Exercise 1 (Part 1)

1. Determine whether the statement forms are logically equivalent. In each case, construct a truth table to justify your answer.

   (a) $(p \wedge q) \to r$, $\quad (p \to r) \wedge (q \to r)$

   (b) $p \to (q \to r)$, $\quad (p \to q) \to r$

   (c) $p \to q \vee r$, $\quad p \wedge \sim q \to r$, $\quad p \wedge \sim r \to q$.

2. Determine whether or not the statement $\sim q \ \wedge p \ \to \sim q$ is a tautology.

3. Let p and q be statements such that $p \leftrightarrow q$ is true. Find the truth values of each of the followings statement forms and provide some justifications.
   (a) $p \to q$ (b) $\sim p \to \sim q$ (c) $\sim p \wedge q$ (d) $p \vee \sim q$ (e) $\sim p \leftrightarrow q$

4. Consider the following statement.

   If its color is green and it is edible, then it is a vegetable or it is a fruit.

   (a) Write the **negation** of the above statement.

   (b) Write the **contrapositive**, **inverse**, and **converse** of the above statement.

5. Use truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer.

   (a)
   $p \to q$
   $q \to p$
   $\therefore \ p \vee q$

   (b)
   p
   $p \to q$
   $\sim q \vee r$
   $\therefore \ r$

   (c)
   If it rains, then I stay home.
   If it does not rain, then I go shopping.
   $\therefore$ I stay home or I go shopping.

   (d)
   If it rains, then I stay home.
   If it does not rain, then I go shopping.
   I do not go shopping.
   $\therefore$ It rains.

6. Suppose that you discover a note written by a pirate. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a-e below) and challenged the reader to use them to figure out the location of the treasure.

   a. If this house is next to a lake, then the treasure is not in the kitchen.
   b. If the tree in the front yard is a lemon tree, then the treasure is in the kitchen.
   c. This house is next to a lake.
   d. The tree in the front yard is a lemon tree or the treasure is buried under the flagpole.
   e. If the tree in the back yard is a coconut tree, then the treasure is in the garage.

   Where is the treasure hidden?