October 7, 2025

Production in the Long-Run

(L) (K)
Labor, Capital

The Long Run: No Fixed Factors

- All inputs are variable
- In making these choice, the profit maximizing firm will try to be technically efficient by using no more of all inputs than necessary
- **Technical efficiency** – when a given number of inputs are combined in such a way as to maximize the level of output
- Technical efficiency is not enough… In order to maximize its profit, the firm must choose from among the many technically efficient options, the one that produces a given level of output at lowest cost

Profit Maximization and Cost Minimization

- Any firm that is trying to maximize its profits in the long run should select the production method that produces its output at the **lowest possible cost**

- This implication of the hypothesis of profit maximization is called cost minimization

Isoquant Analysis

MRTS = Marginal rate of technical substitution

Production In The Long Run

- ***Isoquant:*** the set of all input combinations that yield a given level of output.

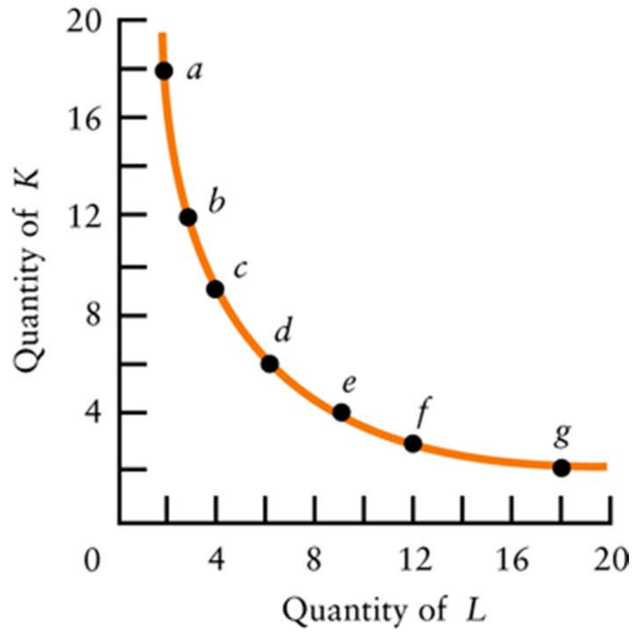
- ***Marginal rate of technical substitution (MRTS):*** the rate at which one input can be exchanged for another without altering the total level of output.

Holding output constant, the less we have of one input, the more we must add to the other input to compensate for a one-unit reduction in the first input.

