} = n+d$$

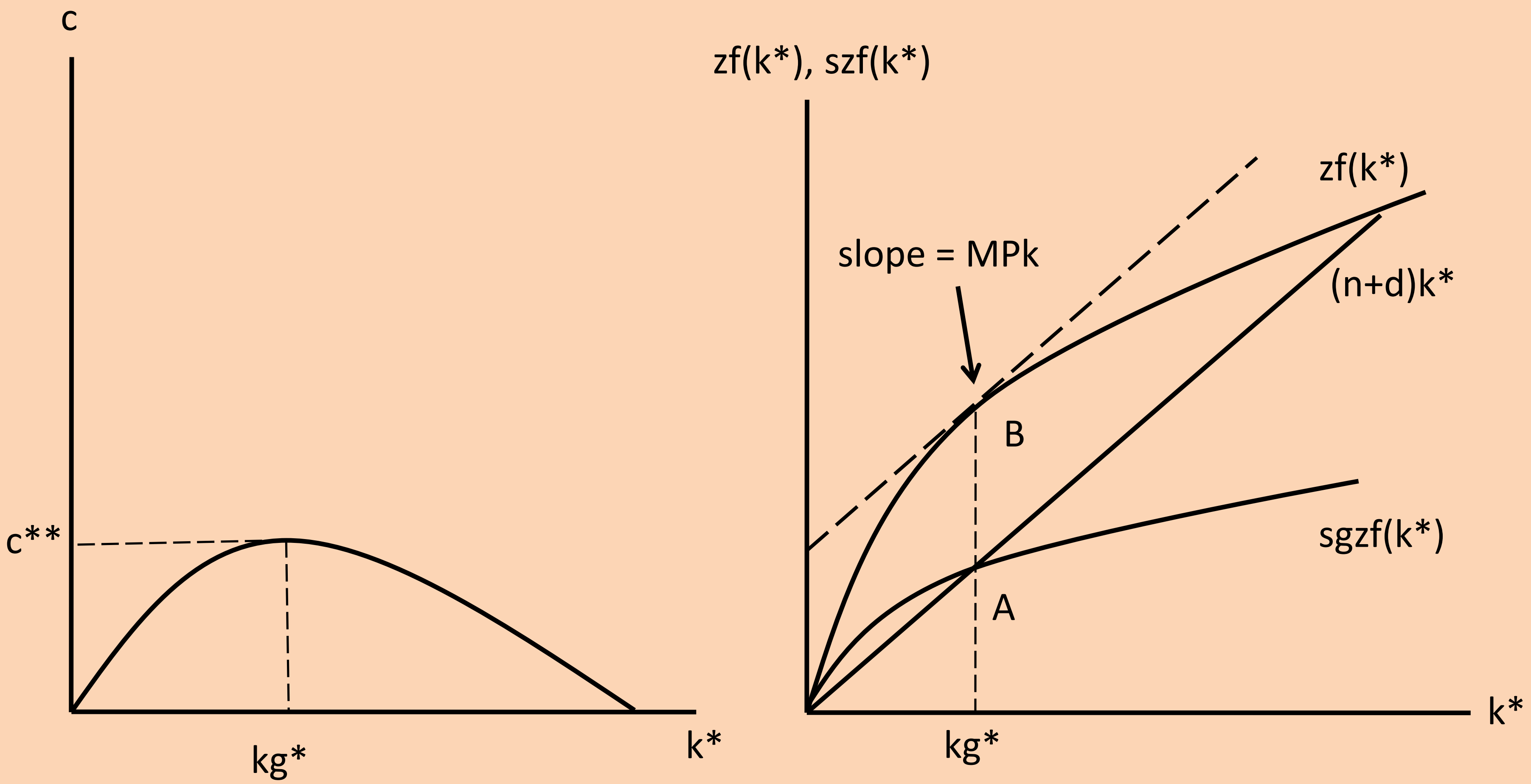
$$MP_k = n+d$$

Golden-rule s_g

- The rate with max. c** is the **'golden-rule' savings rate** (s_g).



Maximized c** and golden-rule s_g



Golden-rule s_g and policy?

