1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^*, C^*, Y_d^*.

$Y^* \to Y^*_d = Y^* - T_0$

$\to C^*_0 = a + bY^*_d$

1. plug all Equations into the Equilibrium condition

2.) $Y = a + bY_d + I_0 + G_0$

$= a + b(Y - T_0) + I_0 + G_0$

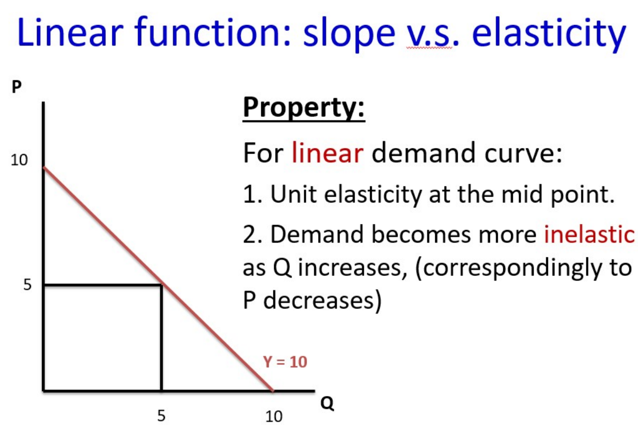
$= bY + a - bT_0 + I_0 + G_0 \Rightarrow Y = \dfrac{a - bT_0 + I_0 + G_0}{1 - b}$

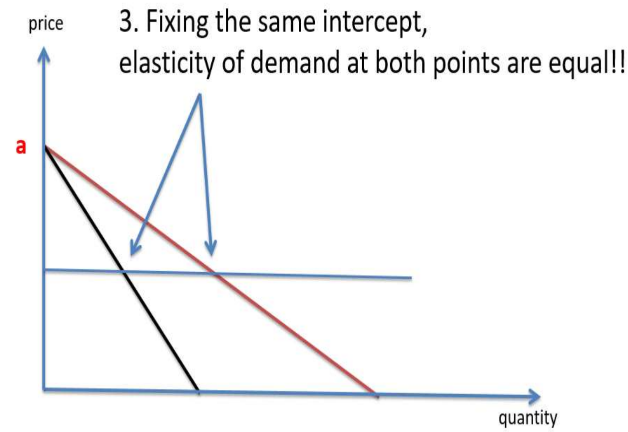
- Numerically, if a = 1, T_0 = \$0 I_0 = \$1, G_0 = \$1 and b = 0.5, this yields us,

$$Y^* = \frac{a - bT_0 + I_0 + G_0}{1 - b}$$

$$= \frac{1 - 0.5(0) + 1 + 1}{1 - 0.5} = \frac{3}{0.5}$$

$$= 6$$





Exercise 2.A:

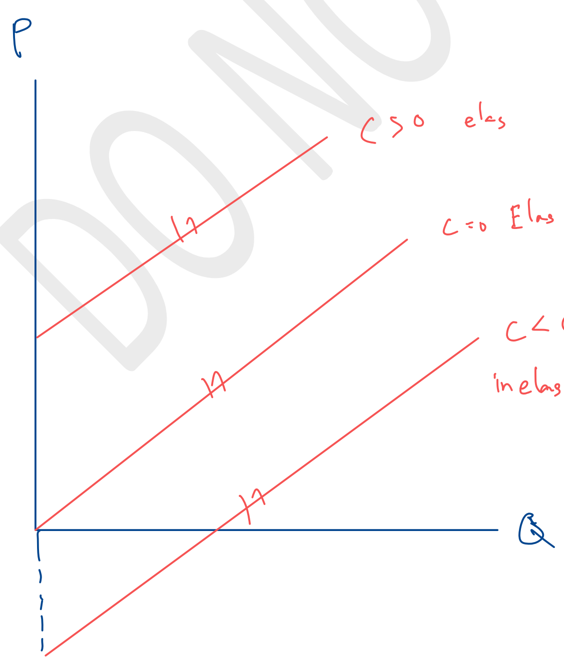
2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

(i) elasticity of supply is always greater than 1 if c > 0,

(ii) elasticity of supply is always eqaul to 1 if c = 0,

(iii) elasticity of supply is always less to 1 if c < 0.



