

1.a

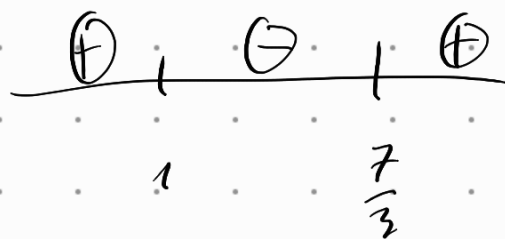
$$y = x^3 - 5x^2 + 7x - 5$$

$$y' = 3x^2 - 10x + 7$$

$$3x^2 - 10x + 7 = 0 \quad \text{to find critical points}$$

$$(3x - 7)(x - 1) = 0$$

$$x = \frac{7}{3}, 1$$



→ substitutes into first derivative

Domain where  $y$  is increasing:  $(-\infty, 1) \cup (\frac{7}{3}, \infty)$

1.b Domain of  $y = (-\infty, \infty)$

no, it is not concave

for all domain since

the turning point suggest that

there is convexity.

$$y'' = 6x - 10$$

$$= 6\left(\frac{7}{3}\right) - 10 = 4$$

∴ convex at  $\frac{7}{3}$

## Question 2

$$a.) \quad MP_L = \frac{\Delta Q}{\Delta L} = 12L - 3L^2 \quad \#$$

$$AP_L = \frac{Q(L)}{L} = \frac{6L^2 - L^3}{L} = \cancel{6L} - \cancel{L^2} = 6L - L^2 \quad \#$$

$$b.) \quad \frac{dAP}{dL} = 6 - 2L \Rightarrow 6 - 2L = 0$$

$$6 = 2L$$

$$L = 3 \quad \#$$

$$\frac{d^2 AP}{dL^2} = -2 < 0 \quad \text{globally concave function}$$

maximum point

$$c.) \quad MP_L = \frac{\Delta Q}{\Delta L} = 12L - 3L^2$$

Maximum point

$$12L - 3L^2 = 0$$

$$L(12 - 3L) = 0 \rightarrow L = 0, 4$$

MP, AP

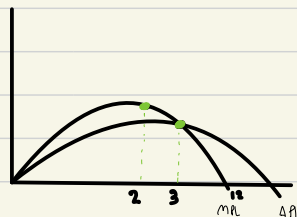
$$\frac{dMP_L}{dL} = 12 - 6L = 12 - 6L$$

$$12 - 6L = 0$$

$$12 = 6L$$

$$2 = L$$

sub L in  $6L^2 - L^3$ ;  $Q = 32$



$$d.) \quad 2P = 100 - Q$$

$$P = 50 - 0.5Q$$

$$\text{Revenue} = P(Q) \cdot (Q)$$

$$= 50 - 0.5Q \cdot Q$$

$$= 50Q - 0.5Q^2$$

$$MRP = \frac{dR}{dQ} = 50 - Q \quad \#$$

3.)

$$\begin{aligned} \text{a.) Monopoly: } R(Q) &= P(Q) \cdot Q \\ &= \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right) \cdot Q \\ &= 40Q + 105 - \frac{3}{2}Q^3 \end{aligned}$$

level of revenue - maximizing output:  $MR = 0$

$$-\frac{9}{2}Q^2 + 40 = 0$$

$$Q = \frac{4\sqrt{5}}{3} \approx 3 \quad \#$$

elasticity of demand at the level of output:  $\epsilon = \frac{\% \Delta Q}{\% \Delta P} = \frac{dQ}{dP} \cdot \frac{P}{Q}$

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

$$\frac{dP}{dQ} = -105Q^{-2} - 3Q$$

$$= \frac{-105}{Q^2} - 3Q$$

$$Q=3; \quad \frac{dP}{dQ} = \frac{-62}{3}$$

$$Q=3; \quad \frac{P}{Q} = \left(40 + \frac{105}{Q} - \frac{3}{2}Q^2\right) \div Q$$

$$= \frac{40}{Q} + \frac{105}{Q^2} - \frac{3}{2}Q$$

$$= \frac{40}{3} + \frac{105}{9} - \frac{3}{2}(3) = \frac{41}{2}$$

$$\Rightarrow \epsilon = \frac{\% \Delta Q}{\% \Delta P} = \frac{dQ}{dP} \cdot \frac{P}{Q}$$

$$= \frac{1}{-62/3} \cdot \frac{41}{2}$$

$$= \frac{-123}{124} \quad \#$$

3b.)

$$\begin{aligned} \pi(Q) &= R(Q) - C(Q) \\ P(Q) \cdot Q - C(Q) &= \left(40Q + 105 - \frac{3}{2}Q^3\right) - (6Q^3 - 81Q^2 - 175Q + 10) \end{aligned}$$

3.c)

$$\begin{aligned}\pi(Q) &= R(Q) - C(Q) \\ &= (40Q + 105 - \frac{3}{2}Q^2) - (6Q^3 - 81Q^2 - 175Q + 10) \\ &= 40Q + 105 - \frac{3}{2}Q^2 - 6Q^3 + 81Q^2 + 175Q - 10 \\ &= \frac{-15}{2}Q^3 + 81Q^2 + 215Q + 95\end{aligned}$$

$$\text{F.O.C. } \frac{d\pi}{dQ} = -\frac{45}{2}Q^2 + 162Q + 215$$

$$\begin{aligned}\hat{Q} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} ; \quad Q = \frac{-162 \pm \sqrt{162^2 - 4(-\frac{45}{2})(215)}}{2(-\frac{45}{2})} \\ &= \frac{-162 \pm 213.528}{-45} \\ &= 6.345, \quad -1.45\end{aligned}$$

$$\text{S.O.C. } \frac{d^2\pi}{dQ^2} = -45Q + 162$$

$$\text{Check: } Q^* = 6.345 ; \quad -45(6.345) + 162 = -213.53 < 0$$

concave

$\therefore 6.345$  is the profit-maximizing output.