- With the golden-rule s_g, c** is maximized and steady in all periods.
  - Other s results in different c* which is not max.
  - So, an increase in "s" may not lead to an increase if s is already higher than s_g.
- Should we achieve s_g if the current $s \neq s_g$?
  - A sacrifice of current c to build up a larger capital stock in the future is needed; is it worth?
  - s depends on individuals' preference and the market for investment.

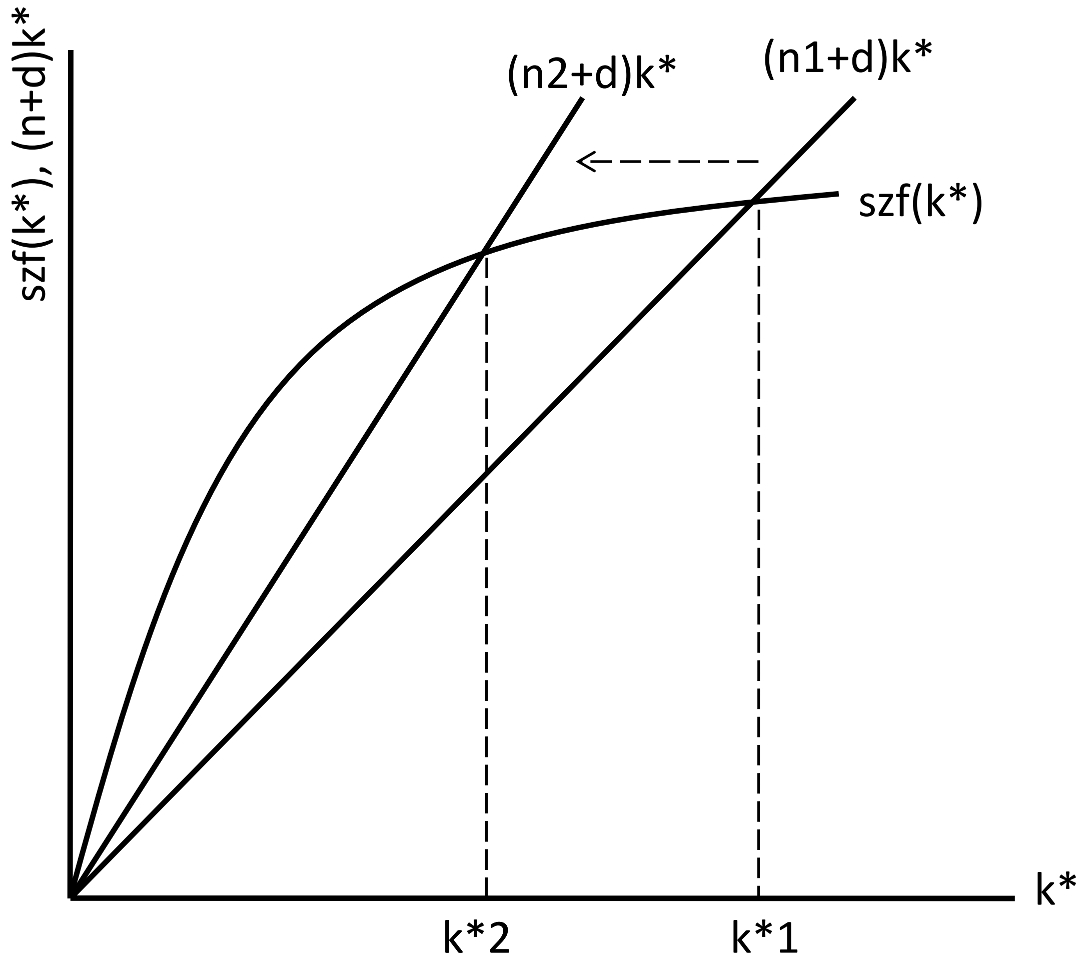
Effect of an increase in n

- The increase in population growth (n1 to n2) rotates (n+d)k* upwards.
- Decreased steady-state capital (k*) and output per worker (y*).
  - More workers (N*) produce larger output (Y*).
  - But falling productivity of labor results in lower output per worker (y*).
- The steady-state growth rate is higher at n2 for the capital stock (K) and total output (Y).

A higher n with lower k*, and hence y*

- Higher population growth (n) results in lower k* and y*.

