Generalized Method of Moment

Estimate Standard Errors

The CAPM model can be stated as:

$$y_t = \alpha + \beta x_t + u_t \qquad (1)$$

where: y_t = Excess return of stock at time t.

x_t = Market excess return at time t.

u_t = Disturbance term at time t.

```
. tsset time, monthly
        time variable:  time, 1980m1 to 1999m12
                delta:  1 month

. reg rendcyco rendmark

      Source |       SS       df       MS              Number of obs =     240
-------------+------------------------------           F(  1,   238) =  241.34
       Model |  7414.37703     1  7414.37703           Prob > F      =  0.0000
    Residual |   7311.8773   238  30.7221735           R-squared     =  0.5035
-------------+------------------------------           Adj R-squared =  0.5014
       Total |  14726.2543   239  61.6161269           Root MSE      =  5.5428

------------------------------------------------------------------------------
    rendcyco |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    rendmark |   1.171128   .0753864    15.54   0.000     1.022619    1.319638
       _cons |   -.447481   .3629428    -1.23   0.219    -1.162472    .2675096
------------------------------------------------------------------------------


. ivregress gmm rendcyco rendmark

Instrumental variables (GMM) regression               Number of obs =     240
                                                      Wald chi2(1)  =  297.26
                                                      Prob > chi2   =  0.0000
                                                      R-squared     =  0.5035
GMM weight matrix: Robust                             Root MSE      =  5.5196

------------------------------------------------------------------------------
             |               Robust
    rendcyco |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    rendmark |   1.171128   .0679256    17.24   0.000     1.037997     1.30426
       _cons |   -.447481   .3421433    -1.31   0.191     -1.11807    .2231076
------------------------------------------------------------------------------
(no endogenous regressors)
```

GMM vs 2SLS

Griliches (1976)'s wage model can be stated as:

$$LW = \alpha + \beta S + \gamma IQ + \delta' \mathrm{h} + \varepsilon \qquad (2)$$

and $\mathrm{h} \equiv (EXPR,\ TENURE,\ RNS, SMSA, YEAR\ DUMMIES)$

where: LW = Log wages

S = Completed years of schooling

IQ = IQ Score

$EXPR$ = Experience in years

$TENURE$ = Tenure in years

RNS = Dummy for residency in the southern states

$SMSA$ = Dummy for residency in the metropolitan areas

$YEAR\ DUMMIES$ = Dummy for years

IQ score and years of completed schooling might be correlated with error term, thus, instrumental variables can be employed. Instrumental variables to be used include:

MED = Mother's education in years

KWW = Score on the "Knowledge of the World of Work" test

MRT = Dummy for marital status (1 if married)

AGE = Age of the individual

Create Dummy Variable

```
. xi i.year
i.year            _Iyear_66-73        (naturally coded; _Iyear_66 omitted)
```

OLS

```
. reg lw s iq expr tenure rns smsa _I*

      Source |       SS       df       MS              Number of obs =     758
-------------+------------------------------           F( 12,   745) =   46.86
       Model |  59.9127611    12  4.99273009           Prob > F      =  0.0000
    Residual |  79.3733888   745  .106541461           R-squared     =  0.4301
-------------+------------------------------           Adj R-squared =  0.4210
       Total |   139.28615   757  .183997556           Root MSE      =  .32641

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .0619548   .0072786     8.51   0.000     .0476658    .0762438
          iq |   .0027121   .0010314     2.63   0.009     .0006873    .0047369
        expr |   .0308395   .0065101     4.74   0.000     .0180592    .0436198
      tenure |   .0421631   .0074812     5.64   0.000     .0274763    .0568498
         rns |  -.0962935   .0275467    -3.50   0.001    -.1503719   -.0422151
        smsa |   .1328993   .0265758     5.00   0.000     .0807268    .1850717
   _Iyear_67 |  -.0542095   .0478522    -1.13   0.258    -.1481506    .0397317
   _Iyear_68 |   .0805808   .0448951     1.79   0.073    -.0075551    .1687168
   _Iyear_69 |   .2075915   .0438605     4.73   0.000     .1214867    .2936963
   _Iyear_70 |   .2282237   .0487994     4.68   0.000      .132423    .3240245
   _Iyear_71 |   .2226915   .0430952     5.17   0.000     .1380889     .307294
   _Iyear_73 |   .3228747   .0406574     7.94   0.000     .2430579    .4026915
       _cons |   4.235357   .1133489    37.37   0.000     4.012836    4.457878
------------------------------------------------------------------------------
```

