Discussion handout 4 (Review session)

Question 1 Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 10 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \quad \text{and}$$

$$P = 20 + k_4 T + 2Q_2^s,$$

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are positive.

Use the information given to answer the following questions:

1.1) Derive the market demand equation

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and

(ii) $1 + k_1 P_x + k_2 Y > 5 + k_3 W$

1.2) What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

1.3) Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	1	1	1	1

Variables	Y	P_x	w	T
Value	5	5	10	5

1.4) Solve for the equilibrium price *and* quantity.

1.5) Suppose that the government provides a subsidy of $5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

Question 2

Consider the following IS-LM model:

Commodity market:

$$Y = C + I + G_0, \qquad (G_0 > 0)$$
$$C = a + bY_d, \qquad (0 < b < 1)$$
$$Y_d = Y - T,$$
$$T = tY, \qquad (0 < t < 1)$$
$$I = I_0 - kr + iY, \qquad (I_0 > 0, k > 0, 0 < i < 1)$$

Money market:

$$M_s = M_0$$
$$M_D = mY - hr, \qquad (m > 0, h > 0)$$

2.1 ***(5 points)*** Write a matrix form of the IS-LM equations with Y and r as the endogenous variables.

2.2 ***(5 points)*** State the condition for the existence of the equilibrium national income and interest rate.

2.3 ***(10 points)*** Suppose that $a = 700, G_0 = 350,$ and $I_0 = 800.$ Solve for the equilibrium level of national income and interest rate by using Cramer's rule. (No point will be given if you do not use Cramer's rule!)

2.4 ***(5 points)*** Determine the rate of change of equilibrium interest rate with respect to money supply $\left(\frac{dr^*}{dM_0}\right)$, assuming that everything else remains constant.

Question 3 Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - \frac{3}{2}Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 - 175Q + 10.$$

Consider the following questions.

3.1) Determine the level of revenue-maximizing output, and calculate the value of the elasticity of demand at that level of output?

3.2) construct the profit function

3.3) Determine the profit-maximizing level of output. Confirm your result that the proposed solution is correct.

3.4) Calculate the level of maximized profit.

3.5) Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

Question 4

Consider a market for SUV cars. Market survey has shown that there are two groups of consumers in the market, namely, A and B. Demand equation for each group can be given by:

- Type A: $\mathrm{P} = 5 - \frac{1}{2}\mathrm{q}$

- Type B: $q = 13 + \frac{Y}{10} - P$.

Let q be the quantity in *thousands* of SUV cars that each type purchases and P be the unit price of each car in *millions Baht*. Y is the level of average (monthly) income, measured in the unit of *thousands Baht*.

Use this information to answers the following questions.

4.1 Is the car considered as normal or inferior goods to both groups of consumers? Briefly explain the reason.

4.2 Suppose that average income (Y) is equal to 70,000 Baht. Derive the market demand equation.

4.3 Continue with the assumption that average income is 70,000 Baht. Suppose further that the market supply of SUV cars is given by:

$$P = \frac{1}{9}Q + 2,$$

where Q is the quantity in *thousands* of SUV cars produced by firms.

Solve for the market equilibrium price and quantity in the market when k = 9. Under the equilibrium, is there any type of consumers being excluded from the market? Why, or why not?

4.4 Continue to suppose that income is 70,000 Baht, and the supply equation is still given by the same functional form as above. Now, suppose that government imposes an ad-valorem tax of 800% on the sale of SUV cars. This tax is imposed on producers.

What would be the after-tax equilibrium price and quantity? Under the new equilibrium, is there any group of consumers excluded from the market?

4.5 From (4.4), calculate the total tax revenue that government can collect. Determine per-unit tax burden for consumers and producers, respectively.

Question 5

A monopolist is operating under a cost function given by,

$$TC = 2Q^3 - 18Q^2 + 99Q + 50,$$

where Q is the quantity of output.

Suppose that the market demand for the goods produced by this monopolist is given by:

$P = 81 - 3Q^2$, where P is the price per unit of output.

Consider the following problems.

5.1 Determine the absolute value of the price elasticity of demand when P = $54.

5.2 Determine the revenue-maximizing level of output.

5.3 Derive the profit function, and solve for the profit-maximizing level of output. Verify your answer by using the second-order derivative.

5.4 Would the monopolist change the level of output if government taxes the monopolist based on the profit level that he earns?