- **Match with Stylized-fact #3**

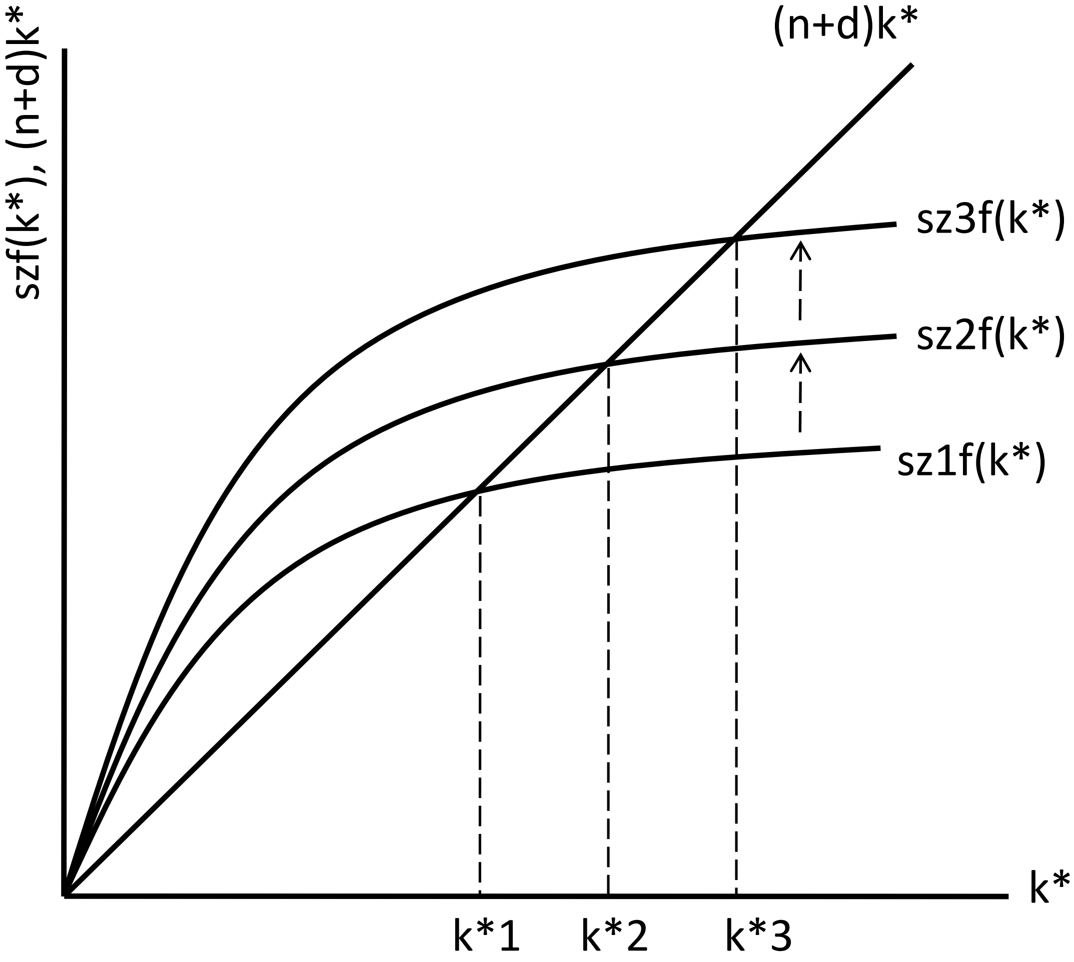


Effect of an increase in z

- A rising s or falling n raises steady-state output per worker (living standards).
  - But the improvement will cease at some point (s cannot exceed 1; n cannot fall indefinitely).
- An increase in total factor productivity (z) raises steady-state capital (k*) and output per worker (y*).
  - Sustained increases in z cause sustained increases in output per worker (y).

Sustained increases in z

- Sustained increases in z cause sustained improvement in y*.



