CHAPTER 6

Optimization without Constraints:

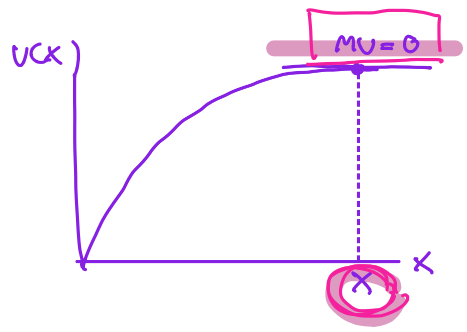
One Independent Variable Case

Topics:

- Maximize profits — Max π (over Q)
  - Competitive market case
  - Monopoly case
- Effects of taxes — Max π_{AT} (over Q)
  - Lump-sum tax
  - Profit tax
  - Excise tax
- Maximization of tax revenue — Max Tax Revenue (over t)

Our attention will be turned to the study of goal equilibrium, in which the equilibrium state is defined as the optimum position for a given economic unit(e.g. a household, a business firm) and in which the said economic unit will be deliberately striving for attainment of that equilibrium.

We will use the optimization to study how consumers, producers, or public policy makers choose their best alternatives given their limited resources, as the essence of the optimization problem is to choose the BEST alternative available.



Utility Maximization:

$$\max_{x} U(x)$$

FOC: $\boxed{MU(x) = 0}$ → solve for x*

Profit Maximization:

$$\max_{Q} \pi(Q)$$

FOC: $MR(Q) = MC(Q)$
The optimal point is Q_1^*.

Cost Minimization:

$$\min_{Q} AC(Q)$$

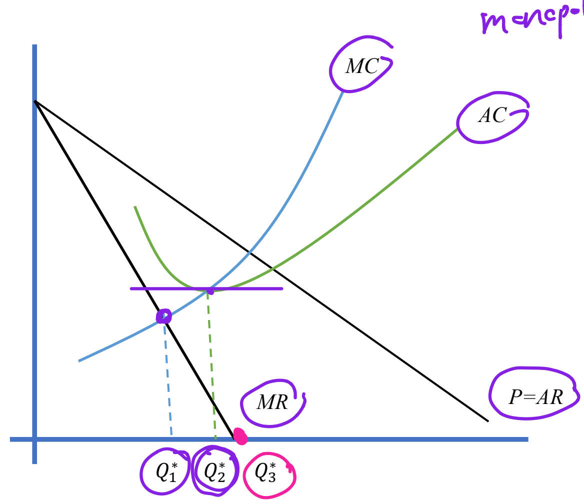
FOC: $\frac{dAC(Q)}{dQ} = 0$
The optimal point is Q_2^*.

Total Revenue Maximization:

$$\max_{Q} TR(Q)$$

FOC: $MR(Q) = 0$
The optimal point is Q_3^*.

monopolist firm



Tax Revenue Maximization:

$$\max_{t} \boxed{Tax\ Revenue(t)}$$

H.W. Find critical values and the maximum/minimum of the following functions:

- $y = f(x) = x^3 - 12x^2 + 36x + 8$
- $y = f(x) = \frac{1}{3}x^3 - x^2 + x + 10$
- $y = f(x) = (2 - x)^4$
- Let $TC = f(Q) = Q^3 - 4Q^2 + 8Q$. Find Q that minimizes AC

$\boxed{\text{Get the correct objective fn}}$

Optimization problem and producers

Objective:

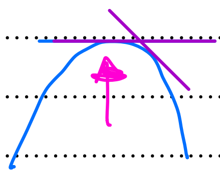
$$\boxed{\pi(q) = TR(q) - TC(q)} : \text{Profit fn}$$

Environment:

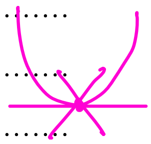
TR	*TC*
A competitive firm A monopolist ~~An oligopolist~~ A monopolistic competition	Production function Structure of market of factors of productions