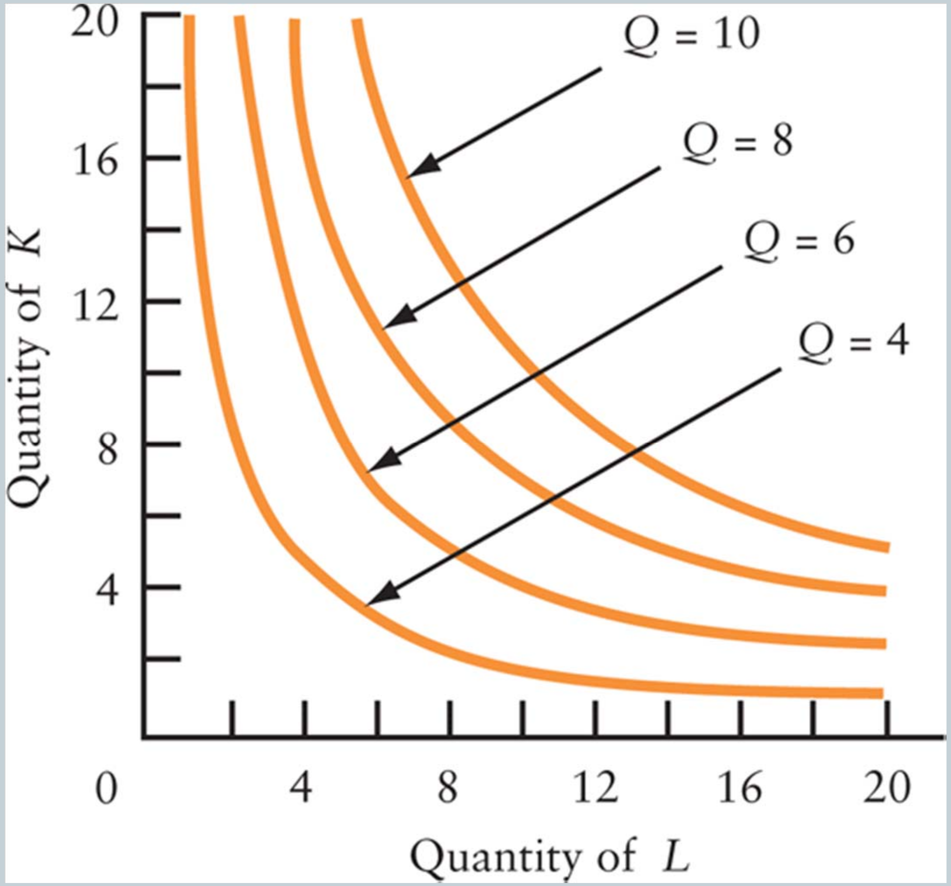
An Isoquant

Alternative Methods of Producing a Given Level of Output

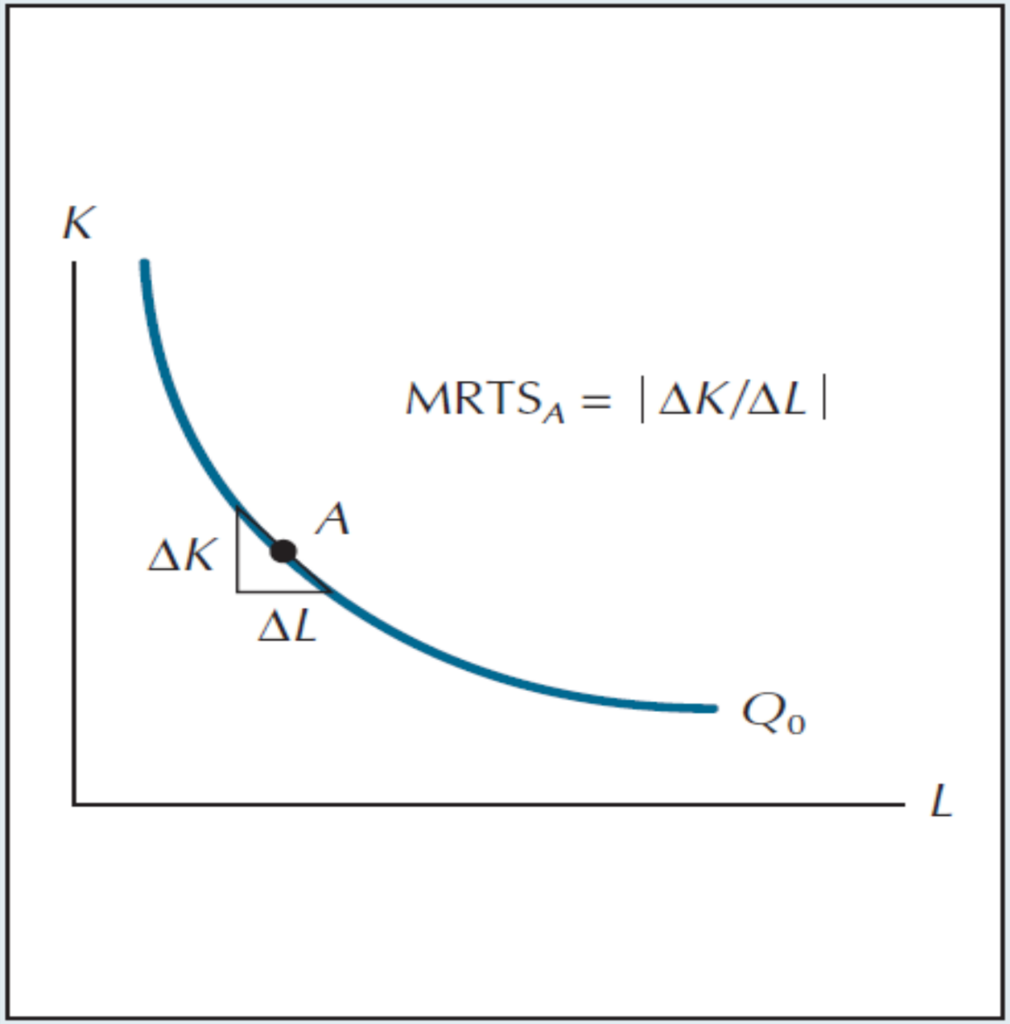
Method	K	L	ΔK	ΔL	Marginal Rate of Substitution (absolute value of $\Delta K/\Delta L$)
a	18	2			
			−6	1	6.00
b	12	3			
			−3	1	3.00
c	9	4			
			−3	2	1.50
d	6	6			
			−2	3	0.67
e	4	9			
			−1	3	0.33
f	3	12			
			−1	6	0.17
g	2	18			



An Isoquant Map



The Marginal Rate of Technical Substitution



Suppose we reduce K by ΔK and augment L by an amount ΔL just sufficient to maintain the original level of output.