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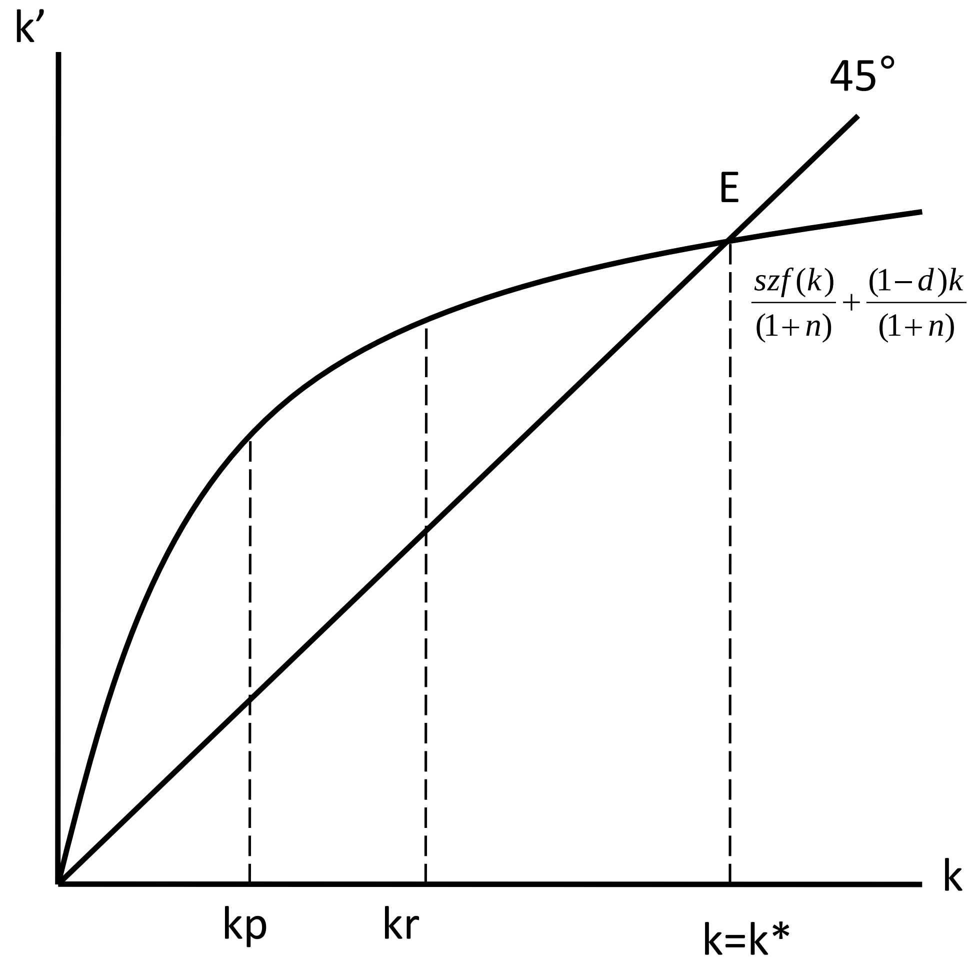
Solow growth predictions: income convergence

- If two countries start with:
  - the same population growth rate (n), savings rate (s) and total factor productivity (z),
  - but different per capita incomes (y), e.g., rich versus poor countries;
  - they will converge to the same steady-state k*, y* and c* --- **Absolute convergence**.
- The poor country will have temporary higher growth and catch up with the rich.