2SLS with IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress 2sls lw s expr tenure rns smsa _I* (iq = med kww mrt age)

Instrumental variables (2SLS) regression              Number of obs =     758
                                                      Wald chi2(12) =  560.57
                                                      Prob > chi2   =  0.0000
                                                      R-squared     =  0.4255
                                                      Root MSE      =  .32491

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          iq |   .0001747   .0039035     0.04   0.964     -.007476    .0078253
           s |   .0691759   .0129366     5.35   0.000     .0438206    .0945312
        expr |    .029866   .0066393     4.50   0.000     .0168533    .0428788
      tenure |   .0432738   .0076271     5.67   0.000     .0283249    .0582226
         rns |  -.1035897    .029481    -3.51   0.000    -.1613715   -.0458079
        smsa |   .1351148   .0266573     5.07   0.000     .0828674    .1873623
   _Iyear_67 |   -.052598   .0476924    -1.10   0.270    -.1460734    .0408774
   _Iyear_68 |   .0794686   .0447194     1.78   0.076    -.0081797    .1671169
   _Iyear_69 |   .2108962   .0439336     4.80   0.000     .1247878    .2970045
   _Iyear_70 |   .2386338   .0509733     4.68   0.000     .1387281    .3385396
   _Iyear_71 |   .2284609   .0437436     5.22   0.000     .1427251    .3141967
   _Iyear_73 |   .3258944   .0407181     8.00   0.000     .2460884    .4057004
       _cons |    4.39955   .2685443    16.38   0.000     3.873213    4.925887
------------------------------------------------------------------------------
Instrumented:  iq
Instruments:   s expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69
               _Iyear_70 _Iyear_71 _Iyear_73 med kww mrt age

. estat overid

  Tests of overidentifying restrictions:

  Sargan (score) chi2(3) =  87.6552  (p = 0.0000)
  Basmann chi2(3)        =   97.025  (p = 0.0000)
```

2SLS with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress 2sls lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental variables (2SLS) regression              Number of obs =     758
                                                      Wald chi2(12) =  459.55
                                                      Prob > chi2   =  0.0000
                                                      R-squared     =  0.2280
                                                      Root MSE      =  .37665

------------------------------------------------------------------------------
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1724253   .0207381     8.31   0.000     .1317794    .2130712
          iq |  -.0090988   .0047044    -1.93   0.053    -.0183193    .0001216
        expr |   .0492895   .0081546     6.04   0.000     .0333068    .0652722
      tenure |   .0422171   .0088429     4.77   0.000     .0248854    .0595488
         rns |  -.1017935   .0341765    -2.98   0.003    -.1687781   -.0348088
        smsa |   .1261109   .0309275     4.08   0.000     .0654942    .1867277
   _Iyear_67 |  -.0596171   .0552955    -1.08   0.281    -.1679942      .04876
   _Iyear_68 |   .0486796   .0520161     0.94   0.349    -.0532701    .1506292
   _Iyear_69 |   .1528176    .051563     2.96   0.003      .051756    .2538792
   _Iyear_70 |   .1744361   .0597576     2.92   0.004     .0573133    .2915588
   _Iyear_71 |    .091666    .054144     1.69   0.090    -.0144543    .1977863
   _Iyear_73 |   .0932398   .0571819     1.63   0.103    -.0188347    .2053142
       _cons |    4.03351   .3154215    12.79   0.000     3.415295    4.651725
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age

. estat overid

  Tests of overidentifying restrictions:

  Sargan (score) chi2(2) =  13.2683  (p = 0.0013)
  Basmann chi2(2)        =  13.2375  (p = 0.0013)
```