Firm's Profit Maximization:

$$\max_{q} \pi(q)$$



FOC: $\boxed{\frac{d\pi(q)}{dq} = 0}$



$$\boxed{\frac{d}{dq}\left(TR(q) - TC(q)\right) = 0}$$

$\Rightarrow MR - MC = 0$

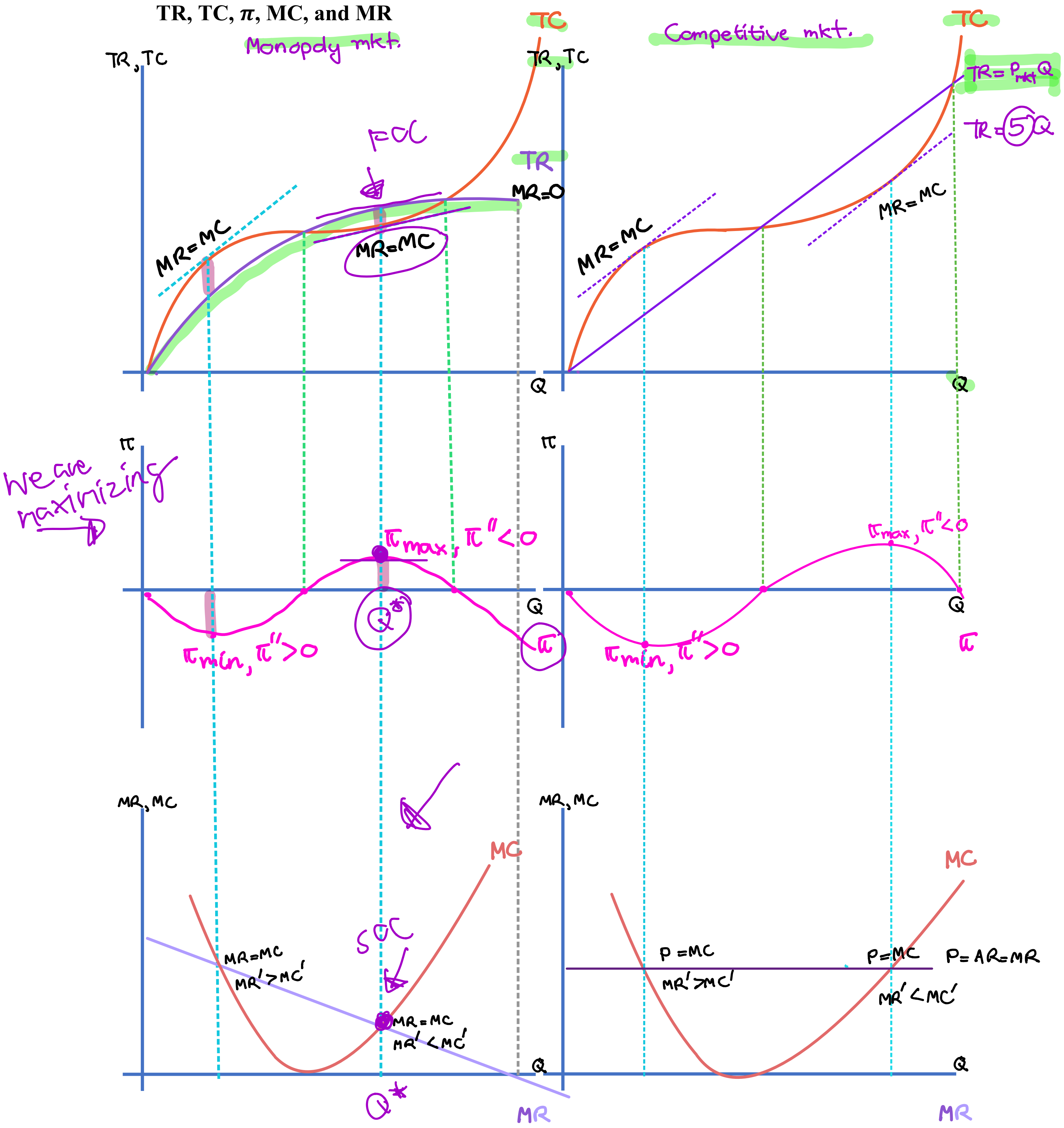
$\boxed{MR = MC}$ $\Rightarrow$ find critical value of q, q^*

SOC: $\frac{d^2\pi}{dq^2} < 0$

$$\frac{d}{dq}(MR - MC) = MR' - MC' < 0$$

$\boxed{MR' < MC'}$ $\Rightarrow$ at q^* that gives π_{max}

TR, TC, π, MC, and MR



➲ Producers in Competitive Market (skip)

Recap: Characteristics of competitive market
1. Many Buyers, many sellers
2. Homogeneous product
3. Perfect information among buyers and Sellers
4. Free entry and perfect mobility

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Firm's Profit Maximization:

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Example: Suppose Peter decides to open a company selling shirts in a competitive market. He has total cost $C = 100 + Q^2$, where Q is his level production. If market price for this shirt is 60 baht per shirt, how many shirt should he produce?