If MP_{KA} denotes the marginal product of capital at A, then the reduction in output caused by the loss of ΔK is equal to $MP_{KA} \Delta K$.

Using MP_{LA} denotes the marginal product of L at A, it follows similarly that the gain in output resulting from the extra ΔL is equal to $MP_{LA} \Delta L$.

MP_K	Marginal product of capital
MP_L	Marginal product of labor

Finally, since the reduction in output from having less K is exactly offset by the gain in output from having more L, it follows that

$$MP_{KA} \Delta K = MP_{LA} \Delta L$$

Cross-multiplying, we get

$$\frac{MP_{LA}}{MP_{KA}} = \frac{\Delta K}{\Delta L}$$

which says that the MRTS at A is simply the ratio of the marginal product of L to the marginal product of K.

Long-Run Cost Minimization

A firm is not minimizing costs if it is possible to substitute one factor for another to keep output constant while reducing total cost:

- The firm should substitute one factor for another factor as long as the marginal product of one factor **per dollar spent on it** is greater than the marginal product of the other factor **per dollar spent on it**.

Using ***K*** and ***L*** to represent capital and labor, and $\boldsymbol{p_L}$ and $\boldsymbol{p_K}$ to represent the prices for the two factors, cost is minimized when:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L}$$

Whenever the ratio of the MP of each factor to its price is not equal for all factors, there are possibilities for factor substitutions that will reduce costs (for a given level of output)

$MP_K = 40$
$P_K = \$10$

Example

$MP_L = 20$
$P_L = \$2$

Suppose the marginal product of capital is 40 units of output and the price of one unit of capital is \$10. The marginal product of labor is 20 units of output and the price of one unit of labor is \$2.

$$\frac{MP_K}{P_K} = \frac{40}{10} = 4 < \frac{MP_L}{P_L} = \frac{20}{2} = 10$$

In this case, the firm can reduce the cost of producing its current level of output by using more labor and less capital.

Another Interpretation

Rearranging terms:

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L} \quad \rightarrow \quad \frac{MP_K}{MP_L} = \frac{P_K}{P_L}$$

The ratio of the marginal products on the left side compares the contribution of output to the last unit of capital and the last unit of labor.

The right hand side shows **how the cost of an additional unit of capital compares to the cost of an additional unit of labor**

Example

$$\frac{MP_K}{MP_L} = \frac{40}{20} = 2 \qquad \frac{P_K}{P_L} = \frac{10}{2} = 5$$

The left side of the equation equals 2 but the right hand side equals to 5. The last unit of capital is twice as productive as the last unit of labor but it is five times as expensive.

It will pay the firm to switch to a method of production that uses less capital and more labor.

Only when the ratio of marginal products is exactly equal to the ratio of factor prices is the firm using the cost minimizing production method

