GROUP 8

Q1

a) $Q = A[K^n + L^n]$

$= A[(tK)^n + (tL)^n]$

$= t^n \cdot A \cdot (K^n + L^n)$

$= t^n \cdot Q$

$2n < 1$ for it to be decreasing RTS

b)

MPK: $\frac{\partial f(K,L)}{\partial K} = AnK^{n-1}$

$\frac{\partial MPK}{\partial K} = (n-1)AnK^{n-2} < 0$. K increase MPK decrease

MPL: $\frac{\partial f(K,L)}{\partial L} = AnL^{n-1}$

$\frac{\partial MPL}{\partial L} = (n-1)AnL^{n-2} < 0$ L increase MPL decrease

c) $MRTS = \frac{MPL}{MPK} = \frac{AnL^{n-1}}{AnK^{n-1}}$

$= \left(\frac{L}{K}\right)^{n-1}$

Q1 cont

d) $\frac{d\,MRTS}{dL} = n-1\left(\frac{L}{k}\right)^{n-2}\cdot(1)$

$2n < 1$

$n < \frac{1}{2}$ $\quad (n-1) < 0$

$\therefore$ MRTS decrease as L increase

e. $Q = A[K^n + L^n]$

FOC

$\frac{\partial Q}{\partial K} = AnK^{n-1}$

$\frac{\partial Q}{\partial L} = AnL^{n-1}$

$H = \begin{bmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{bmatrix}$ $\quad$ S.O.C.

$H = \begin{bmatrix} (n-1)AnK^{n-2} & 0 \\ 0 & (n-1)AnL^{n-2} \end{bmatrix}$

$|H_1| = (n-1)AnK^{n-2}$; from d.) $n < \frac{1}{2}$

$|H_1| < 0 \ \forall (K,L)$

$|H_2| = [(n-1)AnK^{n-2}][(n-1)AnL^{n-2}] > \forall (K,L)$

H is negative definite $\forall (K,L)$

Q is globally concave

Q_1 cont.

f.) $Q = f(K, L)$; $\frac{dQ}{dt} = \frac{\partial Q}{\partial K} K'(t) + \frac{\partial Q}{\partial L} L'(t)$

$= AnK^{n-1} \cdot dK + AnL^{n-1} \cdot dL$

$= AnK^{n-1}(t+2) + AnL^{n-1}(e^t)$

$\therefore$ Q is incresing overtime

g.) $\left.\frac{dQ}{dt}\right|_{t=0} \rightarrow$ $t=0$

$K(t) = 0 + 0 + 3 = 3$

$L(t) = e^0 + 3 = 4$

$\left.\frac{dQ}{dt}\right|_{t=0} = AnK^{n-1}(t+2) + AnL^{n-1}(te^{t-1})$

$= AnK^{n-1}(0+2) + AnL^{n-1}(0e^{0-1})$

$= An(3)^{n-1} \cdot 2 + An4^{n-1} \cdot 0$

$= 2An(3)^{n-1}$

Q4

Market A

D: $P_A^D = 10 - 2Q_A$

S: $P_A^S = 1 + Q_A$

Market B

D: $P_B^D = 20 - Q_B$

S: $P_B^S = 2 + Q_B$

a) Market A

$10 - 2Q_A = 1 + Q_A$

$9 = 3Q_A$

$Q_A^* = 3$

$P_A^* = 4$

Market B

$20 - Q_B = 2 + 2Q_B$

$18 = 3Q_B$

$Q_B = 6$

$P_B = 14$

Q4 cont

b) Mkt A

$$P^S_A = P^D_A - t_A$$

$$1 + Q_A = 10 - 2Q_A - t_A$$

$$3Q_A = 9 - t_A$$

After tax $$Q_A = 3 - \frac{t_A}{3}$$

$$P^S_A = 1 + 3 - \frac{t_A}{3} = 4 - \frac{t_A}{3}$$

$$P^D_A = 10 - 2\left(3 - \frac{t_A}{3}\right) = 4 + \frac{2t_A}{3}$$

Mkt B

$$P^S_B = P^D_B - t_B$$

$$2 + 2Q_B = 20 - Q_B - t_B$$

$$3Q_B = 18 - t_B$$

After tax $$Q_B = 6 - \frac{t_B}{3}$$

$$P^S_B = 2 + 2\left(6 - \frac{t_B}{3}\right) = 14 - \frac{2t_B}{3}$$

$$P^D_B = 20 - \left(6 - \frac{t_B}{3}\right) = 14 + \frac{t_B}{3}$$

Q4. cont

c.) (R) Tax revenue $= Q_A \cdot t_A + Q_B \cdot t_B$

$$= \left(3 - \frac{t_A}{3}\right) t_A + \left(5 - \frac{t_B}{3}\right) t_B$$

$$= 3t_A - \frac{t_A^2}{3} + 6t_B - \frac{t_B^2}{3}$$

d.) max t_A: $\frac{\partial R}{\partial t_A} = 3 - 2\frac{t_A}{3} = 0$, $t_A^* = 4.5$

max t_B $\frac{\partial R}{\partial t_B} = 6 - 2\frac{t_B}{3} = 0$, $t_B^* = 9$

Q.5

a) $\pi = P \cdot Q - C(Q) - A$

$= p(2000 + 4\sqrt{A} - 20p) - 2Q - 1000 - A$

$= 2000p + 4p\sqrt{A} - 20p^2 - 5000 - 8A^{\frac{1}{2}} - A$

$= 2040p + 4p\sqrt{A} - 20p^2 - 5000 - 8A^{\frac{1}{2}} - A$

b) $p: 2040 + 4\sqrt{A} - 40p = 0$

$40p - 4\sqrt{A} = 2040$

$$p^* = \frac{2040 + 4\sqrt{A}}{40}$$

$A: 2pA^{-\frac{1}{2}} - 4A^{-\frac{1}{2}} - 1 = 0$

$\frac{2p}{A^{\frac{1}{2}}} - \frac{4}{A^{\frac{1}{2}}} = 1$

$2p - 4 = A^{\frac{1}{2}}$

$4p^2 - 16 = A$

$$p^* = \frac{2040 + 4(4p^2 - 16)^{\frac{1}{2}}}{40}$$

$$= \frac{2040 + 4(2p - 4)}{40}$$

$40p = 2040 + 8p - 16$

$32p = 2024$

$p^* = 63.25$

$A^* = 4(63.25)^2 - 16$

$= 15986.25$

Q4

c)

$$H = \begin{bmatrix} \pi_{PP} & \pi_{PA} \\ \pi_{AP} & \pi_{AA} \end{bmatrix} = \begin{bmatrix} -40 & 2A^{-\frac{1}{2}} \\ 2A^{-\frac{1}{2}} & A^{-\frac{3}{2}}(2-P) \end{bmatrix}$$

$$|H_1| = -40 < 0 \rightarrow \forall (P, A)$$

$$|H_2| = -40 \left(A^{-\frac{3}{2}}(2-P)\right) - \left(2A^{-\frac{1}{2}}\right)^2 < 0 \quad \forall (P, A)$$

H is negative definite $\forall (P, A)$

π is globally concave

$\rightarrow$ P^*, A^* are our global solution

true profit maxization bundle