

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

$$Y_i - \hat{Y}_i = \hat{u}_i$$

$$2.1 - \hat{Y}_i = \hat{u}_i$$

- 1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

- 1.2 Find \hat{Y}_i and \hat{u}_i and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

- 1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1),$ and $var(\hat{\beta}_2)$ OLS

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of \hat{Y}_i and \hat{u}_i . Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass (\bar{X}, \bar{Y}) ?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

1.

Student	Y_i	X_i	x_i	y_i	$x_i y_i$	x_i^2	X_i^2	\hat{u}_i^2
			$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$		
1	2.8	63	-14.625	-0.0125	6.0388	213.8906	3969	0.0074
2	3.4	72	-5.625	0.1875	-1.0547	31.6406	5184	0.1439
3	3.0	78	0.975	-0.2125	-0.0797	0.1406	6084	0.0508
4	3.5	81	3.975	0.2875	0.9703	11.3906	6561	0.0297
5	3.6	87	4.975	0.9875	3.6328	87.8906	7569	0.0046
6	3.0	75	-2.625	-0.2125	0.5578	6.8906	5625	0.0151
7	2.7	75	-2.625	-0.5125	1.3453	6.8906	5625	0.1789
8	3.7	90	12.975	0.4875	6.0928	153.1406	8100	0.0043

$$\bar{Y}_i = 3.2125 \quad \sum x_i y_i = 17.4375 \quad \sum \hat{u}_i^2 = 0.434703$$

$$\bar{x}_i = 77.625 \quad \sum x_i^2 = 511.8748 \quad \sum X_i^2 = 48,717$$

Interpretation

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{17.4375}{511.8748} = 0.0341$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 3.2125 - (0.0341)(77.625) = 0.5655$$

\therefore As $Y_i = 0.5655 + 0.0341 X_i + u_i$, for any given level of exam point (X_i), the slope of estimated GPA is 0.0341 with estimated intercept on Y -axis is 0.5655 and the error (u_i) is nearly zero.

$\bullet \hat{\beta}_1 = 0.5655 \Rightarrow$ As if the student has got no point at all, the maximum GPA they can get is 0.5655.

$\bullet \hat{\beta}_2 = 0.0341 \Rightarrow$ As one point earned, the GPA is raised up by 0.0341.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x$$

$$\hat{Y}_1 = 0.5655 + 0.0341(63) = 2.7138$$

$$\hat{Y}_2 = 0.5655 + 0.0341(72) = 3.0207$$

$$\hat{Y}_3 = 0.5655 + 0.0341(78) = 3.2253$$

$$\hat{Y}_4 = 0.5655 + 0.0341(81) = 3.3276$$

$$\hat{Y}_5 = 0.5655 + 0.0341(87) = 3.5322$$

$$\hat{Y}_6 = 0.5655 + 0.0341(75) = 3.1230$$

$$\hat{Y}_7 = 0.5655 + 0.0341(75) = 3.1230$$

$$\hat{Y}_8 = 0.5655 + 0.0341(90) = 3.6345$$

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 x + \hat{u}_i$$

$$Y_i = \hat{Y}_i + \hat{u}_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$$

$$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$$

$$\hat{u}_3 = 3.0 - 3.2253 = -0.2253$$

$$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$$

$$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$$

$$\hat{u}_6 = 3.0 - 3.1230 = -0.123$$

$$\hat{u}_7 = 2.7 - 3.1230 = -0.423$$

$$\hat{u}_8 = 3.7 - 3.6345 = 0.0655$$

$$\sum \hat{u}_i = \hat{u}_1 + \hat{u}_2 + \hat{u}_3 + \hat{u}_4 + \hat{u}_5 + \hat{u}_6 + \hat{u}_7 + \hat{u}_8$$

$$= -0.0001$$

$$\approx 0$$

$$\text{Var}(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{6} = 0.0725$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum X_i^2 \hat{\sigma}^2}{n \sum (x_i - \bar{x})^2} = \frac{48717 (0.0725)}{8 (511.8748)} = \frac{3531.982}{4095} = 0.8625$$

$$\text{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{0.0725}{511.8748} = 0.000141$$

2)

X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$x_i y_i$	x_i^2	x_i^2	\hat{u}_i^2
10	0	-10	-9.1	91	100	100	0.0210
12	2	-8	-7.1	56.8	64	144	0.0041
14	5	-6	-4.1	24.6	36	196	1.6205
16	6	-4	-3.1	12.4	16	256	0.2323
18	7	-2	-2.1	4.2	4	324	0.0955
22	10	2	0.9	1.8	4	484	0.7939
24	10	4	0.9	3.6	16	576	7.1931
26	15	6	5.9	35.4	36	676	0.2377
28	16	8	6.9	55.2	64	784	0.0697
30	20	10	10.9	109	100	900	3.7830

$$\bar{x} = 20 \quad \sum x_i y_i = 394 \quad \sum u_i^2 = 14.0908$$

$$\bar{y} = 9.1 \quad \sum x_i^2 = 440 \quad \sum X_i^2 = 4,440$$

2.1)