The Principle of Substitution

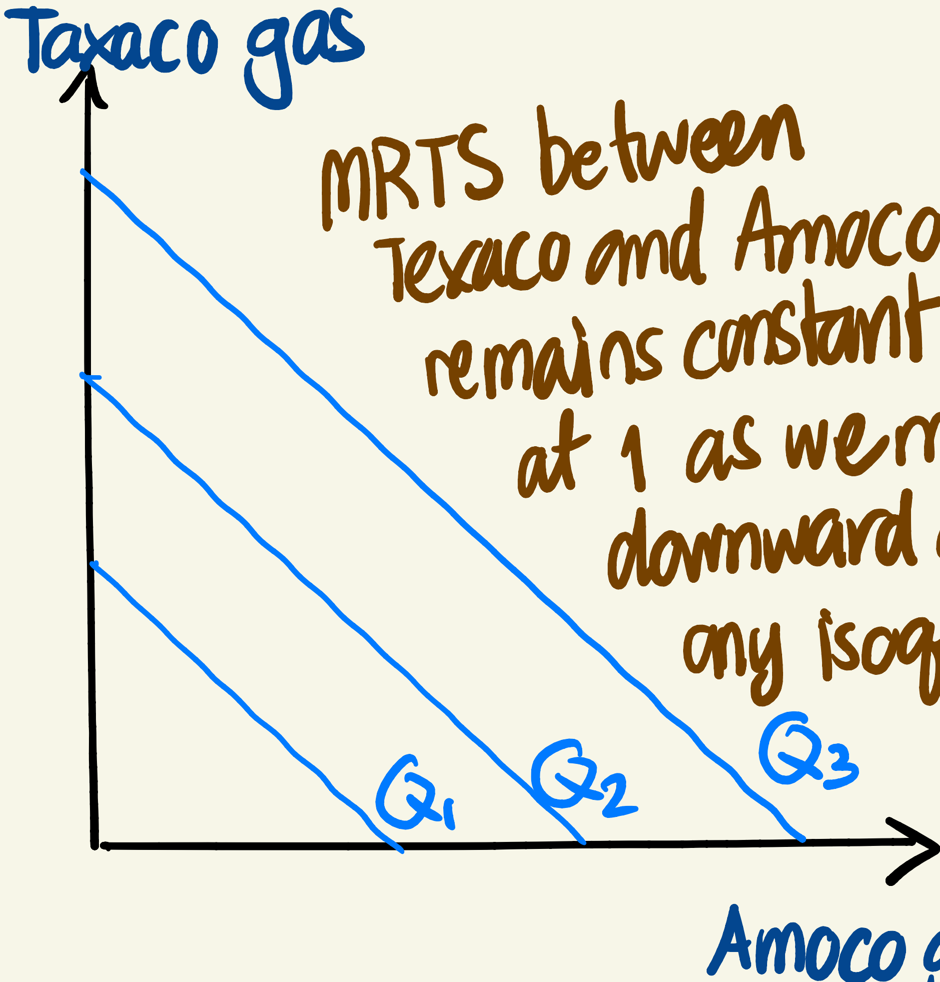
The **principle of substitution**: firms adjust the quantities of factors in response to changing relative factor prices

Firms use more of the cheaper factor and less of the more expensive factor

The principle plays a central role in resource allocation because it relates to the way in which individual firms respond to changes in relative factor prices that are caused by the changing relative scarcities of factors in the economy as a whole.

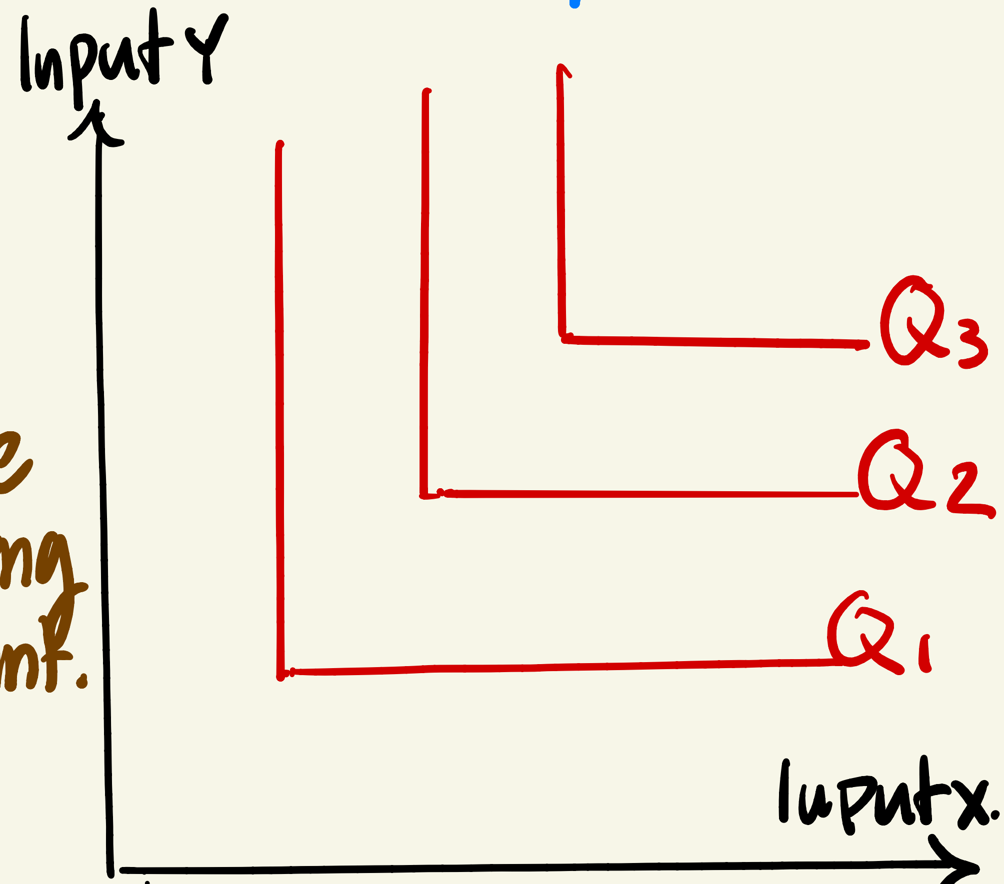
Isoquants maps for perfect substitutes and perfect complements

