Notes on Solving a model on credit

Lecture 8 addition

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A simple model of credit with monitoring cost

- k = investment amount
- $f(k)$ = gross return (from investment)
- w = wealth
- $k - w$ = loan borrowing (credit amount)
- i = interest rate for a borrower to pay to a lender
  - Borrower needs to pay $i(k - w)$ at the end
- h = cost of not repaying loan (proportion to investment)
- r = cost of capital (a lender pays return/interest back to a depositor)
- c = cost of monitoring to try to make sure of borrower's repayment
- Loan will be provided if only the borrower repay back the loan. That is when the borrower's net return from repayment is higher than that from default:
  - $f(k) - i(k - w) \geq f(k) - hk$
  - $i(k - w) = hk$ (1)
- The lender need to set an interest rate i high enough to cover the cost (max profit of lending).

- $\underset{i}{Max}\ i(k-w)-r(k-w)-c$

- Suppose we have a competitive market, zero profit condition yields
  - $i(k-w)=r(k-w)+c$ (2)
- Solve for $k, i, (k-w)$ from (1) and (2)
- We have (1) = (2): $hk=r(k-w)+c$
  - $hk=rk-rw+c$
  - $rw-c=(r-h)k$
  - $k=\frac{rw-c}{r-h}$ (3)
- From (2), $i=\frac{r(k-w)}{(k-w)}+\frac{c}{(k-w)}$
  - $i=r+\frac{c}{(k-w)}$ (4)
- From (1), let $L\equiv k-w$. Then, we have $L=\frac{hk}{i}$
  - Substituting k from (3), and i from (4), we have $L=\frac{h(\frac{rw-c}{r-h})}{r+\frac{c}{k-w}}$
  - $L\left(r+\frac{c}{L}\right)=h\left(\frac{rw-c}{r-h}\right)$
  - $rL+c=h\left(\frac{rw-c}{r-h}\right)$
  - $L=\frac{1}{r}\left[h\left(\frac{rw-c}{r-h}\right)-c\right]$
  - $L=\frac{h}{r}\left(\frac{rw-c}{r-h}\right)-\frac{c}{r}$
  - $L=\frac{hrw}{r(r-h)}-\frac{hc}{r(r-h)}-\frac{c}{r}$
  - $L=\frac{hw}{(r-h)}-\left[\frac{hc}{r(r-h)}+\frac{c(r-h)}{r(r-h)}\right]$
  - $L=\frac{hw}{(r-h)}-\left[\frac{hc+cr-ch}{r(r-h)}\right]$
  - $L=\frac{hw}{(r-h)}-\frac{cr}{(r-h)}$
  - $L=k-w=\frac{hw-c}{r-h}$ (5)
- Substituting (5) into (4), $i=r+c\cdot\frac{r-h}{hw-c}$ (6)