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 9.1 - (0.8955)(20)$$

$$= 9.1 - 17.91$$

$$= -8.81$$

Interpretation

• $\hat{\beta}_1 = -8.81 \Rightarrow$ the y-intercept when $x=0$ is at $(0, -8.81)$

• $\hat{\beta}_2 = 0.8955 \Rightarrow$ As x is increased by 1 unit, y increases by 0.8955

$$2.2) \hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x$$

$$\begin{aligned} \hat{y}_1 &= -8.81 + 0.8955(10) = 0.145 \\ \hat{y}_2 &= -8.81 + 0.8955(12) = 1.936 \\ \hat{y}_3 &= -8.81 + 0.8955(14) = 3.727 \\ \hat{y}_4 &= -8.81 + 0.8955(16) = 5.518 \\ \hat{y}_5 &= -8.81 + 0.8955(18) = 7.309 \\ \hat{y}_6 &= -8.81 + 0.8955(22) = 10.891 \\ \hat{y}_7 &= -8.81 + 0.8955(24) = 12.682 \\ \hat{y}_8 &= -8.81 + 0.8955(26) = 14.473 \\ \hat{y}_9 &= -8.81 + 0.8955(28) = 16.264 \\ \hat{y}_{10} &= -8.81 + 0.8955(30) = 18.055 \end{aligned}$$

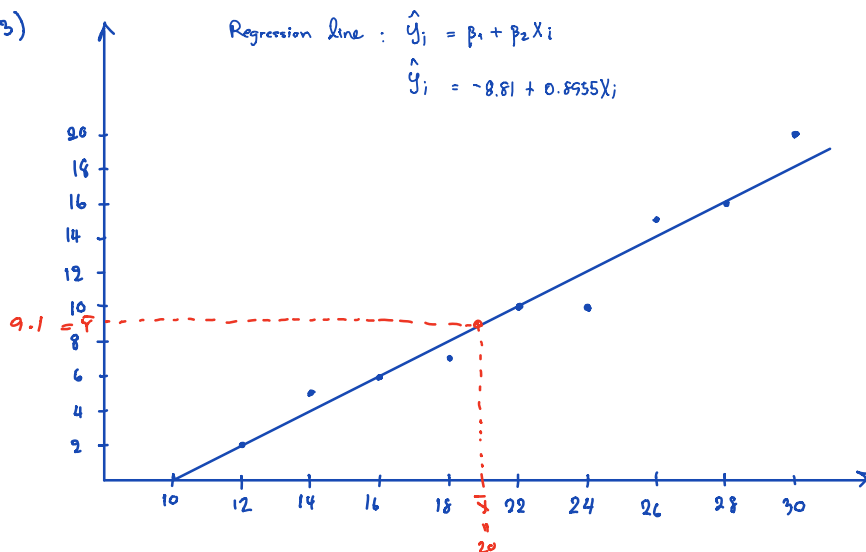
$$y_i = \hat{\beta}_1 + \hat{\beta}_2 x + \hat{u}_i$$

$$y_i = \hat{y}_i + \hat{u}_i \Rightarrow \hat{u}_i = y_i - \hat{y}_i$$

$$\begin{aligned} \hat{u}_1 &= 0 - 0.145 = -0.145 \\ \hat{u}_2 &= 2 - 1.936 = 0.064 \\ \hat{u}_3 &= 5 - 3.727 = 1.273 \\ \hat{u}_4 &= 6 - 5.518 = 0.482 \\ \hat{u}_5 &= 7 - 7.308 = -0.309 \\ \hat{u}_6 &= 10 - 10.891 = -0.891 \\ \hat{u}_7 &= 10 - 12.682 = -2.682 \\ \hat{u}_8 &= 15 - 14.473 = 0.527 \\ \hat{u}_9 &= 16 - 16.264 = -0.264 \\ \hat{u}_{10} &= 20 - 18.055 = 1.945 \end{aligned}$$

$$\sum \hat{u}_i = 0$$

2.3)



\therefore The Regression line passes the $(\bar{x}, \bar{y}) = (20, 9.1)$

2.4) If $X_i = 18$, $\hat{y}_i = ?$

$$\begin{aligned}\hat{y}_i &= \beta_1 + \beta_2 X_i \\ &= -8.81 + \left(\frac{394}{440}\right)(18) \\ &= 7.3081\end{aligned}$$

$$2.5) \text{Var}(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0908}{8} = 1.7614$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum X_i^2 \sigma^2}{n \sum (x_i - \bar{x})^2} = \frac{4440(1.7614)}{10(440)} = 1.7614$$

$$\text{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{(1.7614)^2}{440} = 0.0071$$

3.)

$$\begin{aligned}\hat{\beta}_2 &= \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2} \\ &= \bar{Y} - \hat{\beta}_2 \bar{X}\end{aligned}$$

Proof that this is an unbiased estimator

$$\begin{aligned}E(\hat{\beta}_1) &= E(\bar{Y} - \hat{\beta}_2 \bar{X}) \\ &= E(\bar{Y}) - \hat{\beta}_2 E(\bar{X}) \\ &= \beta_1 + \cancel{\beta_2 \bar{X}} - \bar{X} \cancel{\beta_2}\end{aligned}$$

$$\therefore E(\hat{\beta}_1) = \beta_1$$

\therefore Assumption 1, 7, 8