Perfect substitutes



MRTS between Texaco and Amoco remains constant at 1 as we move downward along any isoquant.

Perfect Complements



Inputs are most effectively combined in fixed proportion.

We can get the same number of trips from a given total quantity of gasoline, no matter how we mix the two brands.

Amoco and Texaco are perfect substitutes in the production of automobile trips

Returns to scale

The technical property of the production function used to describe the relationship between scale and efficiency

The term tells us what happens to output when all inputs are increased by exactly the same proportion.

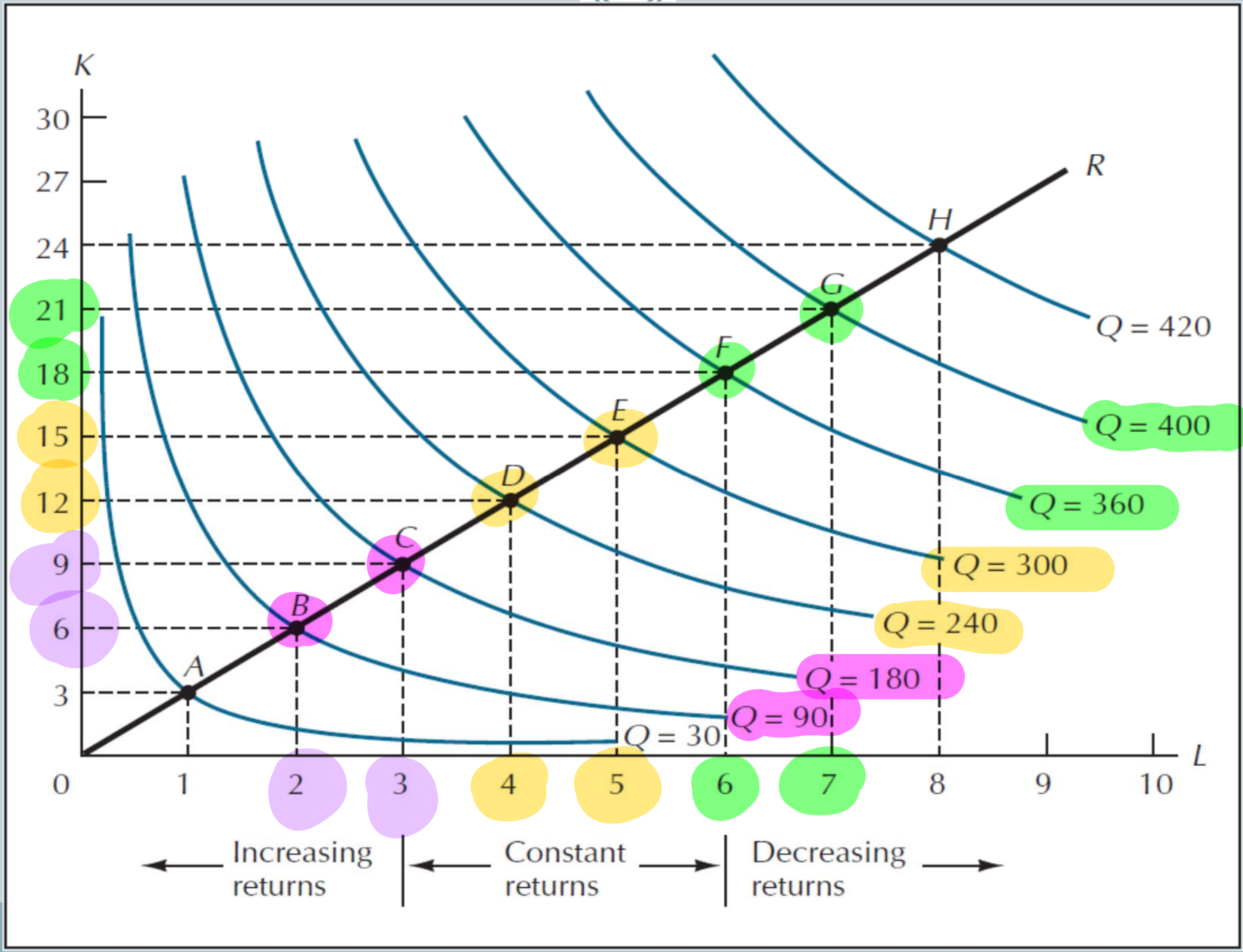
The concept of returns to scale is an inherently long run concept