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If $Q = 2L$, how many employee should Peter hire so that he can maximize his profit?

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➲ A monopolist

Recap: Characteristics of monopoly market
1. One producer
2. Inelastic market demand
3. Economy of scale in production
4. Barrier to entry

* Market Power! to choose price or quantity *

* Demand that firm considers is mkt demand *

Let market demand be $Q = a - bP$. , $a, b > 0$

inverse demand : $\underbrace{P = \frac{a}{b} - \frac{1}{b} Q}$

Total Revenue:TR

$$TR = P(Q)\, Q$$

$$= \left(\frac{a}{b} - \frac{1}{b} Q\right) Q$$

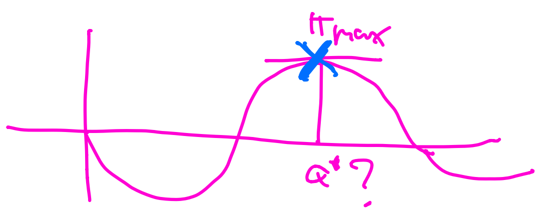
$$TR = \frac{a}{b} Q - \frac{1}{b} Q^2$$

Total Cost: TC

$$TC = C(Q)$$

Objective: Profit fn

$\pi =$ TR − TC



Find Q that maximizes profit

$$\max_{Q} \pi = \boxed{\overbrace{\frac{a}{b} Q - \frac{1}{b} Q^2}^{TR} - C(Q)}$$

FONC:

$\frac{d\pi}{dQ} = 0$: $\frac{d\pi}{dQ} = \boxed{\overbrace{\frac{a}{b} - \frac{2Q}{b}}^{MR} - \overbrace{C'(Q)}^{MC}} = 0 \Rightarrow Q^*$

$MR = MC$

; $\boxed{MR = P\left(1 + \frac{1}{E_p}\right)}$

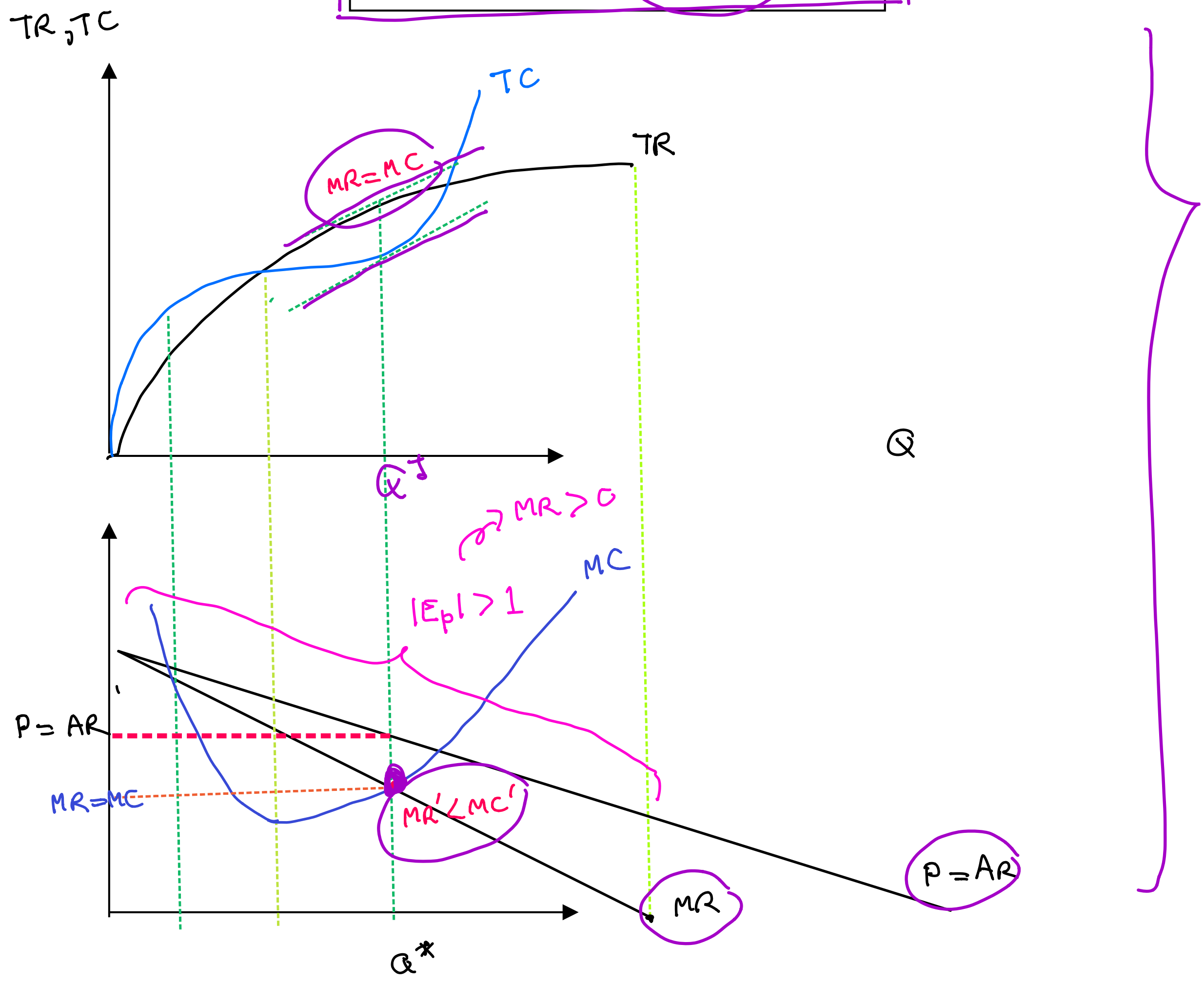
SOSC:

slope of profit fn must be decreasing

$\frac{d^2\pi}{dQ^2} < 0$: $\overbrace{-\frac{2}{b}}^{MR'} - \overbrace{C''(Q)}^{MC'} < 0$

Rule of Thumb for Pricing:

$$\boxed{MC = MR = p\left(1 + \frac{1}{E_p}\right)}$$



Measuring Monopoly Power

$$\boxed{L = \frac{(P - MC)}{P} = -\frac{1}{E_p}}$$

L is called "Lerner index" measuring Lerner's Degree of Monopoly Power.

The Lerner Index is between 0 and 1 since the range of demand that a monopolist chooses to produce has $1 < |E_p| < \infty$

For producers in a competitive market, Lerner Index is 0, since P=MC.

The higher the Lerner index is, the higher the monopoly power is.

The effect of tax on profit maximization

Before tax:

$\pi_{BT} = TR - TC$

After tax:

$\pi_{AT} = TR - TC - T$, T = Tax

Three types of tax:

neutral tax

Lumsum tax $T = \bar{T}$

$\max_{Q} \pi_{AT} = TR - TC - \bar{T}$

FOC : $\frac{d\pi_{AT}}{dQ} = \frac{d}{dQ}(TR - TC - \bar{T})$

$= MR - MC - 0 = 0$

Profit tax: $T = t\pi$

$\max_{Q} \pi_{AT} = (1-t)(TR - TC)$ neutral tax

FOC : $\frac{d\pi_{AT}}{dQ} = (1-t)(MR - MC) = 0$

$MR - MC = 0$

Specific tax: $\boxed{T = tQ}$

$\max_{Q} \pi_{AT} = TR - TC - \overbrace{\boxed{tQ}}^{T}$ distortionary tax

FOC : $\frac{d\pi_{AT}}{dQ} = MR - MC - t = 0$

$\boxed{MR = MC + t}$ different condition than before tax (and from lumsum tax & profit tax)

$Q^*_{BT} \neq Q^*_{AT}$

Let market demand be

inverse mkt demand

$\boxed{P = a - bQ}$

For a monopolist,

$TR = P(Q)Q = (a - bQ)Q$

$TR = aQ - bQ^2$

Let total cost be:

$$TC = C(Q) = \boxed{cQ + f}$$

$$\max_{Q} \pi_{BT}(Q) = \boxed{aQ - bQ^2 - cQ - f}$$

$$\text{FOC}: \frac{d\pi_{BT}}{dQ} = a - 2bQ - c = 0$$

$$Q^*_{BT} = \frac{a-c}{2b}$$

$$\text{SOC}: \frac{d^2\pi_{BT}}{dQ^2} < 0$$

$$\frac{d^2\pi_{BT}}{dQ^2} = -2b < 0 \quad \therefore \text{ at } Q^*_{BT}$$

π_{BT} is maximized.