$$\frac{\%\Delta Q^s}{\%\Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q} = \frac{1}{d}\left(\frac{c+dQ}{Q}\right)$$

$$\frac{\Delta P}{\Delta Q} = d \to \frac{\Delta Q}{\Delta P} = \frac{1}{\frac{\Delta P}{\Delta Q}} = \frac{1}{d}$$

$$= \frac{1}{d}\left(\frac{c}{Q}+d\right)$$

$$= \frac{c}{d\cdot Q} + 1$$

$\varepsilon^s_{Q,P} = \frac{c}{d\cdot Q} + 1$ $<0; c<0$

if $Q>0 \to Q \to$ large $\frac{c}{dQ} \to$ smaller

if $c=0 \Rightarrow \varepsilon_{QP} = 1$

$c>0 \to \varepsilon^s_{QP} > 1$

$c<0 \to \varepsilon^s_{QP} < 1$

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

revenue function $TR(Q) = P(Q) \times Q$

$= (10 - Q) \times Q$

$= 10Q - Q^2$

slope = $\frac{dTR}{dQ} = 10 - 2Q$

$= 10 - 2Q$

maximum occurs when $\frac{dTR}{dQ} = 0$

$10 - 2Q = 0$

$Q = 5$

$R(Q) = 10(5) - (5)^2 = 50 - 25 = 25$ ฿

At $Q = 5$, TR is max

TR = 25

- What is the break-even output?

Definition break even output: $R = 0$; $C = 0$, $\pi = 0$, π max

1 2 3 4

→ Q^* → Ensure sufficient level of revenue to cover the cost } $R = C$ → $\pi = 0$

$\pi = R - C$

$= (10Q - Q^2) - (4Q) = 10Q - Q^2 - 4Q$

$= 6Q - Q^2$; Q such that $\pi(Q) = 0$

$6Q - Q^2 = 0$

$Q(6 - Q) = 0 \rightarrow Q = 0, 6$

- What is the profit-maximizing level of output?

$\pi(Q) = 6Q - Q^2$

$a = -1 \quad b = 6 \quad c = 0$

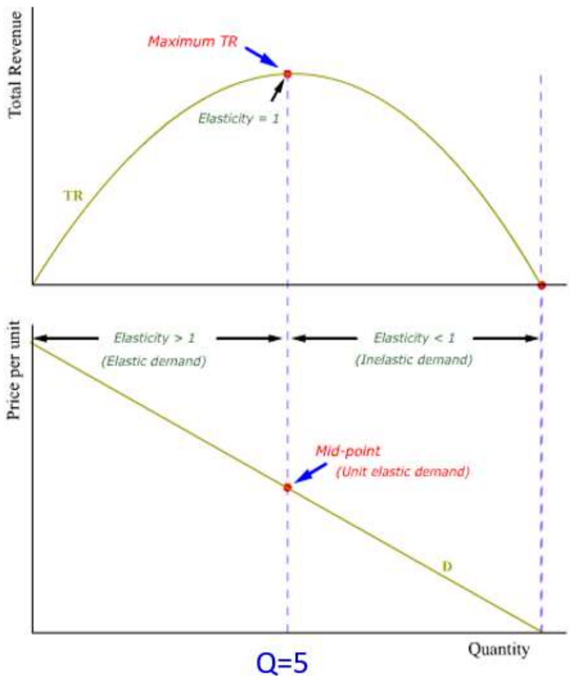
$[Q] = -\frac{b}{2a} = \frac{-6}{2(-1)} = 3$ units

maximized Profit: $\pi(3) = 6(3) - 3^2 = 18 - 9 = 9$ ฿

Price that monopolist should charge → plug $Q = 3$ into the market demand

$P = 10 - Q = 10 - 3 = 7$ ฿ / unit.

Quadratic function: revenue function/break even analysis



$$\left|\frac{\%\Delta Q}{\%\Delta P}\right| : \begin{array}{lll} >1 & ; & elastic \\ =1 & ; & unit\ elastic \\ <1 & ; & inelastic \end{array}$$

Revenue maximizing output: **unit** elasticity

Profit-maximizing output: under region with **elastic** demand

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Exrcise 2B. Consider a function that relates tax revenues R, in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to $60 billion? 0.3

(b) What tax rate(s) is consistent with raising tax revenues equal to $61.25 billion? 0.35

(c) What tax rate is consistent with the maximum tax revenue?