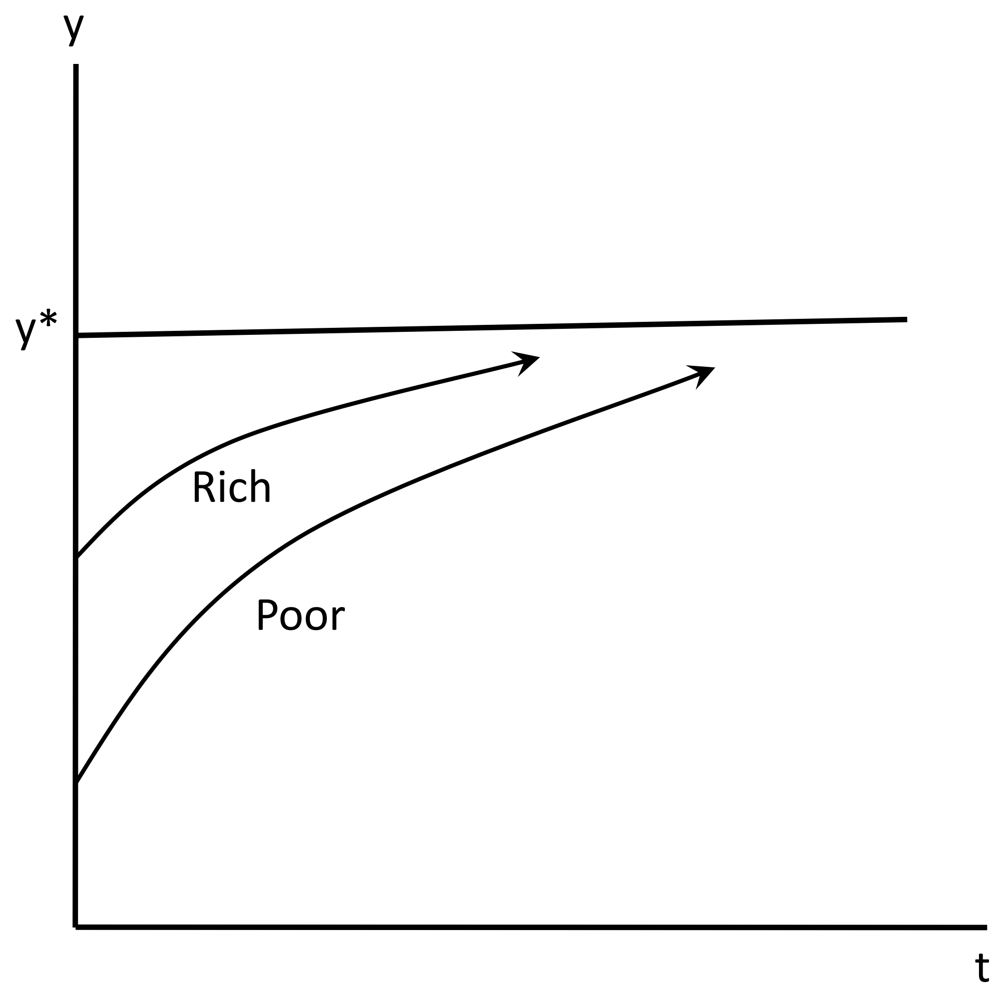
Absolute convergence

- **The rich** starts at kr while **the poor** starts at kp (with the same s, n and z).
- They converge to k* and y* in the long run.



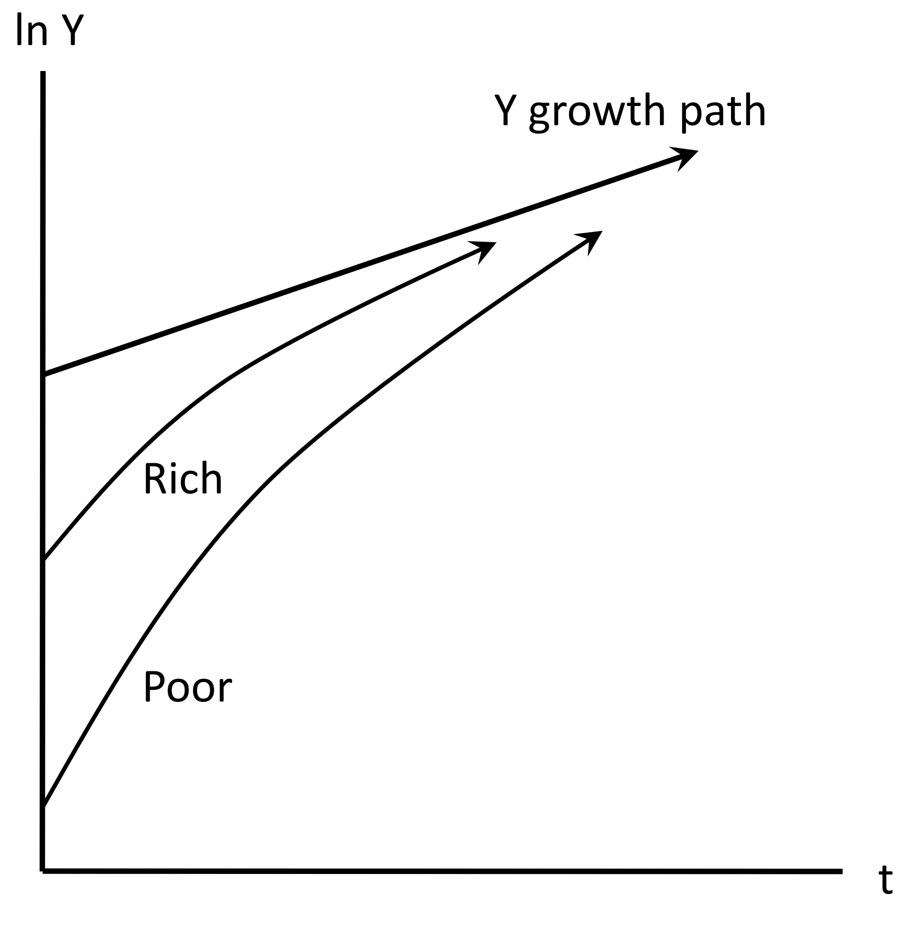
Convergence in per capita income

- The rich and the poor converge to the same level of y*.