A production function for which any given proportional change in all inputs leads to a more than proportional change in output is said to exhibit “increasing returns to scale”.

A production function for which a proportional change in all inputs causes outputs to change by the same proportion is said to exhibit “constant returns to scale”.

A production function for which a proportional change in all inputs causes a less than a proportional change in output is said to exhibit “decreasing returns to scale”.

Returns to Scale Shown on the Isoquant Map



From B to C (Increasing Return to Scale)

Input

K From 6 to 9

$$\frac{9-6}{6} \times 100 = \frac{3}{6} \times 100 = \frac{1}{2} \times 100 = 50\%$$

L From 2 to 3

$$\frac{3-2}{2} \times 100 = \frac{1}{2} \times 100 = 50\%$$

Output

90 to 180 =

$$\frac{180-90}{90} \times 100 = \frac{90}{90} \times 100 = 100\%$$

From D to E (Constant Return to Scale)

Input

K From 12 to 15 $= \frac{15-12}{12} \times 100 = \frac{3}{12} \times 100$

$= \frac{1}{4} \times 100 = 25\%$

L From 4 to 5 $= \frac{5-4}{4} \times 100 = \frac{1}{4} \times 100 = 25\%$

Output

240 to 300 $= \frac{300-240}{240} \times 100 = \frac{60}{240} \times 100$

$= \frac{1}{4} \times 100 = 25\%$

From F to G (Decreasing Return to Scale)

Input

K From 18 to 21 = $\frac{21-18}{18} \times 100 = \frac{3}{18} \times 100$

$= \frac{1}{6} \times 100 = 16.67\%$

L From 6 to 7 = $\frac{7-6}{6} \times 100 = \frac{1}{6} \times 100 = 16.67\%$

Output

360 to 400 = $\frac{400-360}{360} \times 100 = \frac{40 \times 100}{360}$

$= 11.11\%$

- The region from A to C exhibits increasing returns to scale

  From B to C both inputs grow by 50 percent while output grows by 100 percent

- The region from C to F exhibits constant return to scale

  When we move from D to E, both inputs grow by 25 percent and output also grows by 25 percent

- The region to the northeast of F exhibits decreasing returns to scale

  When we move from F to G, both inputs grow by 16.7 percent and output also grows by only 11.1 percent

The distinction between diminishing returns and decreasing returns to scale

The distinction between diminishing returns and decreasing returns to scale

Decreasing returns to scale refer to what happens when all inputs are varied by a given proportion.

The law of diminishing returns, by contrast, refers to the case in which one input varies while all others are hold fixed.

A production function for which a proportional change in all inputs causes a less than a proportional change in output is said to exhibit “decreasing returns to scale”.