$$P^*_{BT} = a - bQ^*_{BT} = \frac{a}{2} + \frac{c}{2}$$

$$\pi_{BT} = \ldots$$

Case 1: Lump sum Tax $T = \bar{T}$

Total Profit after Tax

$$\max_{Q} \pi_{AT}(Q) = TR(Q) - TC(Q) - \bar{T}$$
$$\max_{Q} \pi_{AT}(Q) = aQ - bQ^2 - cQ - f - \bar{T}$$

Find the level of output that maximizes profit:

FOC:

$$\frac{d\pi_{AT}}{dQ} = 0$$

$$\frac{d(TR(Q) - TC(Q) - \bar{T})}{dQ} = 0$$

$$\frac{d(aQ - bQ^2 - cQ - f - \bar{T})}{dQ} = 0$$

$$a - 2bQ - c = 0$$

$$Q_{AT}^* = \frac{a-c}{2b}$$

$$P_{AT}^* = a - b\left(\frac{a-c}{2b}\right) = \frac{a}{2} + \frac{c}{2}$$

$$\pi_{AT}(Q_{AT}^*) = aQ_{AT}^* - bQ_{AT}^{*\,2} - cQ_{AT}^* - f - \bar{T}$$

$$\pi_{AT}(Q_{AT}^*) = a\left(\frac{a-c}{2b}\right) - b\left(\frac{a-c}{2b}\right)^2 - c\left(\frac{a-c}{2b}\right) - f - \bar{T}$$

SOC:

$$\frac{d^2\pi_{AT}}{dQ^2} < 0$$

$$\frac{d\,(a - 2bQ - c)}{dQ} < 0$$

$$-2b < 0, \text{ since } b > 0$$

From FONC, at the level of output at maximizing profit

$$Q_{AT}^* = Q_{BT}^*$$

The profit-maximizing price is

$$P_{AT}^* = P_{BT}^*$$

Profit after tax is

$$\pi_{AT}^* = \pi_{BT}^* - \bar{T}$$

Therefore, if lumsum tax is collected, the optimal quantity and price will be as same as before tax, but the level of profit will be reduced by the tax amount $\bar{T}$. Lumsum tax can be called neutral tax.

Case 2 :Profit Tax $T = t\pi_{BT}$

Suppose tax rate t % of the profit is collected.

Total Profit after Tax:

$$\max_{Q} \pi_{AT}(Q) = (1-t)\pi_{BT} = (1-t)\big(TR(Q) - TC(Q)\big)$$
$$\max_{Q} \pi_{AT}(Q) = (1-t)\pi_{BT} = (1-t)(aQ - bQ^2 - cQ - f)$$

Find the level of output that maximizes profit:

FOC:

$$\frac{d\pi_{AT}}{dQ} = 0$$

$$\frac{d\big((1-t)\big(TR(Q) - TC(Q)\big)\big)}{dQ} = 0$$

$$\frac{d((1-t)(aQ - bQ^2 - cQ - f))}{dQ} = 0$$

$$(1-t)(a - 2bQ - c) = 0$$

$$Q_{AT}^* = \frac{a-c}{2b}$$

$$P_{AT}^* = a - b\left(\frac{a-c}{2b}\right) = \frac{a}{2} + \frac{c}{2}$$

$$\pi_{AT}(Q) = (1-t)\left(aQ_{AT}^* - bQ_{AT}^{*\,2} - cQ_{AT}^* - f\right)$$

$$\pi_{AT}(Q) = (1-t)\left(a\left(\frac{a-c}{2b}\right) - b\left(\frac{a-c}{2b}\right)^2 - c\left(\frac{a-c}{2b}\right) - f\right)$$

SOC:

$$\frac{d^2\pi_{AT}}{dQ^2} < 0$$

$$\frac{d\ (1-t)(a - 2bQ - c)}{dQ} < 0$$

$-2b(1-t) < 0$, since $b > 0$, $0 < t < 1$

From FONC, at the level of output at maximizing profit

$$Q_{AT}^* = Q_{BT}^*$$

The profit-maximizing price is

$$P^*_{AT} = P^*_{BT}$$

Profit after tax is

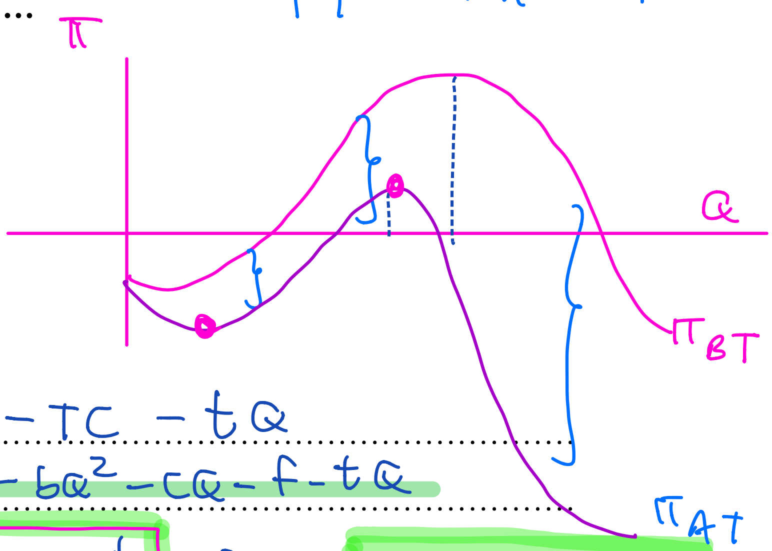
$$\pi^*_{AT} = (1-t)\pi^*_{BT}$$

Therefore, if lumsum tax is collected, the optimal quantity and price will be as same as before tax, but the level of profit will be reduced by the tax amount $t\pi^*_{BT}$. Profit tax can be called neutral tax.

<u>Case 3</u>: Unit Tax $T = tQ$ ……………………………………

T↑ when Q↑

Collect t baht per unit (Specific Tax)



Total Profit after Tax:

$\pi_{AT} = TR - TC - tQ$

Find the level of output that maximizes profit:

$$\max_{Q} \ \pi_{AT} = TR - TC - tQ$$

$$= aQ - bQ^2 - cQ - f - tQ$$

FOC: $\frac{d\pi_{AT}}{dQ} = \boxed{MR - MC - t} = 0 \Rightarrow \boxed{MR = MC + t}$

$$\boxed{a - 2bQ - c - t} = 0$$

$$\boxed{Q^*_{AT} = \frac{a-c-t}{2b}} < Q^*_{BT} = \frac{a-c}{2b}$$

$$p^*_{BT} = a - bQ^*_{AT}$$

$$= a - b\left(\frac{a-c-t}{2b}\right)$$

$$= \frac{a}{2} + \frac{c}{2} + \frac{t}{2} > P^*_{BT} = \frac{a+c}{2}$$

$$\pi^*_{AT} = \dots$$

SOC : $\frac{d^2\pi_{AT}}{dQ^2} < 0$

$\frac{d^2\pi_{AT}}{dQ^2} = MR' - MC' - \cancel{\frac{dt}{dQ}}^{\,0} < 0$

$-2b < 0 \; ; \; b > 0$

at Q^*_{AT} , we get $\pi^*_{AT,max}$

From FONC, at the level of output at maximizing profit

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The profit-maximizing price is

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Profit after tax is

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If unit tax is collected, producer will reduce production and increase price. The level of profit will be reduced. Unit tax is distortion tax.

H.W.: Comparative Static analysis can be done to see the effect of increase in tax rate by figuring out $\frac{\partial Q^*}{\partial t}$, $\frac{\partial P^*}{\partial t}$ and $\frac{\partial \pi^*}{\partial t}$

Tax Revenue maximization

The next question is which unit tax rate will maximize government's tax revenue.

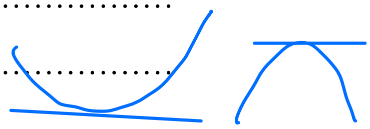
Profit-maximizing output $Q^* = \boxed{\dfrac{a-c-t}{2b}} = Q^*_{AT}$

With unit tax rate t baht per unit, total tax revenue will be

Total Tax Revenue = $t.Q^*$ = $t\left[\dfrac{a-c-t}{2b}\right] = \dfrac{(a-c)t-t^2}{2b}$

$\max_t$ Tax Revenue $= \boxed{\dfrac{(a-c)t-t^2}{2b}}$

FOC : $\dfrac{dT}{dt} = \boxed{\dfrac{a-c-2t}{2b}} = 0$



$t^* = \dfrac{a-c}{2}$ #

SOC : $\dfrac{d^2T}{dt^2} = -\dfrac{2}{2b} = -\dfrac{1}{b} < 0$; $b > 0$

$\therefore$ at t^*, Tax revenue is maximized.

Tax Revenue$_{max}$ $= t^* Q^*_{AT} = \dfrac{(a-c)t^* - t^{*2}}{2b}$

$= \dfrac{(a-c)^2}{8b}$ #