GMM with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress gmm lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental variables (GMM) regression               Number of obs =     758
                                                      Wald chi2(12) =  506.92
                                                      Prob > chi2   =  0.0000
                                                      R-squared     =  0.2168
GMM weight matrix: Robust                             Root MSE      =  .37936

------------------------------------------------------------------------------
             |               Robust
          lw |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           s |   .1757958   .0208514     8.43   0.000     .1349279    .2166637
          iq |  -.0092862   .0049182    -1.89   0.059    -.0189256    .0003533
        expr |   .0502828    .008104     6.20   0.000     .0343992    .0661664
      tenure |   .0425214   .0095601     4.45   0.000     .0237838    .0612589
         rns |  -.1040931     .03373    -3.09   0.002    -.1702026   -.0379835
        smsa |   .1247512   .0309873     4.03   0.000     .0640172    .1854853
   _Iyear_67 |  -.0530432   .0517876    -1.02   0.306    -.1545449    .0484586
   _Iyear_68 |   .0459546   .0500069     0.92   0.358    -.0520572    .1439664
   _Iyear_69 |   .1554801   .0480126     3.24   0.001     .0613772     .249583
   _Iyear_70 |   .1669875   .0613341     2.72   0.006     .0467748    .2872001
   _Iyear_71 |   .0846485    .055817     1.52   0.129    -.0247508    .1940477
   _Iyear_73 |   .0996068   .0612394     1.63   0.104    -.0204201    .2196338
       _cons |   4.003924   .3364754    11.90   0.000     3.344445    4.663404
------------------------------------------------------------------------------
Instrumented:  s iq
Instruments:   expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
               _Iyear_71 _Iyear_73 med kww mrt age

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 11.6015 (p = 0.0030)
```

GMM using Moment Equations

In the study of interest rate structure, the continuous time model can be specified as:

$$r_{t+\Delta t} - r_t = (\alpha + \beta r_t)\Delta t + \varepsilon_{t+\Delta t} \quad (1)$$

where: $E[\varepsilon_{t+\Delta t}] = 0$ and $E[\varepsilon^2_{t+\Delta t}] = \sigma^2 r_t^{2\gamma}\Delta t$

Then, the model can be transformed to be discrete time model by setting $\Delta t = 1$. The discrete time model can be stated as:

$$r_{t+1} - r_t = \Delta r_t = \alpha + \beta r_t + \varepsilon_{t+1} \quad (2)$$

where: $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon^2_{t+1}] = \sigma^2 r_t^{2\gamma}$

The above model consists of four parameters including $\alpha, \beta, \sigma^2, \gamma$. The model can be estimate using GMM. Four moment condition equations can be stated as:

Zero mean condition:	$E(\varepsilon_{t+1}) = 0$
Orthogonality condition:	$E(\varepsilon_{t+1} r_t) = 0$
Variance condition:	$E(\varepsilon^2_{t+1} - \sigma^2 r_t^{2\gamma}) = 0$
Zero covariance condition:	$E\left(\left(\varepsilon^2_{t+1} - \sigma^2 r_t^{2\gamma}\right) r_t\right) = 0$

The above model can be claimed as unrestricted model for other eight interest rate structure models which can be indicated as follows:

Model	α	β	σ^2	γ
Unrestricted				
Merton		0		0
Vasicek				0
CIR SR				0.5
Dothan	0	0		1
GBM	0			1
Brennan & Schwartz				1
CIR VR	0	0		1.5
CEV	0			

Unrestricted model has four unknown parameters to be estimated and four moment condition equations, thus, method of estimate will be method of moment since it is exact identify equations model. Other eight restricted models will be estimated using GMM.

GMM can be performed by using the following command:

Unrestricted model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r^(2*{gamma})) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r)
winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
warning: 1 missing value returned for equation 4 at initial values

Step 1
numerical derivatives are approximate
flat or discontinuous region encountered
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  .00005192  (backed up)
Iteration 2:   GMM criterion Q(b) =  .00004097
Iteration 3:   GMM criterion Q(b) =  3.562e-06
Iteration 4:   GMM criterion Q(b) =  2.174e-07
Iteration 5:   GMM criterion Q(b) =  1.644e-07

Step 2
Iteration 0:   GMM criterion Q(b) =  .01229538
Iteration 1:   GMM criterion Q(b) =   .0055188
Iteration 2:   GMM criterion Q(b) =  .00045234
Iteration 3:   GMM criterion Q(b) =  8.141e-07
Iteration 4:   GMM criterion Q(b) =  2.461e-12
Iteration 5:   GMM criterion Q(b) =  1.451e-23

GMM estimation

Number of parameters =   4
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0009679   .0012837    -0.75   0.451    -.0034839    .0015481
       /beta |   .0014035   .0005181     2.71   0.007     .0003882    .0024189
     /sigma2 |   .0006419   .0001156     5.55   0.000     .0004153    .0008686
      /gamma |   .1273995   .0720773     1.77   0.077    -.0138695    .2686685
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(0) = 2.1e-20 (p =     .)

. est store Unrestricted
```