Convergence in output growth path

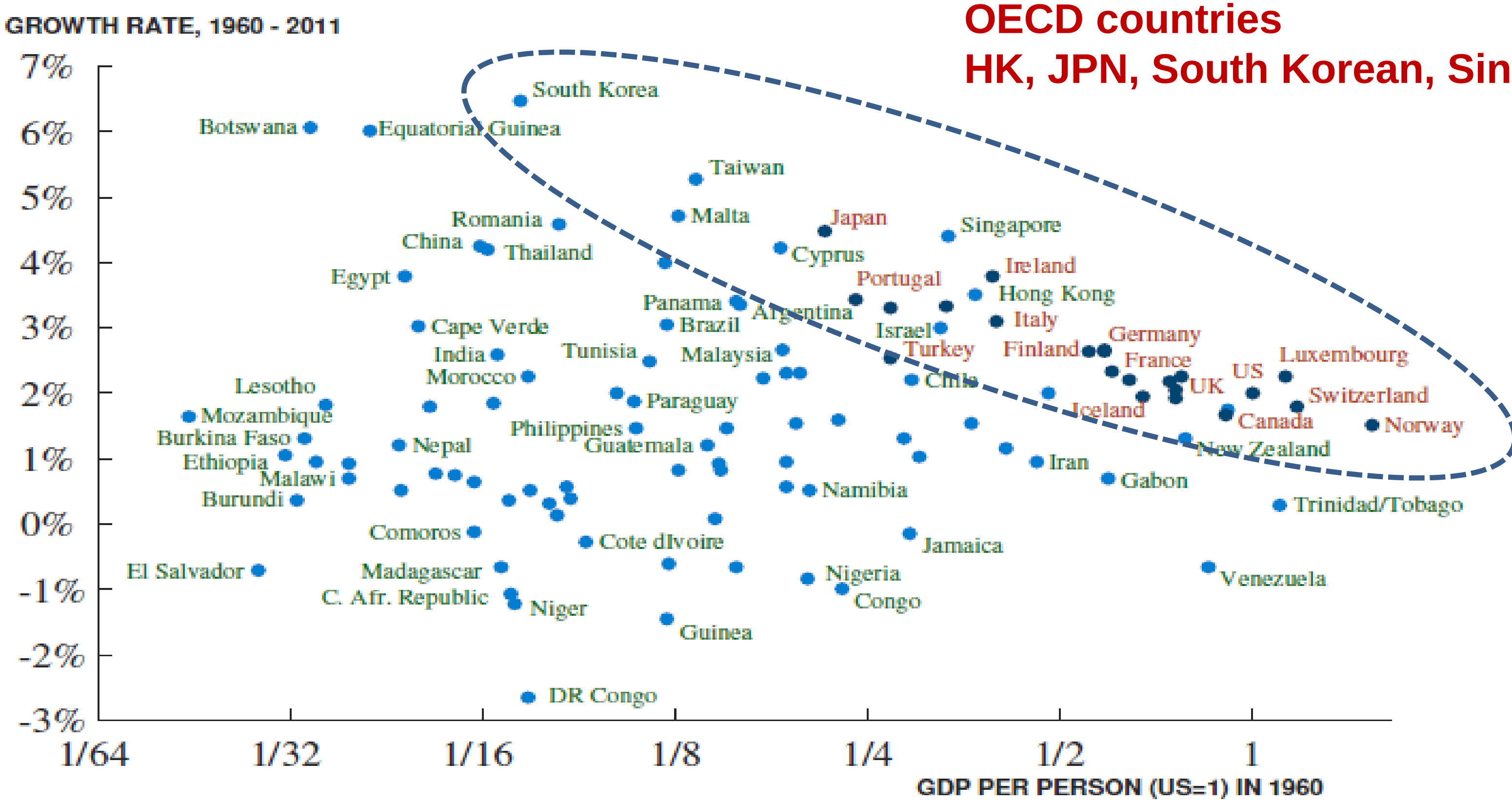
- The rich and the poor converge to the long-run growth rate (n) of aggregate output (Y).



