Assignment 2: Due date: February 17, 2022 before 2.00 pm

Question 1 (30 Points) Score....................

At this moment, all of your assets are invested in asset A with the following return and risk characteristics:

$$E(r_A) = 10\%$$

$$\sigma_A = 10\%$$

Another asset (call it"B") becomes avaiable; the characteristics of B are as follows;

$$E(r_B) = 20\%$$

$$\sigma_B = 25\%$$

. Furthermore, the correlation of A's and B's return patterns is -1. $\rho_{AB} = -1$

Questions:

(1) Using MATLAB to write down the syntax (.m file) for determining the optimal weight (w) of asset A and B in order to achieve the lowest variance, or, in other words, determining the optimal weight for the minimum-variance portfolio. w^*

(2) Find out the Expected return and its variance of the min-variance portfolio using the MATLAB.

(3) By reallocating your portfolio to include some of asset B, how much additional return could you expect to receive if you wanted to maintain your portfolio's risk at $\sigma_p = 10\%$. (Hint: Solve for W_B, not for the W_A). $\sigma_A = 10\%$

Note: You must submit both the.m file and your answer in the next page's supplied space.

Question 1

Refer to Mattab, the optimal weight for minimum-variance portfolio

$W_A = 0.7143 = 71.43\%$ invest in A

$W_B = 0.2857 = 28.57\%$ invest in B

Question 2

The expected return of the minimum-variance portfolio

$$R_{mv} = \frac{alpha}{delta} = \frac{\alpha}{\delta} = 1.1286$$

The variance of the minimum-variance portfolio

$$\sigma^2_{mv} = \frac{1}{delta} = \frac{1}{\delta} = 7.0805e-19$$

Question 3

$$\sigma_P^2 = W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2W_AW_B\sigma_A\sigma_B\rho_{AB} \qquad \text{Note : } W_B = 1 - W_A$$

Set $\sigma_P^2 = 10\% = 0.01$

$$0.01 = (1-W_B)^2\sigma_A^2 + W_B^2\sigma_B^2 + 2(1-W_B)W_B\sigma_A\sigma_B\rho_{AB}$$

$$0.01 = 0.01(1-W_B)^2 + 0.0625W_B^2 + 2(1-W_B)W_B(0.1)(0.25)(-1)$$

$$0.01 = 0.01[1-2W_B+W_B^2] + 0.0625W_B^2 - 0.05W_B + 0.05W_B^2$$

$$\cancel{0.01} = \cancel{0.01} - 0.02W_B + 0.01W_B^2 + 0.0625W_B^2 - 0.05W_B + 0.05W_B^2$$

$$0.1225W_B^2 = 0.07W_B$$

$$\therefore W_B = 0.5714 ※$$

$$W_A = 1 - W_B = 0.4286 ※$$

New expected return : $E(R_P) = W_A E(r_A) + W_B E(r_B)$

$= 0.4286(0.1) + 0.5714(0.2)$

$= 0.15714 = 15.714\%$

Additional return $= E(R_P^{new}) - E(R_P^{old})$

$15.714\% - 10\% = 5.714\%$ ※

```
clc;
clear all;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Two assets : Expected return (R_bar)
c =[1.1 1.2]

%% the variance-covariance matrix (V)
cov = -1*0.1*0.25
var_Ra = 0.1^2
var_Rb = 0.25^2
V = [var_Ra cov; cov var_Rb]
V =[0.1^2 -0.025;-0.025 0.25^2]

V_inv = inv(V)
e =ones(size(c))

alpha =e*V_inv*transpose(c)
sigma =c*V_inv*transpose(c)
delta =e*V_inv*transpose(e)

R_bar =alpha/delta
variance =1/delta
std =sqrt(variance)

%% Question 1
%% the optimal weight for the minimum-variance portfolio
w = variance*V_inv*transpose(e)

%% Question 2
%% the expected return of the the minimum-variance portfolio
R_mv =alpha/delta

%% the variance of the the minimum-variance portfolio
var_mv =1/delta
```

```
c = 1x2
      1.1000    1.2000

cov = -0.0250
var_Ra = 0.0100
var_Rb = 0.0625
V = 2x2
      0.0100   -0.0250
     -0.0250    0.0625

V = 2x2
      0.0100   -0.0250
     -0.0250    0.0625

Warning: Matrix is close to
singular or badly scaled.
Results may be inaccurate. RCOND
=  1.132881e-17.
V_inv = 2x2
10^17 ×
      7.2058    2.8823
      2.8823    1.1529

e = 1x2
        1     1

alpha = 1.5939e+18
sigma = 1.7988e+18
delta = 1.4123e+18
R_bar = 1.1286
variance = 7.0805e-19
std = 8.4146e-10
w = 2x1
      0.7143
      0.2857

R_mv = 1.1286
var_mv = 7.0805e-19
```