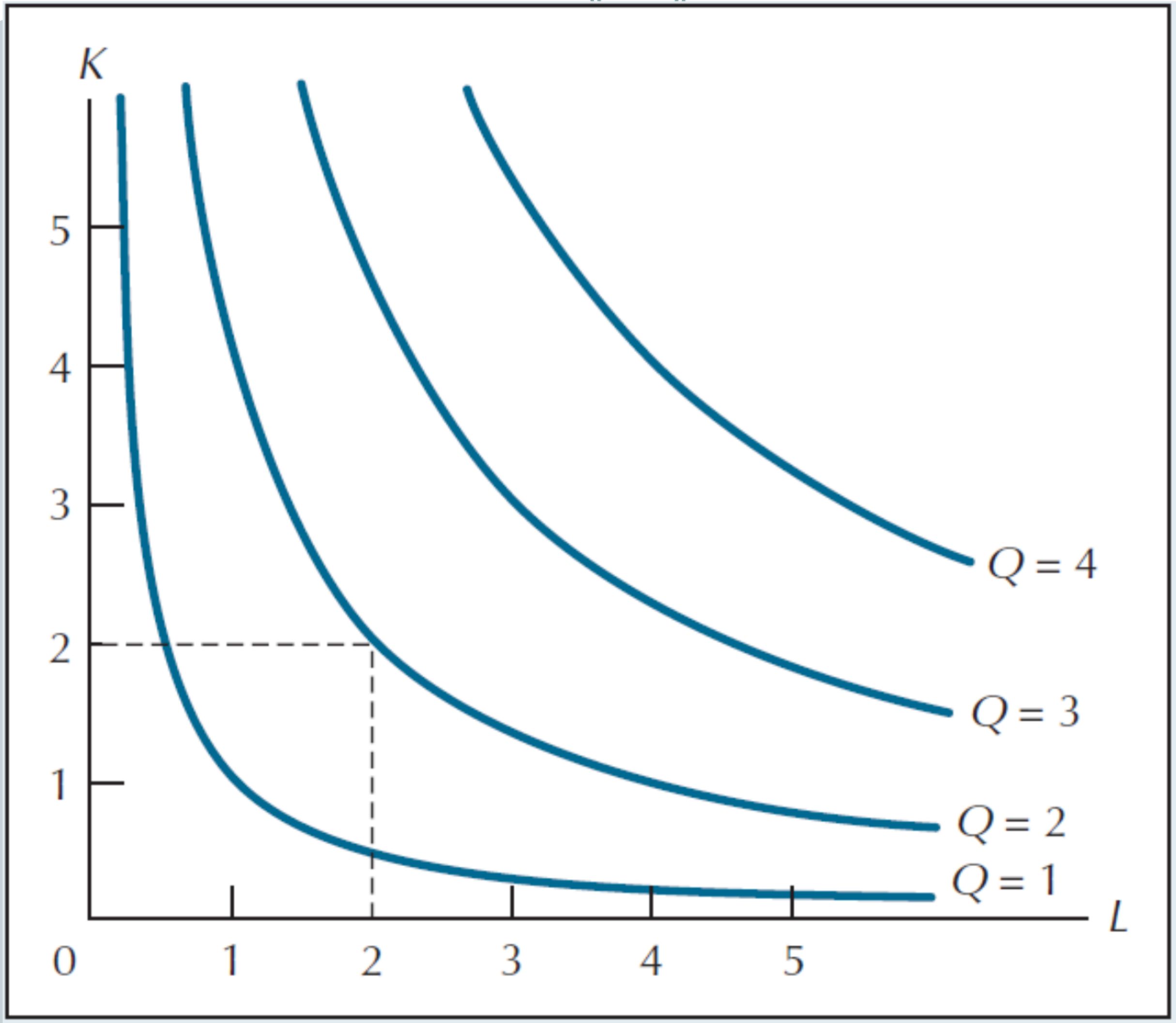
Diminishing Marginal Product

The law of diminishing returns:

As more workers are added to a production process, each can specialize on one task, and the workers' marginal product initially rises.

But if there is a fixed amount of physical capital, eventually the marginal product is likely to fall.

Example: Isoquant Map for the Cobb-Douglas Production Function $Q = K^{1/2}L^{1/2}$



$$Q = mK^{\alpha}L^{\beta}$$

where α and β are number between 0 and 1

m can be any positive number.

To generate an equation for the Q_0 isoquant, we fix Q at Q_0 and then solve for K in terms of L.