Facts: no convergence worldwide!

Figure 26: The Lack of Convergence Worldwide

OECD countries
HK, JPN, South Korean, Singapore



Source: The Penn World Tables 8.0.

Conditional convergence

- With **differences in n, z and s**, the steady-state k*, y*, c* are different.
  - Each country has its own steady state.
  - The steady-state growth rate of aggregates (K, Y) is still 'n' for each country.

- **Disparity among countries due to *different values of n, z and s.***

Growth facts

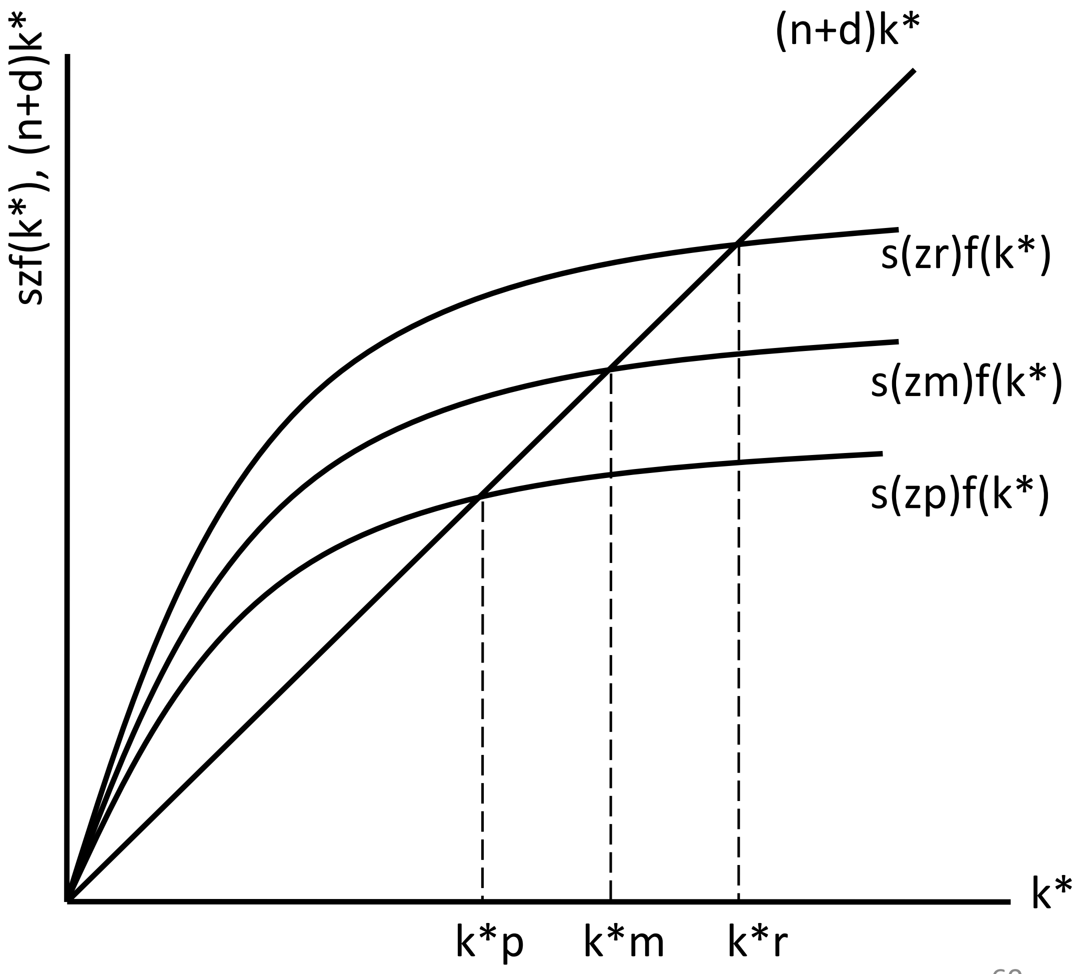
- Absolute convergence has occurred among rich countries.
- No absolute convergence between rich and poor countries.
  - Exception is East Asia.
- No absolute convergence among poor countries.
  - Great diversity among poor countries.

