

Assignment 6

1. Consider the relation f defined from X to Y , $X, Y \subseteq \mathbb{R}$,

$$f = \left\{ (x, y) \in X \times Y \mid x^3 + yx^2 - y = 0 \right\}.$$

Note: It might be useful to write y as a function of x , i.e. $y = f(x)$, before answering some questions.

- (1.1) Find the domain of f .
- (1.2) Find x -intercepts and y -intercepts (if any).
- (1.3) Determine the symmetry of f .
- (1.4) Find the horizontal and vertical asymptotes for f (if any).
- (1.5) Show that $y = -x$ is a slant asymptote of f .
- (1.6) Find the critical number of f . Determine the intervals on which f is increasing and decreasing. Determine the extrema (maximum and minimum) of f .
- (1.7) Determine the intervals on which f is concave up and concave down. Find the points of inflection (if any).
- (1.8) Sketch the curve of f .

Optional Problems

1. Find the critical numbers of the given functions on $(-\infty, \infty)$.
 - (a) $f(x) = x^3 - 3x^2 + 3x - 1$
 - (b) $f(x) = \frac{x^2}{x^2+2}$
 - (c) $f(x) = e^{-x} + 2x$
 - (d) $f(x) = -x + \sin(x)$
 - (e) $f(x) = x^2 - 8 \ln(x)$
2. Determine the intervals on which the given function f is increasing and the interval on which f is decreasing.
 - (a) $f(x) = x^2 + 6x - 1$
 - (b) $f(x) = x^4 - 4x^3 + 9$
 - (c) $f(x) = x^2 e^{-x}$
 - (d) $f(x) = x^2 - 2|x|$
3. Use the *First Derivative Test* to find the relative extrema of the given function.
 - (a) $f(x) = x^3 - 3x$
 - (b) $f(x) = x^3 + x - 3$
 - (c) $\frac{x^2+3}{x+1}$
4. For each given function,
 - (i) use the Second Derivative Test, when applicable, to find the relative extrema;
 - (ii) find intercepts, vertical/horizontal asymptotes, and points of inflection, (if any);
 - (iii) find the intervals on which it is increasing and the interval on which it is decreasing;
 - (iv) find the intervals on which it is concave up and the intervals on which it is concave down;
 - and (v) sketch the graph.
 - (a) $f(x) = x^3 + 3x^2 + 3x + 1$
 - (b) $f(x) = 6x^5 - 10x^3$
 - (c) $f(x) = \frac{x}{x^2+2}$
 - (d) $f(x) = 2x - x \ln(x)$
 - (e) $\frac{x^2+3}{x+1}$
5. Let $X, Y_1, Y_2 \subseteq \mathbb{R}$. Consider the relations f_1, f_2 defined by

$$f_1 = \left\{ (x, y) \in X \times Y_1 \mid y = \sqrt{\frac{x+1}{2x-3}} \right\}, \quad f_2 = \left\{ (x, y) \in X \times Y_2 \mid (x, -y) \in f_1 \right\}.$$

Sketch the curve of the relation $f = f_1 \cup f_2$.
6. Find the vertical and horizontal asymptotes (if any) of $f(x) = \frac{x^2-1}{2x}$ and show that $y = \frac{x}{2}$ is a slant asymptote of f .