In the Cobb-Douglas case, this yields

$$K = \left(\frac{m}{Q_0}\right)^{-1/\alpha} L^{-\beta/\alpha}$$

For the particular Cobb-Douglas function $Q = K^{1/2}L^{1/2}$, the Q_0 isoquant will be

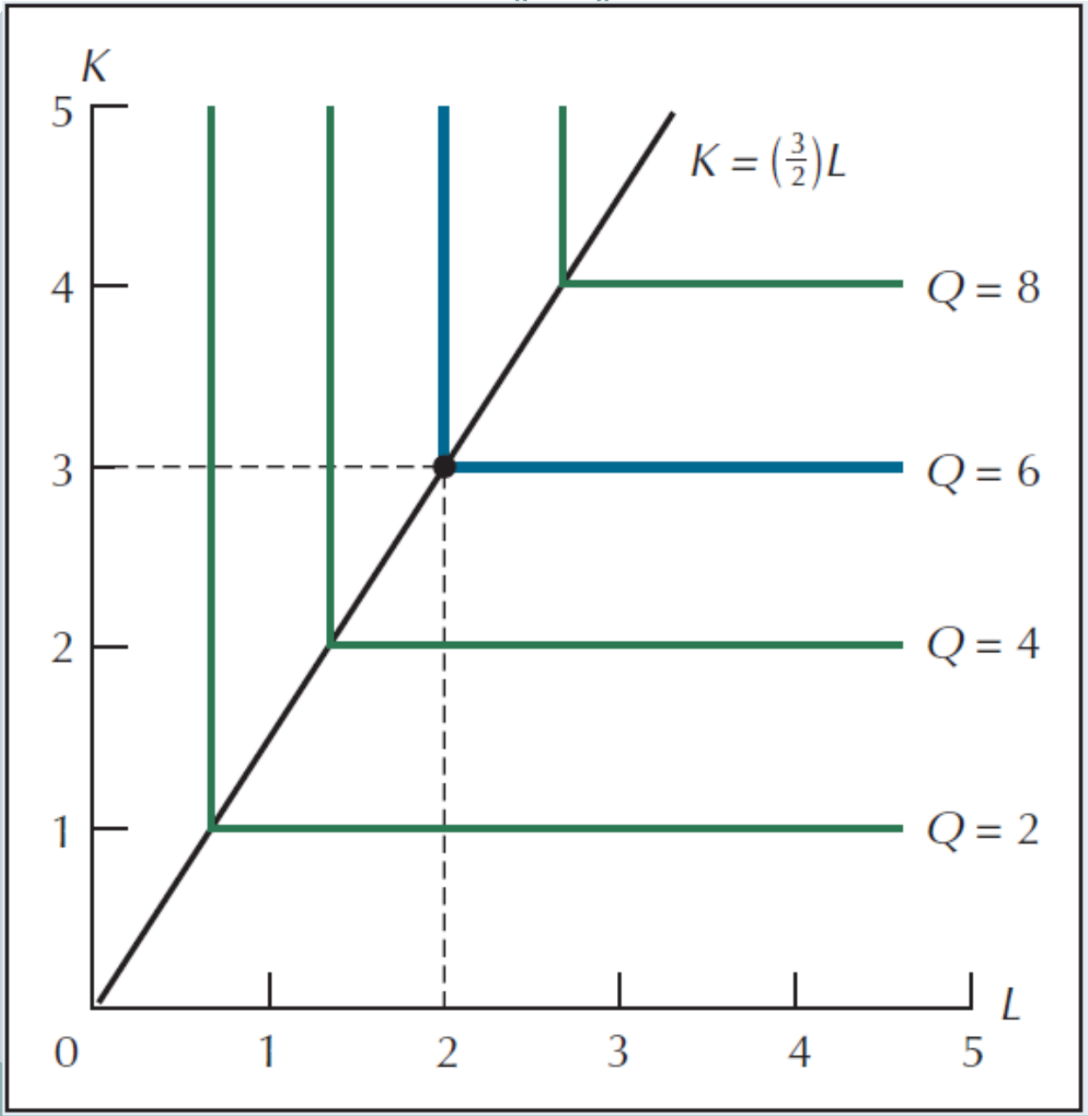
$$K = \frac{Q_0^2}{L}$$

A portion of the isoquant map for this particular Cobb-Douglas production function is shown in Figure.

$$MP_K = \frac{\partial Q}{\partial K} = \alpha m K^{\alpha-1} L^{\beta}$$

$$MP_L = \frac{\partial Q}{\partial L} = \beta m K^{\alpha} L^{\beta-1}$$

Example: Isoquant Map for the Leontief Production Function Q = min (2*K*,3*L*)



Sources:

- Lipsey, Ragan, and Storer (2008)
- Frank, R.H. (2010)