D) The equilibrium of both the price and the output will remain constant. The only change will be the decrease in the profit.

4a)

$$\pi = p \cdot q - \text{STC} = pq - 0.5q^2 - 10q - 5$$

$$q^* = p - 10$$

$$\text{FOC} \quad \frac{d\pi}{dq} = p - q - 10 = 0$$

$$q = p - 10$$

$$\text{SOC} \quad \frac{d^2\pi}{dq^2} = -1 < 0$$

$\pi$  is global concave  
 $q$  is max profit

$$\text{STC} = 0.5q^2 + pq + 5$$

$$\text{SVC} = 0.5q + 10$$

$$\text{SFC} = 5$$

$$\text{AVC} = \frac{\text{SVC}}{q} = \frac{0.5q^2 + 10q}{q} = 0.5q + 10$$

$$\text{min (AVC); } q = 0$$

$$\text{AVC} > 0$$

$$q^s = q^* = \begin{cases} p - 10, & p > 10 \\ 0, & p \leq 10 \end{cases}$$

$$n = 100$$

$$\sum_{i=1}^n q_i^s(p) - q^s = 100(p - 10)$$

$$> 100p - 1000$$

$$q^s = \begin{cases} 100p - 1000, & p > 10 \\ 0, & p \leq 10 \end{cases}$$

4b.

$$Q^d = \begin{cases} 1100 - 50p, & p < 22 \\ 0, & p \geq 22 \end{cases}$$

$$q^s_{\text{market}} = \begin{cases} 100p - 1000, & p > 10 \\ 0, & p \leq 10 \end{cases}$$

$$1100 - 50p = 100p - 1000$$

$$2100 = 150p$$

$$p^* = 14$$

$$Q^d = 1100 - 50(14)$$

$$Q^* = 400$$

400/100 for each firm

$$\pi = (14)(4) - 0.5(4^2) - 10(4) - 5$$

$$= 3$$

4c.

$$\text{tax} = \text{€}3$$

$$P^s + t = P^d \text{ and } P^d - t = P^s$$

$$1100 - 50P^d = Q^d$$

$$P^d = \frac{1100 - Q^d}{50}$$

$$100P^s - 1000 = Q^s$$

$$P^s = \frac{Q^s + 1000}{100}$$

$$P^d - t = P^s$$

$$P^d - 3 = P^s$$

$$\frac{1100 - Q^d}{50} - 3 = \frac{Q^s + 1000}{100}$$

$$\frac{1100 - Q^d - 150}{50} = \frac{Q^s + 1000}{100}$$

$$4500 = 150Q^d$$

$$Q^d = 300$$

$$\text{after tax } P^s = \frac{300 + 1000}{100} = 13$$

$$P^d = P^s + t = 13 + 3 = 16$$

4d.

	B/F tax		A/A tax
Q	400		300
pd	14	+2	16
ps	14	-1	13
Tax	0		900

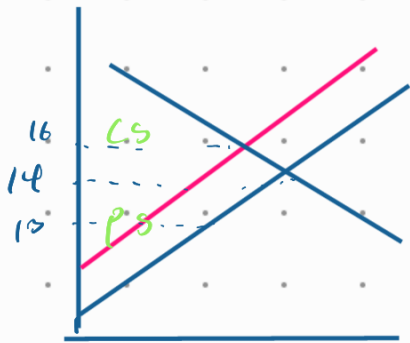
$$\text{Tax revenue} = 3 \times 300 = 900$$

$$\text{producer pay} : (16 - 14) 300 = 600 \rightarrow 66.6\%$$

$$\text{consumer pay} : (14 - 13) 300 = 300 \rightarrow 33.3\%$$

$$\text{Burden}$$
$$\text{Producer} : \frac{600}{300} = \$2/\text{unit}$$
$$\text{Consumer} : \frac{300}{300} = \$1/\text{unit}$$

4e.



$$A/P \text{ tax} = \frac{1}{2}(3)(300) = 450$$

$$B/P \text{ tax} = \frac{1}{2}(4)(400) = 800$$

$$\Delta \text{ Producer surplus} = -350$$

$$\begin{aligned} \text{profit a/p tax} &= PQ - STC \\ &= (13)(3) - (0.5)(3)^2 - 10(3) - 5 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} \text{profit for industry with 100 identical firms} \\ &= -0.5 \cdot 100 \\ &= -50 \end{aligned}$$

$$\begin{aligned} \text{profit before tax} &= (14)(4) - 0.5(4)^2 - 10(4) - 5 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{profit with 100 firms} \\ &= 3 \cdot 100 = 300 \end{aligned}$$

$$\Delta X = \text{profit a/p} - \text{profit w/p}$$

$$= -50 - 300$$

$$= -350$$