B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Semester 2/2013)

Practice Problem 10

(Integrations and Its Applications) [1]

Suggested Answers

1. In the manufacture of a product, the marginal cost of producing x units is C'(x) and fixed cost are C(0). Find the total cost function C(x) when:

(a) $C'(x) = 3x + 4$, $C(0) = 40$. Ans. $C(x) = \frac{3}{2}x^2 + 4x + 40$

(b) $C'(x) = ax + b$, $C(0) = C_0$. Ans. $C(x) = \frac{1}{2}ax^2 + bx + C_0$

2. (a) Find F(x) if $F'(x) = \frac{1}{2}e^x - 2x$ and $F(0) = \frac{1}{2}$.

Ans. $F(x) = \frac{1}{2}e^x - x^2$

(b) Find F(x) if $F'(x) = x(1 - x^2)$ and $F(0) = \frac{5}{12}$.

Ans. $F(x) = \frac{1}{2}x^2 - \frac{1}{4}x^4 + \frac{5}{12}$

3. Compute the area A bounded by the graph of $f(x) = \frac{1}{x^3}$, the x-axis, and the two lines x = -2 and x = -1. Make a drawing. (Hint: f(x) < 0 in [-2, -1].)

Ans. $A = 3/8$.

[1] All questions are from Sydsaeter and Hammond, 2008.

4. Compute the area A bounded by the graph of $f(x) = \frac{1}{2}(e^x - e^{-x})$, the x-axis, and the two lines x = -1 and x = 1.

Ans. $A = e + \frac{1}{e} - 2$

5. (a) The profit of a firm as a function of its output x is given by

$$f(x) = 4000 - x - \frac{3000000}{x}, \ x > 0$$

Find the level of output that maximizes profit. Sketch the graph of f.

Ans. $x = 1000\sqrt{3}$

(b) The actual output varies between 1000 and 3000 units. Compute the average profit

$I = \frac{1}{2000}\int_{1000}^{3000} f(x)dx.$

Ans. $I = 2000 - 1500\ln 3 \approx 352$

6. Let $K(t)$ denote the capital stock of an economy at time t. Then net investment at time t, denoted by $I(t)$, is given by the rate of increase $\dot{K}(t)$ of K(t). [Note: $\dot{K}(t) = \frac{dK}{dt}$.]

(a) If $I(t) = 3t^2 + 2t + 5, t \geq 0$, what is the total increase in the capital stock during the interval from t = 0 to t = 5?

Ans. K(5) – K(0) = 175.

(b) If $K(t_0) = K_0$, find an expression for the total increase in the capital stock from time $t = t_0$ to $t = T$ when the investment function $I(t)$ is as in part (a).

Ans. $K(T) - K_o = (T^3 - t_0^3) + (T^2 - t_0^2) - 5(T - t_o)$

7. Suppose that the demand and supply curves are $P = f(Q) = 200 - 0.2Q$ and $P = g(Q) = 20 + 0.1Q$, respectively. Find the equilibrium quantity and compute the consumer and producer surplus.

Ans. (Q*, P*) = (600, 80). Consumer surplus = 36000, and producer surplus = 18000.

8. Suppose that the demand and supply curves are $P = f(Q) = \frac{6000}{Q+50} P = g(Q) = Q + 10$. Find the equilibrium price, and compute the consumer and producer surplus.

Ans. (Q*, P*) = (50, 60). Consumer surplus = 6000ln2 - 3000, and producer surplus = 1250.

9. Evaluate the following integrals by using integrations by parts (r ≠ 0).

(a) $\int_0^T bte^{-rt}dt$ Ans. $br^{-2}[1 - (1 + rT)e^{-rT}]$

(b) $\int_0^T (a + bt)e^{-rt}dt$ Ans. $ar^{-1}(1 - e^{-rT}) + br^{-2}[1 - (1 + rT)e^{-rT}]$

10. Evaluate the following integrals by using integrations by substitution:

(a) $\int_0^1 x\sqrt{1 + x^2}dx$

Ans. Let $u = \sqrt{1 + x^2}$. Thus, $\int_0^1 x\sqrt{1 + x^2}dx = \int_1^{\sqrt{2}} u^2 du = \frac{1}{3}(2\sqrt{2} - 1)$.

(b) $\int_1^e \frac{lny}{y} dy$

Ans. Let u = lny. $\int_1^e \frac{lny}{y} dy = \frac{1}{2}$.