Why no absolute convergence?

- Countries have different s, n and z.
  - Each country has different steady-state k*, y*, c*.
  - Each country is moving towards its own steady-state --- **Conditional convergence**.
- However, empirically, differences in s and n are not large enough to explain all international disparity.
- **Difference in access to technology (z)?**

No convergence with different z's

- Countries with different z's will not converge to the same k* and y*.
  - zp = poor
  - zm = medium
  - zr = rich



Disparity due to different z's

- Different levels of total factor productivity (z) will perpetuate differences in capital per worker (k*) and per capita income (y*) …
  - despite the same savings rate (s) and population growth rate (n).

Barriers to technology adoption

- **Labor legislation**: strong labor unions obstruct adoption of new technology.
- **Trade protectionism**: domestic firms with market power lack incentives to innovation.
- **Political corruption**: government's protection of inefficient firms.
- **Undeveloped financial system**: poor resource allocation mechanism.

How to catch up: Growth policies

- Promotion of more competition among firms.
  - **Liberalization and competition policy.**
  - More pressure and incentive for firms to innovation.
- **Free trade** for greater international competition.
- **Privatization** of state enterprises.
  - State enterprises guarantee employment at the expense of efficiency.

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Sources of sustained growth

- GDP growth can be derived from
  - Growth from increases in **productive inputs**:
    - Physical capital accumulation, F(K, N).
  - Growth from **total factor productivity** (z):
    - Technical progress, inventions, better management and organization.

Engine of growth in Thailand

- Using Solow model, we can back out the contribution of each factor on long-term growth

- Let assume that Thailand’s production function given by $Y = zK^{\alpha}N^{1-\alpha}$
  - The function is CRTS.
  - α = Share of capital income to GDP = 1/3 (approx.)

Engine of growth in Thailand

- We know that

$$\%\Delta Y = \%\Delta z + \alpha\%\Delta K \quad + (1-\alpha)\%\Delta N$$

- Given the capital growth, labor growth, and GDP growth, we can back out the $\%\Delta z$
  - This term is called the Solow residual
  - This term is commonly used to capture the **"total factor productivity"**

Engine of growth in Thailand

	1999-2006	2007-2009	2010 - 2016
K	2.596	2.512	2.54
N	1.508	1.348	0.516
Z	2.484	1.132	1.372
Y	4.7	3.236	3.204

- A decline in long-term growth! Why?
  - Capital investment averagely grows at a stable rate over the past 20 years!
    - Compared to period before that, the rate is lower
  - Demographic issue has suppressed growth process
    - Lower labor force growth!
  - After the GFC, Thailand has experienced a decline in TFP growth!

Appendix

Mathematical analysis of Solow growth model

Numerical example

- Consider a Cobb-Douglass production

$$Y = zF(K, N) = zK^{\alpha}N^{1-\alpha}$$

- With the per-capita formulation, output-per head is given by

$$y = zf(k) = zk^{\alpha}$$

Numerical example

- Given “s”, “n”, “z”, and “d”, future capital-per-head (k’) can be given by

$$k' = \frac{szf(k)}{1+n} + \frac{(1-d)k}{1+n}$$

$$k' = \frac{szk^{\alpha}}{1+n} + \frac{(1-d)k}{1+n}$$

Numerical example

- At the steady-state equilibrium where $k' = k = k^*$, we know that

$$k^* = \frac{sz(k^*)^\alpha}{1+n} + \frac{(1-d)k^*}{1+n} \rightarrow sz(k^*)^\alpha = (n+d)k^*$$

- The steady-state k* is then equal to

$$k^* = \left(\frac{sz}{n+d}\right)^{\frac{1}{1-\alpha}}$$

Numerical example

- Since the golden rule k_g^* requires that

$$MP_k = n + d,$$

this implies that,

$$(\alpha)z\left(k_g^*\right)^{\alpha-1} = (n + d) \rightarrow k_g^* = \left(\frac{\alpha z}{n+d}\right)^{\frac{1}{1-\alpha}}$$

- The corresponding golden rule saving rate s_g is equal to α.