Merton model:

```
. gmm (dr-{alpha}) ((dr-{alpha})*r) ((dr-{alpha})^2-{sigma2}) (((dr-{alpha})^2-
{sigma2})*r) winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
```

```
warning: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  6.386e-07
Iteration 2:   GMM criterion Q(b) =  6.382e-07

Step 2
Iteration 0:   GMM criterion Q(b) =  .00760718
Iteration 1:   GMM criterion Q(b) =  .00652536
Iteration 2:   GMM criterion Q(b) =  .00652532

GMM estimation

Number of parameters =   2
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |   .0021651   .0007043     3.07   0.002     .0007847    .0035456
     /sigma2 |   .0008218   .0000853     9.63   0.000     .0006546     .000989
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(2) = 9.32468 (p = 0.0094)

. est store Merton
```

Vasicek model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}) (((dr-{alpha}-{beta}*r)^2-{sigma2})*r) winitial(identity)

warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
warning: 1 missing value returned for equation 4 at initial values

Step 1
Iteration 0:   GMM criterion Q(b) =  .00005673
Iteration 1:   GMM criterion Q(b) =  3.000e-09
Iteration 2:   GMM criterion Q(b) =  2.107e-09

Step 2
Iteration 0:   GMM criterion Q(b) =  .00263124
Iteration 1:   GMM criterion Q(b) =  .00226079
Iteration 2:   GMM criterion Q(b) =  .00226074

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                       Number of obs  =    1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0003515   .0012376    -0.28   0.776     -.002777    .0020741
       /beta |   .0012479   .0005106     2.44   0.015     .0002471    .0022487
     /sigma2 |   .0007951   .0000862     9.23   0.000     .0006262    .0009641
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
```

```
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 3.23059 (p = 0.0723)

. est store Vasicek
```

CIR SR model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r)*r) winitial(identity) nolog

warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
warning: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) =  .0117512

GMM estimation

Number of parameters =   3
Number of moments    =   4
Initial weight matrix: Identity                         Number of obs   =      1429
GMM weight matrix:     Robust

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /alpha |  -.0031028   .0011644    -2.66   0.008     -.005385   -.0008207
       /beta |   .0018442   .0005066     3.64   0.000     .0008514    .0028371
     /sigma2 |   .0002351   .0000278     8.45   0.000     .0001805    .0002896
------------------------------------------------------------------------------
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

. estat overid

  Test of overidentifying restriction:

  Hansen's J chi2(1) = 16.7924 (p = 0.0000)

. est store CIR_SR

. est table Unrestricted Merton Vasicek CIR_SR, star(0.1 0.05 0.01)

----------------------------------------------------------------------------
    Variable | Unrestricted       Merton          Vasicek         CIR_SR
-------------+--------------------------------------------------------------
alpha        |
       _cons | -.00096793       .00216513***   -.00035145      -.00310284***
-------------+--------------------------------------------------------------
beta         |
       _cons |  .00140354***                    .00124786**     .00184424***
-------------+--------------------------------------------------------------
sigma2       |
       _cons |  .00064191***    .00082182***    .00079513***    .00023507***
-------------+--------------------------------------------------------------
gamma        |
       _cons |  .12739949*
----------------------------------------------------------------------------
                                          legend: * p<.1; ** p<.05; *** p<.01
```