Practice problem set 3
Differential calculus
EE320 Semester 2/2015
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Question 1. Exercise 7.2 questions 1, 2, 4, 7, and 10 in Chiang and Wainwright (2005)

Question 2. Find the marginal and the average functions for each of the following total functions.

a) $TC = 2Q^2 + 5Q + 11$
b) $\pi = Q^2 - 10Q + 68$
c) $TR = 12Q - Q^2$

Question 3. Suppose that market demand is $p = 300 - q^2$

a. Determine the point elasticity of demand where q = 5.
b. For q = 5, is demand elastic, inelastic, or does it have unit elasticity?
c. For what value of q does the demand curve have unit elasticity?

Question 4. Suppose that $P = a + bQ$ as the equation for linear supply curve, where $b \geq 0$. Show the followings,

a. Elasticity of supply is greater than or equal to zero.
b. Elasticity of supply is less than 1 if the supply curve intersects the horizontal axis (i.e., if the vertical intercept a < 0)
c. Elasticity of supply is greater than 1 if the supply curve intersects the vertical axis (if a > 0).

d. Elasticity of supply is equal than 1 if the supply curve originates from the origin (if a = 0).

Question 5. Given the total-product function:

$$Q = 3L + 2L^2 - L^3$$

a) Find the average product function
b) FInd the marginal product function
c) Determine the slopes of the AP and MP functions. What can you conclude about their relative slopes?

Question 6.A monopolist faces the demand curve given by $P = kQ^{-b}, \quad k, b > 0.$ Answer the following questions.

a. Find the absolute value of elasticity of demand. Does the value decrease as the quantity of output increases? Interpret your result.
b. Find the expression for the revenue function
c. What is the average revenue function? Describe the property of average revenue as the quantity of output increases.
d. Derive the marginal revenue function. How does the value of “b” determine the property of marginal revenue function?
e. Based on the functional form of market demand equation, prove/disprove the following argument.The statement is: *“Raising up the price <u>always</u> increases the revenue of firm, since an increase in price would always make-up with the loss in the quantity sold.”* (Hint: your proof has to make uses some mathematical results regarding to the marginal revenue function.)

Question 7. Let the production function be $Q = -3(7-L)^5 + k$, where k is a very large value constant. Determine the level (interval) of L that the production function exhibits the law of diminishing returns.

Question 8. Suppose that the consumption function can be given by,

$$C = 5\left(\frac{2\sqrt{Y^3}+3}{Y+10}\right)$$

a. find the value of MPC and MPS when Y = 100
b. Calculate the elasticity of consumption with respect to income when Y = 100.

Question 9. Suppose that total output of a firm can be given by

$Q = 10\frac{L^2}{\sqrt{L^2+19}}$, where is L is the number of workers hired. Answer the following questions

a. Find the marginal product of labor (worker).
b. Suppose this firm can only charge for a constant fixed price equal to "P". Find the expression for the revenue function (TR) in term of L.
c. Using the function derived above, find the marginal revenue product of labor or $\frac{dTR}{dL}$. (Hint: you may just treat "P" as a constant term in your equation.)

Continue with the same production function given above, but replace the assumption that the firm can only charge for a constant fixed price equal to "P" with that the firm can charge price based on the quantity of output that it sells. Assume the the pricing equation is given by $P = \frac{900}{Q+9}$. Consider the following questions.

d. Find the express of the revenue function (TR) in term of L.

e. Applying the chain rule and derive the expression of marginal revenue product of labor or $\frac{dTR}{dL}$. Then find the value of marginal revenue product of labor when L = 9.

Question 10. Differentials

a. Find the differential of $y = x^3 - 2x^2 + 3x - 4$ and evaluate it when $x_0 = 1$ and $\Delta x = 0.04$

b. Suppose that the profit of producing q units of a product is $\pi = 396q - 2.2q^2 - 400$. Use differentials, find the approximate change in profit if the level of production changes from q = 80 to q = 81. Compare the differential with the true change.

c. The demand equation of a monopolist's product is $p = 0.5q^2 - 66q + 7000$, and the average-cost function is $AC = 500 - q + (40,000/q)$. Use the differential to estimate the profit when 98 units are demanded.